Utilities of Differential Algebraic Equations (DAE) Model of SVC and TCSC for Operation, Control, Planning & Protection of Power System Environments

Bindeshwar Singh[1], Dr. S. P. Singh[2] , Jaswant Singh[3] , Krishan Kumar Singh[4] and Shashank Dadhich[5]
Assistant Professor[1,4], Kamla Nehru Institute of Technology(KNIT), Sultanpur-228118, U.P., India,
Associate Professor[2], Rajakiya Engineering College (REC), Ambedkarnagar, U.P., India,
Assistant Professor[3,5] , Arya College of Engineering & IT (ACEIT), Jaipur-302028 (Raj.), India
bindeshwar.singh2025@gmail.com, singhsurya12@gmail.comand sinjaswant@gmail.com

Abstract-This paper presents the development of the differential Algebraic equation (DAE) model of various FACTS controllerssuch as TCSC and SVC for operation, control, planning &protection of power systems.Also this paper presents the current status on development of the DAEmodel of various FACTS controllerssuch as TCSC and SVC for operation, control, planning &protection of power systems.Authors strongly believe that this article will be very much useful to the researchers for finding out the relevant references in the field of the DAE model of FACTS controllers for operation, control, planning & protection of power systems.

Index Terms:-*Flexible ACTransmission Systems (FACTS), FACTS Controllers,Differential Algebraic Equations (DAE) model, ThyristorControlled Series Capacitor (TCSC), Static Var Compensator (SVC), Power Systems.*

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I. INTRODUCTION

Traditionally, the objective of the reactive power (VAR) planning problem is to provide a minimum number of new reactive power supplies to satisfy only the voltage feasibility constraints in normal and post-contingency states. Various researches have been carried out for this subject [1] and [2]. Recently, due to a necessity to include the voltage stability constraints, a few researches have been reported concerning new formulations considering the voltage stability problem [3] and [4], which provides more realistic solutions for the VAR planning problem. However, the obtained solutions are sometimes too expensive since they satisfy all of the specified feasibility and stability constraints. In [5], a new formulation and solution method are presented for the VAR planning problem including FACTS devices, taking into account the issues just mentioned. TCSC and SVC are used to keep bus voltages and to ensure the voltage stability margin.The TCSC model presented in [2]-[5] and SVC model presented in [6]-[21]. TCSC [2]-[5] and SVC [6]-[21] can be used for power flow control, loop flow control, load sharing among parallel corridors, enhancement of transient stability, mitigation of system oscillations and voltage (reactive power) regulation. Performance analysis and control synthesis of TCSC and SVC requires its steady-state and dynamic models.

This paper is organized as follows: Section II discusses the DAE model of TCSC and SVC. Section IIIpresents the conclusions of the paper.

II.DIFFERENTIAL ALGEBRAIC EQUATION (DAE) MODEL OF TCSC and SVC

A. DAE model of TCSC

1. *Fundamentals of TCSC:*

Thyristor Controlled Series Capacitor (TCSC) provides powerful means of controlling and increasing power transfer level of a system by varying the apparent impedance of a specific transmission line. A TCSC can be utilized in a planned way for contingencies to enhance power system stability. Using TCSC, it is possible to operate stably at power levels well beyond those for which the system was originally intended without endangering system stability [3]. Apart from this, TCSC is also being used to mitigate SSR (Sub Synchronous Resonance).The TCSC module shown in Fig.1.

