Simulative Investigation on Different Multi Input Multi Output Signal Detection Technique

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Abstract—The number of users of wireless communication is increasing day by day, but as the radio spectrum is limited hence the only solution is to increase the data rates to accommodate more users. These data rates can be achieved only by designing more efficient signaling techniques. Multiple Input Multiple Output (MIMO) technology is one of the most promising wireless technologies that can efficiently boost the data transmission rate, improve system coverage, and enhance link reliability. By employing multiple antennas at transmitter and receiver sides, MIMO techniques enable a new dimension called the spatial dimension that can be utilized in different ways to combat the impairments of wireless channels, but Inter Symbol Interference (ISI) is the main problem. To reduce ISI there are different detection techniques used. Detection is a well known technique for combating intersymbol interference. This paper will focus different types of detectors like Minimum Mean Square Error (MMSE), Maximum likelihood (ML), Minimum Mean Square Error Successive Interference Cancellation (MMSE-SIC), Zero Forcing Successive Interference Cancellation (ZF-SIC), Zero Forcing (ZF), Expected Propagation (EP-10), Gaussian Tree Approximation (GTA) and Gaussian Tree Approximation Successive Interference Cancellation (GTA-SIC) detector. These detectors are compared and analyzed for different Signal Error Rate (SER) v/s Signal to Noise Ratio (SNR) in spatial multiplexing domain. A simulation results shows that ZF, MMSE and MMSE-SIC detectors have better performance in terms of SER and SNR and have less computing time as over other detectors.

Keywords—Inter symbol Interference (ISI), Multiple Input Multiple Output (MIMO), Minimum Mean Square Error (MMSE) and Zero Forcing (ZF)

I. INTRODUCTION

Using MIMO technology either the data rate can be increased or SER can be reduced. If we want to increase the data rates spatial multiplexing technique should be used. If different propagation paths can be resolved by multiple antennas then independent data can be transferred through each propagation path at same frequency, and the data rate can be increased. In this technique, different information signals are sent by different transmitters. To reduce the SER, diversity technique is used in which the same information signal is sent from all the transmitters. Demands for capacity in wireless communications, driven by Cellular mobile, Internet and Multimedia services have been rapidly increasing worldwide. On the other hand available radio spectrum is limited and the communication capacity needs cannot be met without a significant increase in communication spectral efficiency. Advances in coding, such as Turbo codes, Low density parity check codes and Space time codes [1], [2] made it feasible to approach the Shannon capacity limit in system with a single antenna link. Significant further advances in spectral efficiency are available though increasing the number of antennas at both transmitter and the receiver which is as MIMO technology. It is one of several forms of smart antenna technology. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or transmit power. It is achieved by higher spectral efficiency (more bits per second per hertz of bandwidth) and link reliability or diversity (reduced fading) [3]. Because of these properties, MIMO is an important part of modern wireless communication system. Spatially distributed channels can be supported simultaneously in the same frequency band by using multiple antennas at both the transmitter and the receiver, and by transmitting data in parallel through these channels the data rate can be increased [4]. Such systems are capable of greatly increasing the spectral efficiency over traditional single channel systems by deployed in a rich scattering environment. The capacity of the flat MIMO Rayleigh fading channel associated with a system with N transmit antennas and M ≥ N receive antennas is given as

\[ C = \log_2(\det(I_M + \rho HH^H)) \text{bit/sec/Hz} \]  

(1)

Where \( I_M \) is the \( M \times M \) identity matrix, \( H \) is the \( M \times N \) matrix whose elements \( \{h_{mn}\} \) represent the channel gains between pairs of transmit and receive antennas, and \( \rho \) is SNR. The achievable data rate depends on the rank of \( H \). For large SNR and large \( N \) and \( M \), the capacity tends to the value \( r \log_2 \rho \), where \( r = \text{rank}(H) \). When the elements of \( H \) are identically distributed and independent, the rank \( r = H \). Hence, in this
ideal scenario of independent fading, the data rate grows linearly with the number of transmit antennas. Ideally, the $M$ receive antennas can provide $M_{th}$ order diversity reception for each of the $N$ transmitted signals in addition to whatever implicit diversity the channel has to offer. Since there is no orthogonal structure imposed on the signals by the transmitter and the received signals contain inter channel interference. The receiver must therefore be able to separate the $N$ signals and at the same time take advantage of the inherent signal diversity. The rule of thumb is that in order to ensure independent fading, the antennas have to be separated by at least half a wavelength at the receiver and as much as several wavelengths at an elevated transmitting base station. In this context, we will discuss the performance of eight detectors namely ML, ZF-SIC, ZF, MMSE, GTA, GTA-SIC, EP-10, ML and MMSE-SIC detectors. We would focus our discussion to the experimental results carried out to MIMO systems and then try to analyze which of the detectors have a better performance in terms of SER for a given SNR. The article is organized as follows; Section II describes the details of MIMO system model, Section III describes simulation results and Section IV concluded the discussion and conclusion.

