Prime $\Gamma$– Radical and Radical $TT$ – Ideal in Ternary $\Gamma$- Semirings

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Abstract—In this paper we investigate some important properties of prime $\Gamma$- radical of an $TT$- ideal in a ternary $\Gamma$– semiring. On some special properties of prime $\Gamma$–radical, radical $TT$-ideal are also obtain in the case when the ideals are $k$-$TT$-ideals and $h$-$TT$-ideals.

Keywords: Ternary $\Gamma$-semiring, radical$TT$-ideal, radical $k$-$TT$-ideal, radical $h$-$TT$– ideal

I. Introduction:

The notion of ternary $\Gamma$– semiring was introduced by M. SajaniLavanyaand D. MadhusudhanaRao in [5, 6] in the year 2015, as a natural generalization of ternary $\Gamma$– ring and $\Gamma$-semiring. The notion of prime radical of an ideal is important to the theory of semigroups, semirings as well as $\Gamma$-semigroups etc. In this paper we study prime $\Gamma$- radicals in ternary $\Gamma$- semiring as mentioned in the abstract.

II. Preliminaries:

Definition 2.1:[5]: The non empty sets $T$ and $\Gamma$ together with a binary operation called addition and ternary multiplication denoted by juxtaposition is said to be a ternary $\Gamma$– semiring if $T$ and $\Gamma$ be two additive commutative semigroups satisfying the following conditions.

(i) $[(x_{1}\alpha x_{2}, x_{3}, x_{4}, x_{5})] = [(x_{1}\alpha x_{2}x_{3}, x_{4}, x_{5})] = [(x_{1}x_{2}\alpha x_{3}, x_{4}, x_{5})]$

(ii) $[(x_{1}, x_{2}\alpha x_{3}, x_{4}, x_{5})] = [(x_{1}\alpha x_{2}, x_{3}, x_{4}, x_{5})]$

(iii) $[(x_{1}, x_{2}, x_{3}, x_{4}, x_{5})] = [(x_{1}, x_{2}, x_{3}, x_{4}, x_{5})]$

(iv) $[(x_{1}, x_{2}, x_{3}, x_{4}, x_{5})] = [(x_{1}, x_{2}, x_{3}, x_{4}, x_{5})]

for all $x_{1}, x_{2}, x_{3}, x_{4}, x_{5} \in T$ and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Definition 2.2:[5]: An element $0$ in ternary $\Gamma$– semiring $T$ such that $0 + a = a$ and $0aflb = a0flb = aabl = 0$ for all $a, b \in T, \alpha, \beta \in \Gamma$. Then $0$ is called the 0 – element or simply zero element of the ternary $\Gamma$– semiring $T$.

Definition 2.3:[5]: An element $a$ of a ternary $\Gamma$– semiring $T$ is said to be an identity provided $aafla = tafla = aafla = t$ for all $t \in T, \alpha, \beta \in \Gamma$.

Definition 2.4:[5]: A ternary $\Gamma$– semiring $T$ is said to be a commutative provided $aaflb = bflca = cflba = baflb = aclfa = aocflb$ for all $a, b, c \in T, \alpha, \beta \in \Gamma$.

Definition 2.5:[5]: An additive subsemigroup $S$ of $T$ is said to be a $TT$– sub semiring if $xflya \in A$ for all $x, y \in T, \alpha, \beta \in \Gamma$.

Definition 2.6:[5]: An additive subsemigroup $A$ of $T$ is said to be a leftTT– ideal of $T$ if $xflya \in A$ for all $x \in T, a, y \in T, \alpha, \beta \in \Gamma$.

Definition 2.7:[5]: An additive subsemigroup $A$ of $T$ is said to be a lateralTT– ideal of $T$ if $xflya \in A$ for all $x \in T, a, y \in T, \alpha, \beta \in \Gamma$.

Definition 2.8:[5]: An additive subsemigroup $A$ of $T$ is said to be a rightTT– ideal of $T$ if $xflya \in A$ for all $x \in T, a, y \in T, \alpha, \beta \in \Gamma$.

Definition 2.9:[5]: An additive subsemigroup $A$ of $T$ is said to be a $TT$– ideal of $T$ if $xflya \in A$ and $x, y \in T, \alpha, \beta \in \Gamma$.