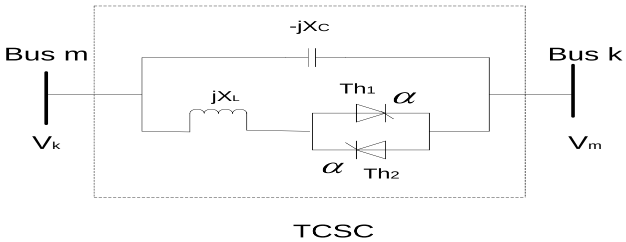

Fig. 1.TCSC module

The steady-state impedance of the TCSC is that of a parallel LC circuit, consisting of fixed capacitive impedance, X_C , and a variable inductive impedance, $X_L(\alpha)$, that is,

$$X_{TCSC}(\alpha) = \frac{X_C X_L(\alpha)}{X_L(\alpha) - X_C} \qquad (1)$$

Where

$$X_L(\alpha) = X_L \frac{\pi}{\pi - 2\alpha - \sin 2\alpha}, X_L \le X_L(\alpha) \le \infty$$

$X_L = \omega L$, and α is the delay angle measured from the crest of the capacitor voltage (or, equivalently, the zero crossing of the line current). The impedance of the TCSC by delay is shown in Fig. 2.

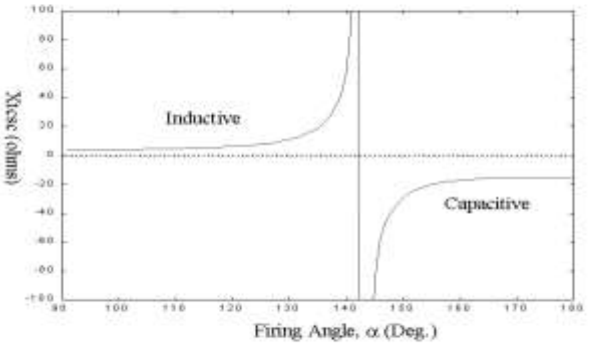

Fig. 2.TCSC equivalent Reactance as a function of firing angle

2. TCSC Controller Model:

The structure of the TCSC is the same as that of a FC-TCR type SVC. The equivalent impedance of the TCSC can be modeled using the following equations (2).

$$X_{TCSC} = X_C \begin{bmatrix} 1 - \frac{k}{k^2-1}.\frac{\sigma+\sin\sigma}{\pi} + \\ \frac{4.k^2.\cos^2(\sigma/2)}{\pi(k^2-1)^2}.(k\tan\frac{k\sigma}{2} - \tan\frac{\sigma}{2}) \end{bmatrix} \quad (2)$$

Where

$\alpha =$ Firing angle delay (after forward vale voltage)

$\sigma =$ Conduction angle= $2(\pi-\alpha)$ and

$k =$ TCSC ratio = $\sqrt{X_C / X_T}$

The TCSC can be continuously controlled in the capacitive or inductive zone by varying firing angle in a predetermined fashion thus avoiding steady state resonance region.

3. Incorporation of TCSC in Multi-machine Power Systems:

The block diagram representation of TCSC shown in *figure. 3.*

Let a TCSC be connected between bus k and bus m as shown in Fig. It has been assumed that the controller is lossless. The power-balance equation and B_{TCSC} are given as [4]

$$P_k = V_k V_m B_{TCSC} \sin(\theta_k - \theta_m) \quad (3)$$

$$Q_k = V_k^2 B_{TCSC} - V_k V_m B_{TCSC} \cos(\theta_k - \theta_m) \quad (4)$$

$$P_m = V_k V_m B_{TCSC} \sin(\theta_m - \theta_k) \quad (5)$$

$$Q_m = V_m^2 B_{TCSC} - V_k V_m B_{TCSC} \cos(\theta_m - \theta_k) \quad (6)$$

$$B_{TCSC} = -\pi(k^4 - 2k^2 + 1)\cos k(\pi-\alpha) / \begin{bmatrix} X_C(\pi k^4 \cos k(\pi-\alpha) \\ -\pi\cos k(\pi-\alpha) - \\ 2k^4\alpha\cos k(\pi-\alpha) \\ +2\alpha k^2\cos k(\pi-\alpha) \\ -k^4\sin 2\alpha\cos k(\pi-\alpha) \\ +k^2\sin 2\alpha\cos k(\pi-\alpha) \\ -4k^3\cos^2\alpha\sin k(\pi-\alpha) \\ -4k^2\cos\alpha\sin\alpha\cos k(\pi-\alpha)) \end{bmatrix} \quad (7)$$

Equation (3) is obtained from (7).