II. MIMO SYSTEM MODELS

A. ZF Signal Detection

In communication system, ZF Equalizer is a linear equalization algorithm, which inverts the frequency response of channel, and it was proposed by Robert Lucky. For restore the signal before the channel, ZF Equalizer uses the inverse of channel to the received signal. This algorithm is named as Zero Forcing, because it achieves zero ISI. This algorithm is widely used in such cases in which ISI is more predominant as compare to noise [5]. Frequency response of ZF is represented as

$$C(f) = 1/F(f)$$

Let Consider a $2 \times 2$ MIMO channel, and Pseudo inverse for a general $m \times n$ matrix and is represented as

$$H^H H = \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}$$

ZF technique nullifies the interference by the following weight matrix

$$W_{ZF} = (H^H H)^{-1} H^H$$

In ZF algorithm, the error performance is directly connected to the power of $(\tilde{Z}_{ZF})$, which is represented by Frobenius Norms of channel. Similarly the post detection noise power can be evaluated by using the concept of Singular Value Decomposition (SVD) such as

$$\|\tilde{Z}_{ZF}\|^2 = \| (H^H H)^{-1} H^H z \|^2$$

$$= \| (V e^{-i} V^H)^{-1} V \in U^H z \|^2$$

$$= \| (V e^{-i} V^H) V \in U^H z \|^2$$

(5)

The final result of ZF is represented by using following expression as

$$E\{\|\tilde{Z}_{ZF}\|^2\} = \sum_{i=1}^{N_T} \sigma_i^2$$

(6)

B. MMSE Signal Detection

MMSE is a common estimation method which is used to measure the estimator quality and minimize the mean square error (MSE). The main feature of MMSE equalizer is that it does not eliminate ISI completely, but it minimizes the total power of noise and ISI components in the output [6]. Let $x$ is the unknown random variable, and $y$ is the known random variable. Now the following mathematics calculations will show the extraction of two symbols which may interfered with each other. In the first time slot, the received signal on the first received antenna will be represented as

$$y_1 = h_{1,1} x_1 + h_{1,2} x_2 + n_1$$

$$= [h_{1,1},h_{1,2}][x_1] + n_1$$

(7)

Similarly 2nd receiving antenna will receive the signal as

$$y_2 = h_{2,1} x_1 + h_{2,2} x_2 + n_2$$

$$= [h_{2,1},h_{2,2}][x_2] + n_2$$

(8)

For the solution of $x$ (transmitted signal), we need a matrix $W$ which satisfies $(WH=I)$. The MMSE decoder met this criteria by using Equation as

$$w = (H^H H + \sigma_x^2)^{-1} H^H$$

(9)

The MMSE weight matrix is given as

$$W_{MMSE} = (H^H H + \sigma_x^2 l)^{-1} H^H$$

(10)

MMSE receiver requires the statistical information of noise($\sigma_0^2$). Using SVD again, the post detection noise power is expressed as

$$\|\hat{z}_{MMSE}\|^2 = \| (H^H H + \sigma_x^2 l)^{-1} H^H z \|^2$$

(11)

Comparing to different equations it is clear that the effect of noise enhancement in MMSE is less critical over ZF detector.
C. ML detection method

ML detection calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H, and finds the one with the minimum distance. Let C and N denote a set of signal constellation symbol points and a number of transmit antennas respectively. ML detection determines the estimate of the transmitted signal vector $x$ as

$$\hat{x}_{ML} = \arg\min_{x \in \mathbb{C}^N} \|y - Hx\|^2$$

(12)

where $\|y - Hx\|^2$ corresponds to the ML metric. The ML method achieves the optimal performance as the maximum a posteriori (MAP) detection when all the transmitted vectors are equally likely. However, its complexity increases exponentially as modulation order and the number of transmit antennas increases. The required number of ML metric calculation is $|C|^N$, that is, the complexity of metric calculation exponentially increases with the number of antennas. Although OSIC detection method requires much lower complexity than the optimal ML detection, yet their performance is significantly inferior to the ML detection.

D. SIC detector

We can improve the performance of system without increasing the complexity significantly by an SIC detection method. It is a bank of linear receivers, each of which detects one of the parallel data streams, with the detected signal components successively canceled from the received signal at each stage. More specifically, the detected signal in each stage is successively canceled from the received signal at each stage. Parallel data streams, with the detected signal components subtracted from the received signal so that the remaining signal with the reduced interference can be used in the subsequent stage. Figure 1 illustrates the OSIC signal detection process for four spatial streams. In the course of OSIC, either ZF method in equation (4) or MMSE method in equation (9) can be used for symbol estimation. Due to the error propagation caused by erroneous decision in the previous stages, the order of detection has significant influence on the overall performance of OSIC detection.