Definition 2.10: A $TT$- ideal $A$ of a ternary $\Gamma$- semiring $T$ is said to be a $k$-$TT$-ideal if for $x, y \in T$, $x + y \in A$ and $x \in A$ then $y \in A$.

Definition 2.11: A $TT$- ideal $A$ of a ternary $\Gamma$- semiring $T$ is said to be a $h$-$TT$-ideal if for $x \in T$, and for $a_{1}, a_{2} \in A$, $x + a_{1} \alpha, t \in T$ implies $x \in A$.

Definition 2.12:[5]: A proper $TT$- ideal $P$ of a ternary $\Gamma$- semiring $T$ is said to be a prime $TT$–ideal of $T$ if for any three $TT$- ideals $A, B, C$ of $T$, $A \Gamma B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ or $C \subseteq P$. 

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Definition 2.13 [6]: A proper TT'-ideal Q of T is said to be a semiprime TT'-ideal of T if ATΓA⊆Q implies A⊆Q for any TT'-ideal A of T.

Definition 2.14: Annonempty subset M of a ternary Γ-semiring T is said to be an m-system if for each a, b, c∈M, there exists elements x_1,x_2,x_3,x_4 of T such that aΓx_1ΓΓΓΓx_2ΓΓΓΓc⊆M or aΓx_1ΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓGamma

III. Prime Γ – Radical of a TT'-ideal:

Definition 3.1: Let T be a ternary Γ-semiring and A be a TT'-ideal of T. Then prime Γ – Radical of A is denoted by rad(A) is defined to be the intersection of all prime TT'-ideals of T each of which contains A.

Definition 3.2: A TT'-ideal N in a ternary Γ-semiring T is said to be a nilpotent TT'-ideal if (NΓ)^n = 0 for some natural number n.

Theorem 3.3: In a ternary Γ-semiring T the following conditions are equivalent.

1. P is a prime TT'-ideal of T.
2. aΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓΓGamma

Theorem 3.7: Let A be a TT'-ideal in a ternary Γ-semiring T then rad(A) = {x∈T/every m-system in T which contains has a non empty intersection with A}

Theorem 3.8: Suppose T is a commutative ternary Γ-semiring and M is a m-system in T containing a. Then there exist an integer n≥0 such that (aa)^n a∈A for a∈Γ.
Note: In this paper we simply called a prime radical TT-ideal to be a radical TT-ideal.

Theorem 3.12: If A is a TT-ideal in a ternary Γ-semiring T then the following are equivalent.
1) rad(A) = A
2) (aa)\(^n\) \(\in\) A implies \(\in\) A for some odd natural number n.

Proof: (1) \(\Rightarrow\) (2): Let (aa)\(^n\) \(\in\) A then by theorem 3.9, a \(\in\) rad(A) = A.
(2) \(\Rightarrow\) (1): We know that A \(\subseteq\) rad(A). Let a \(\in\) rad(A). By theorem 3.7, there exist an odd natural number n such that (aa)\(^n\) \(\in\) A. Hence by hypothesis a \(\in\) A. Hence rad(A) \(\subseteq\) A. Therefore rad(A) = A.

Definition 3.13: A k-TT-ideal in a ternary Γ-semiring T is said to be a radical k-TT-ideal provided it is a radical TT-ideal.

Definition 3.14: A h-TT-ideal in a ternary Γ-semiring T which also is a radical TT-ideal is called a radical h-TT-ideal.

Theorem 3.15: Let A be a radical k-TT-ideal of a commutative ternary Γ-semiring T and P, Q be any two subsets of T then S = \{x \in T | xΓPQ \(\subseteq\) A\} is a radical k-TT-ideal.

Proof: S is clearly a TT-ideal of T. Now, let x + y \(\in\) S and x \(\in\) S, y \(\in\) T. Then (x + y)p,q \(\in\) A and xΓPQ \(\subseteq\) A for all p \(\in\) P and for all q \(\in\) Q. So yΓPQ \(\subseteq\) A for all p \(\in\) P and for all q \(\in\) Q as A is a TT-ideal in T. Hence y \(\in\) S.