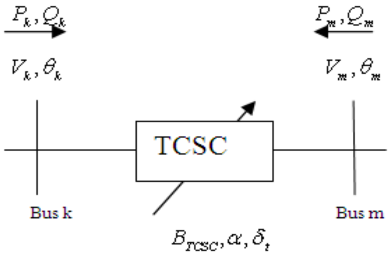

Fig.3.Block diagram representation of TCSC module

There are number of control strategies for TCSC [4]

- Reactance control: $B_{set} - B_{TCSC} = 0$
- Power control: $P_{set} - P = 0$
- Current control: $I_{set} - I = 0$
- Transmission angle control: $\delta_{set} - \delta = 0$

Where the subscript "set" indicates set point.

Any of the above mentioned control strategies can be used to achieve the objectives of TCSC. In this paper, the power control strategy has been used, the block diagram.The line power is monitored and compared to desired power P_{set}. The error is fed to proportional-integral (PI) controller. The output of PI controller is fed through a first order block to get the desired α. The block diagram representation of TCSC with PI controller shown in Fig.4.

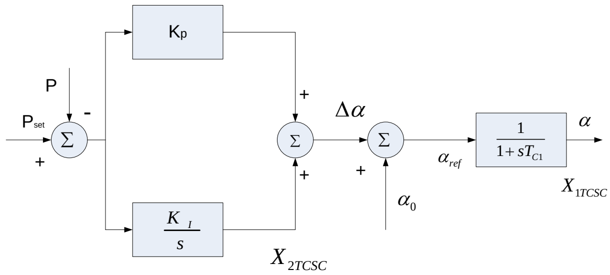

Fig. 4 Block diagram representation of TCSC with PI controller

The controller equations are given as

$$X_{2TCSC} = \frac{K_I}{s}(P_{set} - P)$$

$$\dot{X}_{2TCSC} = K_I P_{set} - K_I P \quad (8)$$

$$\dot{X}_{1TCSC} = \frac{-X_{1TCSC}}{T_{c1}} + \frac{X_{2TCSC}}{T_{c1}} + \frac{K_P P_{set}}{T_{c1}} - \frac{K_P P}{T_{c1}} + \frac{\alpha_o}{T_{c1}} \quad (9)$$

Linearization of (8) and (9) yields

$$\Delta\dot{X}_{1TCSC} = \frac{-\Delta X_{1TCSC}}{T_{c1}} + \frac{\Delta X_{2TCSC}}{T_{c1}} - \frac{K_P \Delta P_k}{T_{c1}}$$

$$\Delta\dot{X}_{2TCSC} = -K_I \Delta P_k \quad (10\text{-}11)$$

As the TCSC controller is assumed lossless, the real power at the two ends of the bus is same. The real power feedback is taken from bus k.

$$\Delta P_k = \Delta V_k V_{mo} B_{TCSCo}\sin(\theta_{ko}-\theta_{mo}) + V_{ko}\Delta V_m B_{TCSCo}\sin(\theta_{ko}-\theta_{mo}) + V_{ko}V_{mo}\Delta B_{TCSC}\sin(\theta_{ko}-\theta_{mo})$$
$$V_{ko}V_{mo}B_{TCSCo}\cos(\theta_{ko}-\theta_{mo})\Delta\theta_k - V_{ko}V_{mo}B_{TCSCo}\cos(\theta_{ko}-\theta_{mo})\Delta\theta_m$$