E. GTA detector

The GTA was first proposed in [7] as a feasible method to improve the MMSE-SIC solution for MIMO detection. The GTA algorithm differs from the ZF, MMSE, MMSE SIC algorithms in several ways. The first difference is that the GTA algorithm utilizes a Markovian approximation of $f(x; z, C)$ instead of an approximation based on a product of independent densities. The second aspect is the use of an optimal tree. GTA is based on an approximation of the exact probability function and is defined as

$$P(x_1, \ldots, x_n/y) \propto \exp\left(-\frac{1}{2\sigma^2}\|Hx - y\|^2\right)$$

(13)

where $x \in \mathbb{A}^n$. Such approach enables a successful implementation of such algorithm which is optimal on connected cycle free factor graphs i.e. trees and is known as Belief Propagation (BP) algorithm. For a given distribution, Chow and Liu [8] proposed a method to find tree approximation that has the minimal Kullback Leibler (KL) divergence to the true distribution and KL is defined as a measure of how one probability distribution diverges from second expected probability distribution. Such approach is based on the two dimensional marginal distribution and use Gaussian distribution. Next step of our approach is to apply the finite set constraint and utilize the Gaussian tree distribution to form a discrete loop free approximation of $P(x/y)$ which can be efficiently globally maximized using the Belief Propagation (BP) algorithm. Using Gaussian approximation, we can also derive the ZF decoding algorithm, which is a simplified version of GTA.

Let $z(y) = (H^T H)^{-1} H^T y$ be the least square estimator and $c = [\sigma^2 (H^T H)^{-1}]$ be its covariance matrix. It can be easily verified that $P(x/y)$ can be written as

$$P(x/y) \propto f(x; z, c)$$

$$= \frac{1}{\sqrt{(2\pi)^n |c|}} \exp\left(-\frac{1}{2} (z - x)^T c^{-1} (z - x)\right)$$

(14)

Where $f(x; z, c)$ is a Gaussian density with mean $z$ and covariance matrix $c$. Now instead of marginalizing the true distribution $P(x/y)$, which is NP hard problem, we approximate it by the product of marginals of Gaussian density $f(x; z, c)$ and is represented as

$$f(x; z, c) \approx \prod_i f(x_i; z_i, c_{ii})$$

$$= \prod_i \frac{1}{\sqrt{2\pi c_{ii}}} \exp\left(-\frac{(x_i - z_i)^2}{2c_{ii}}\right)$$

(15)

From the Gaussian Approximation (15) we can extract a discrete approximation.

$$\hat{P}(x/y) \propto \prod_i \exp\left(-\frac{(x_i - z_i)^2}{2c_{ii}}\right)$$

(16)
Where \( x \in \mathbb{A} \). Since this joint probability function is obtained as a product of marginal probabilities, we can decode each variable separately as

\[
\hat{p}(x_i = a/y) \propto \pi_i \exp\left(-\frac{(x_i-a)^2}{2\sigma_i^2}\right)
\]  
(17)

Where \( a \in A \). By taking the most likely symbol from equation (17), we can obtain sub optimum zero solution.

F. EP Detector

EP [9]–[11], [12] is a technique in Bayesian machine learning for approximating posterior beliefs with exponential family distributions. EP is based on novel interpretation of assumed density filtration. In EP algorithm, we can firstly approximating each observed term \( t_i \) exactly with approximating term \( \hat{t}_i \) and using an exact posterior with \( \hat{\pi}_i \). We can define \( \hat{\pi}_i \) as the ratio of new posterior to the old posterior times as a constant and is represented as

\[
\hat{\pi}_i(x) = Z_i \frac{q(x)}{q^*(x)}
\]  
(18)

It must be noted that if the approximate posterior is an exponential family, then the term approximation will be in the same family. BP algorithm can be interpreted as sequentially computing a Gaussian Approximation \( \hat{\pi}_i(x) \) to every observation term \( t_i(x) \), and then combining these approximations analytically to get Gaussian posterior on \( x \).

STEPS

- Initialize the term approximation \( \hat{t}_i \).
- Compute the posterior of \( x \) from the product of \( \hat{t}_i \), and is represented as
  \[
  q(x) = \prod_i \hat{\pi}_i (x) / \int \prod_i \hat{\pi}_i (x) dx
  \]  
(19)
- Until all \( \hat{\pi}_i \) converge.
- Choose a \( \hat{t}_i \) to refine.
- Remove \( \hat{t}_i \) from the posterior to get an old posterior \( q^*(x) \), by the process of dividing and normalizing, and is represented as
  \[
  q^*(x) \propto \frac{q(x)}{\hat{\pi}_i(x)}
  \]  
(20)
- Combine \( q^*(x) \) and \( t_i(x) \) and minimize KL divergence to get a new posterior \( q(x) \) with normalize \( z_i \).
- Use the normalizing constant of \( q(x) \) as an approximation to \( p(d) \) and is represented as
  \[
  p(d) \approx \int \prod_i \hat{\pi}_i (x) dx
  \]  
(21)