Consequently, S is a k-TT-ideal in T. Let (xΓ)\(^n\) \(\in\) S for some odd natural number n then (xΓ)\(^n\) \(\in\) xΓPQ \(\subseteq\) A for all p \(\in\) P and for all q \(\in\) Q which implies ((xΓ)\(^n\))^\(\in\) x(ΓP)^\(n\) \(\in\) A for all p \(\in\) P and for all q \(\in\) Q as A is a TT-ideal in T. Therefore (xΓ)\(^n\) \(\in\) xΓPQ \(\subseteq\) A for all p \(\in\) P and for all q \(\in\) Q. So xΓPQ \(\subseteq\) A for all p \(\in\) P and for all q \(\in\) Q as A is a radical TT-ideal. Thus xΓPQ \(\subseteq\) A and so x \(\in\) S. Hence by theorem 3.12, S is also a radical TT-ideal.

Theorem 3.16: Let A be a radical h-TT-ideal of a commutative ternary Γ-semiring T and P, Q are two subsets of T then S = \{x \in T | xΓPQ \(\subseteq\) A\} is a radical h-TT-ideal.

Proof: ClearlyS is an TT-ideal of T. Now, let x \(\in\) T and x + a, a = at \(\in\) T for t \(\in\) T and for a \(\in\) A \(\subseteq\) S. Then(x + a)t \(\in\) TΓPQ = (a,t)ΓPQ for all p \(\in\) P and for all q \(\in\) Q. Therefore xΓPQ + aΓPQ \(\subseteq\) TΓPQ = aΓPQ + ΓPQ where p \(\in\) P \(\subseteq\) T and aΓPQ \(\subseteq\) A, aΓPQ \(\subseteq\) Q. So aΓPQ \(\subseteq\) A for all p \(\in\) P and for all q \(\in\) Q as A is a h-TT-ideal of T. Hence x \(\in\) S.

Consequently, S is a h-TT-ideal. The proof of the part that S is a radical TT-ideal is similar to that in theorem 3.15.

7) By condition (1), A \(\subseteq\) rad(A). So by condition (3), rad(A) \(\subseteq\) rad[rad(A)]. Let a \(\in\) rad[rad(A)] and \{P, \_\} be the family of prime TT-ideals of T such that A \(\subseteq\) P, for all i \(\in\) \(\Delta\). Then by definition rad(A) \(\subseteq\) P, for all i \(\in\) \(\Delta\). Hence rad[rad(A)] \(\subseteq\) P. Therefore a \(\in\) P, for all i \(\in\) \(\Delta\) implies that a \(\in\) rad(A). Therefore rad[rad(A)] = rad(A).

Theorem 3.17: In a ternary Γ-semiring intersection of any collection of radical TT-ideals is again a radical TT-ideal.

Definition 3.18: Suppose T is a ternary Γ-semiring with a ternary Γ-subsemiring A and a TT-ideal I, P = I\(\cap\)A is a TT-ideal. If there is another TT-ideal I such that I \(\subseteq\) J and P = J\(\cap\)A, then we say I can be enlarged to be aTT-ideal in T which also contracts to P.

Theorem 3.19: Let A be an m – system and N be a TT-ideal of a ternary Γ-semiring T such that N\(\cap\)A = \(\emptyset\) then there exist a maximal TT-ideal M of T containing A such that M\(\cap\)A = \(\emptyset\) moreover M is a prime TT-ideal of T.

Theorem 3.20: Let T be a commutative ternary Γ-semiring and A be a ternary Γ-subsemiring of T. Let I be a radical TT-ideal of T such that aaβc \(\in\) I, a, c \(\in\) A, b \(\in\) T, a, β ∈ Γ imply either a \(\in\) I or b \(\in\) I or c \(\in\) I. Then P = I\(\cap\)A is a prime TT-ideal in A. Also I can be expressed as an intersection of prime TT-ideals each os which contracts to P.

Proof: Let a, b \(\in\) A, a, b \(\in\) Γ such that aaβc \(\in\) P. Then aaβc \(\in\) I. Therefore by hypothesis either a \(\in\) I or b \(\in\) I or c \(\in\) I.Hence either a \(\in\) P or b \(\in\) P or c \(\in\) P. So P becomes a prime TT-ideal corollary 3.4.

IV. Conclusion:
In this paper mainly we studied about radical TT-ideals in ternary Γ-semirings.

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