Substituting value of ΔP_k into (10) and (11)

$$\dot{\Delta X}_{1TCSC} = \frac{-\Delta X_{1TCSC}}{T_{c1}} + \frac{\Delta X_{2TCSC}}{T_{c1}} - \frac{K_P}{T_{c1}} \begin{bmatrix} \Delta V_k V_{mo} B_{TCSCo} \sin(\theta_{ko}-\theta_{mo}) + V_{ko} \Delta V_m B_{TCSCo} \sin(\theta_{ko}-\theta_{mo}) \\ +V_{ko} V_{mo} \Delta B_{TCSC} \sin(\theta_{ko}-\theta_{mo}) + V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) \Delta\theta_k \\ -V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) \Delta\theta_m \end{bmatrix}$$

$$\dot{\Delta X}_{2TCSC} = -K_I \begin{bmatrix} \Delta V_k V_{mo} B_{TCSCo} \sin(\theta_{ko}-\theta_{mo}) + V_{ko} \Delta V_m B_{TCSCo} \sin(\theta_{ko}-\theta_{mo}) \\ +V_{ko} V_{mo} \Delta B_{TCSC} \sin(\theta_{ko}-\theta_{mo}) + V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) \Delta\theta_k \\ -V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) \Delta\theta_m \end{bmatrix}$$

Writing above equations in matrix notation

$$\nabla \dot{X}_{TCSC} = A_{TCSC} \nabla X_{TCSC} + B_{TCSC} \begin{bmatrix} \nabla\theta_k \\ \nabla V_k \\ \nabla\theta_m \\ \nabla V_m \end{bmatrix} \qquad (12)$$

Where $\nabla X_{TCSC} = \begin{bmatrix} \nabla X_{1TCSC} \\ \nabla X_{2TCSC} \end{bmatrix}$

a. Details of Constant FFCONS:

Linearizing the equations (13) gives

$$\Delta B_{TCSC} = (FFCONS) \Delta X_{1TCSC}$$

Details of FFCONS are obtained as shown below.

Linearization of (13) gives

$$\Delta B_{TCSC}(FF1) + \Delta X_{1TCSC}(FF2) = -k\pi(k^2-1)^2 \sin(k\pi - kX_{1TCSCo}) \Delta X_{1TCSC}$$

or

$$\Delta B_{TCSC} = \left[\frac{-FF2 - k\pi(k^2-1)^2 \sin(k\pi - kX_{1TCSCo}) \Delta X_{1TCSC}}{FF1} \right] \Delta X_{1TCSC}$$

or

$$\Delta B_{TCSC} = (FFCONS) \Delta X_{1TCSC}$$

Where

$$FFCONS = \left[\frac{-FF2 - k\pi(k^2-1)^2 \sin(k\pi - kX_{1TCSCo}) \Delta X_{1TCSC}}{FF1} \right]$$

FF1 and FF2 are computed

$$\begin{aligned}
FF2 = {} & k^5 B_{TCSCo} X_c \pi \sin(k\pi - kX_{1TCSCo}) \\
& -k B_{TCSCo} X_c \pi \sin(k\pi - kX_{1TCSCo}) \\
& -2k^4 B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& -2k^5 X_{1TCSCo} B_{TCSCo} X_c \sin(k\pi - kX_{1TCSCo}) \\
& +2k^2 B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& +2k^3 X_{1TCSCo} B_{TCSCo} X_c \sin(k\pi - kX_{1TCSCo}) \\
& -2k^4 \cos^2(X_{1TCSCo}) B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& +2k^4 \sin^2(X_{1TCSCo}) B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& -2k^5 \sin(X_{1TCSCo}) \cos(X_{1TCSCo}) B_{TCSCo} X_c \sin(k\pi - kX_{1TCSCo}) \\
& +8k^3 \sin(X_{1TCSCo}) \cos(X_{1TCSCo}) B_{TCSCo} X_c \sin(k\pi - kX_{1TCSCo}) \\
& +4k^4 \cos^2(X_{1TCSCo}) B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& +4k^4 \sin^2(X_{1TCSCo}) B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& -4k^2 \cos^2(X_{1TCSCo}) B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& -4k^3 \sin(X_{1TCSCo}) \cos(X_{1TCSCo}) B_{TCSCo} X_c \sin(k\pi - kX_{1TCSCo}) \\
& +2k^2 \cos^2(X_{1TCSCo}) B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& -2k^2 \sin^2(X_{1TCSCo}) B_{TCSCo} X_c \cos(k\pi - kX_{1TCSCo}) \\
& +2k^3 \sin(X_{1TCSCo}) \cos(X_{1TCSCo}) B_{TCSCo} X_c \sin(k\pi - kX_{1TCSCo})
\end{aligned}$$