III. SIMULATION RESULTS

A. SIMULATION SETUP FOR 2 × 2 MIMO DETECTOR WITH CONSTELLATION SIZE 8

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>SIMULATION PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTx</td>
<td>2</td>
</tr>
<tr>
<td>NRx</td>
<td>2</td>
</tr>
<tr>
<td>Signal constellation size</td>
<td>8,16,32</td>
</tr>
<tr>
<td>Noise</td>
<td>Gaussian Noise</td>
</tr>
<tr>
<td>Channel</td>
<td>AWGN channel and Rayleigh fading channel</td>
</tr>
<tr>
<td>SNR</td>
<td>0-50</td>
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<tr>
<td>Modulation</td>
<td>Pulse amplitude modulation (PAM)</td>
</tr>
<tr>
<td>Detectors</td>
<td>ZF, MMSE, ML, MMSE - SIC, ZF-SIC, GTA, GTA-SIC, EP-10</td>
</tr>
</tbody>
</table>

B. RESULTS

Figure 2 shows the SNR and SER for various MIMO detectors in 2 × 2 MIMO system with constellation size 8 and achieves a better SER for ZF, MMSE and MMSE-SIC i.e. 0.007655, 0.00725 and 0.003345 respectively over other detectors. Similarly Figure 3 shows computing time for various MIMO detectors and shows that ZF, MMSE and MMSE-SIC has less computing time i.e. 0.04337, 0.02226 and 6.655 respectively over other detectors.

Figure 4 shows the SNR and SER for various MIMO detectors in 2 × 2 MIMO system with constellation size 16 and achieves a better SER for ZF, MMSE and MMSE-SIC i.e. 0.02403, 0.01303 and 0.02353 respectively as compare to other detectors. Similarly Figure 5 shows computing time for various MIMO detectors and shows that ZF, MMSE and MMSE-SIC has less computing time i.e. 0.01304, 0.009664 and 3.743 respectively as compare to other MIMO detectors.

Figure 2. COMPARISON BETWEEN SNR AND SER FOR VARIOUS MIMO DETECTORS IN 2 × 2 MIMO SYSTEM WITH CONSTELLATION SIZE 8.
Figure 3. COMPARISON BETWEEN COMPUTING TIME FOR VARIOUS MIMO DETECTORS IN 2 × 2 MIMO SYSTEM WITH CONSTELLATION SIZE 8

Figure 4. COMPARISON BETWEEN SNR AND SER FOR VARIOUS MIMO DETECTORS IN 2 × 2 MIMO SYSTEM WITH CONSTELLATION SIZE 16.

Figure 5. COMPARISON BETWEEN COMPUTING TIME FOR VARIOUS MIMO DETECTORS IN 2 × 2 MIMO SYSTEM WITH CONSTELLATION SIZE 16.

Figure 6. COMPARISON BETWEEN SNR AND SER FOR VARIOUS MIMO DETECTORS IN 2 × 2 MIMO SYSTEM WITH CONSTELLATION SIZE 32.

Figure 7. COMPARISON BETWEEN COMPUTING TIME FOR VARIOUS MIMO DETECTORS IN 2 × 2 MIMO SYSTEM WITH CONSTELLATION SIZE 32

IV. CONCLUSION

To conclude this paper provides the complete knowledge of the key issues in the field of mobile communication. The data transmission at high bit rate is essential for many services such as video, high quality audio and mobile integrated service digital network. When data is transmitted at high bit rates over mobile radio channels, the channel impulse response can extend over many symbol periods which leads to ISI. To reduce ISI there are different detection techniques used. Detection is a well known technique for combating intersymbol interference. The ultimate goal is to provide universal personal and multimedia communication without regard to mobility or location with a high data rates. This paper focused on different types of detectors like MMSE, ML, MMSE-SIC, ZF-SIC, ZF, EP-10, GTA and GTA-SIC detector. These detectors are compared and analyzed for different SER vs SNR in spatial multiplexing domain. In this paper, 2 × 2 MIMO system analyzed with different detection schemes under AWGN and flat fading Rayleigh channel with different constellation size and it is concluded that, as we increase the signal constellation size of system, then the computing time of each detector will also get increased. A
simulation results shows that ZF, MMSE and MMSE-SIC detectors have better performance in terms of SER and SNR and have less computing time as other detectors. From the simulation, it is clear that ZF, MMSE, MMSE-SIC detectors are better than ZF-SIC, GTA, GTA-SIC and EP-10 because of having less computing time.

REFERENCES