$$\begin{aligned}
FF1 = {} & X_c \pi k^4 \cos(k\pi - kX_{1TCSCo}) \\
& -X_c \pi \cos(k\pi - kX_{1TCSCo}) \\
& -2k^4 X_{1TCSCo} \cos(k\pi - kX_{1TCSCo}) X_c \\
& +2k^2 X_{1TCSCo}) \cos(k\pi - kX_{1TCSCo}) X_c \\
& -2k^4 \sin(X_{1TCSCo}) \cos(X_{1TCSCo}) \cos(k\pi - kX_{1TCSCo}) X_c \\
& -4k^3 \cos^2(X_{1TCSCo}) \sin(k\pi - kX_{1TCSCo}) X_c \\
& -4k^2 \sin(X_{1TCSCo}) \cos(X_{1TCSCo}) \cos(k\pi - kX_{1TCSCo}) X_c \\
& +2k^2 \sin(X_{1TCSCo}) \cos(X_{1TCSCo}) \cos(k\pi - kX_{1TCSCo}) X_c
\end{aligned}$$

Linearizing the equations (13-14) gives

$$\Delta P_k = \Delta V_k V_{mo} B_{TCSCo} \sin(\theta_{ko}-\theta_{mo}) + V_{ko} \Delta V_m B_{TCSCo} \sin(\theta_{ko}-\theta_{mo}) + V_{ko} V_{mo} \Delta B_{TCSC} \sin(\theta_{ko}-\theta_{mo}) + V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) \Delta\theta_k - V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) \Delta\theta_m$$

$$\Delta Q_k = 2V_{ko} B_{TCSCo} \Delta V_k + V^2_{ko} \Delta B_{TCSC} - \Delta V_k V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) - V_{ko} \Delta V_m B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) - V_{ko} V_{mo} \Delta B_{TCSC} \cos(\theta_{ko}-\theta_{mo}) + V_{ko} V_{mo} B_{TCSCo} \sin(\theta_{ko}-\theta_{mo}) \Delta\theta_k - V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{ko}-\theta_{mo}) \Delta\theta_m$$

$$\Delta P_m = \Delta V_k V_{mo} B_{TCSCo} \sin(\theta_{mo}-\theta_{ko}) + V_{ko} \Delta V_m B_{TCSCo} \sin(\theta_{mo}-\theta_{ko}) + V_{ko} V_{mo} \Delta B_{TCSC} \sin(\theta_{mo}-\theta_{ko}) + V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{mo}-\theta_{ko}) \Delta\theta_m - V_{ko} V_{mo} B_{TCSCo} \cos(\theta_{mo}-\theta_{ko}) \Delta\theta_k$$

$$\Delta Q_m = 2V_{mo} B_{TCSCo} \Delta V_m + V^2_{mo} \Delta B_{TCSC} - \Delta V_k V_{mo} B_{TCSCo} \cos(\theta_{mo}-\theta_{ko}) - V_{ko} \Delta V_m B_{TCSCo} \cos(\theta_{mo}-\theta_{ko}) - V_{ko} V_{mo} \Delta B_{TCSC} \cos(\theta_{mo}-\theta_{ko}) + V_{ko} V_{mo} B_{TCSCo} \sin(\theta_{mo}-\theta_{ko}) \Delta\theta_m - V_{ko} V_{mo} B_{TCSCo} \sin(\theta_{mo}-\theta_{ko}) \Delta\theta_k$$

The above equation can be written in matrix notation

$$\begin{bmatrix} \nabla P_k \\ \nabla Q_k \\ \nabla P_m \\ \nabla Q_m \end{bmatrix} = C_{TCSC} \nabla X_{1TCSC} + D_{TCSC} \begin{bmatrix} \nabla\theta_k \\ \nabla V_k \\ \nabla\theta_m \\ \nabla V_m \end{bmatrix}$$

Incorporation of (25), (26), and (5) gives DAE model of multi-machine power system with TCSC incorporated in the system. After reordering, final form of DAE model with TCSC is given as

$$\begin{bmatrix} \nabla \dot{X} \\ \nabla \dot{X}_{TCSC} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{1\text{mod}} & P_{1t\,csc} & A_{2new} & A_{new} \\ P_{2t\,csc} & A_{TCSC} & B_{t\,csc\,1new} & B_{t\,csc\,new} \\ K_2 & P_{4t\,csc} & K_{1new} & C_{4new} \\ G_1 & C_{TCSC} & D_{1new\_t\,csc} & D_{2new\_t\,csc} \end{bmatrix} \begin{bmatrix} \nabla X \\ \nabla X_{TCSC} \\ \nabla z \\ \nabla v \end{bmatrix} + \begin{bmatrix} E \\ 0 \\ 0 \end{bmatrix} \nabla U$$

Equation (27) can be written as

$$\nabla \dot{X}_{SYS\_TCSC} = A_{SYS\_TCSC} \nabla X_{SYS\_TCSC} + E_{TCSC} \nabla U$$

The System matrix with TCSC given as

$$A_{SYS\_TCSC} = A_{TC1} - (A_{TC2} * (inv(A_{TC4})) * A_{TC3})$$

Where

$$A_{TC1} = \begin{bmatrix} A_{1\text{mod}} & P_{1t\,csc} \\ P_{2t\,csc} & A_{TCSC} \end{bmatrix}$$

$$A_{TC2} = \begin{bmatrix} A_{2new} & A_{new} \\ B_{t\,csc\,1new} & B_{t\,csc\,new} \end{bmatrix}$$

$$A_{TC3} = \begin{bmatrix} K_2 & P_{4t\,csc} \\ G_1 & C_{TCSC} \end{bmatrix}$$

$$A_{TC4} = \begin{bmatrix} K_{1new} & C_{4new} \\ D_{1new\_t\,csc} & D_{2new\_t\,csc} \end{bmatrix}$$

b. DAE model of SVC

1. Fundamentals of SVC :

Static VAR Compensator (SVC) is a shunt connected FACTS controller whose main functionality is to regulate the voltage at a given bus by controlling its equivalent reactance. Basically it consists of a fixed capacitor (FC) and a thyristor controlled reactor (TCR). Generally they are two configurations of the SVC.

a) SVC total susceptance model. A changing susceptanceBsvc represents the fundamental frequency equivalent susceptance of all shunt modules making up the SVC as shown in Fig. 5 (a).

b) SVC firing angle model. The equivalent reactance XSVC, which is function of a changing firing angle α, is made up of the parallel combination of a thyristor controlled reactor (TCR) equivalent admittance and a fixed capacitive reactance as shown in Fig. 5 (b). This model provides information on the SVC firing angle required to achieve a given level of compensation.

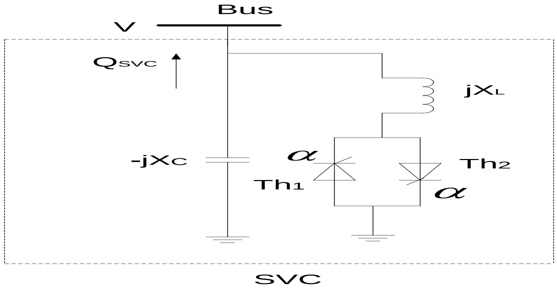

Fig. 5(a) SVC firing angle model

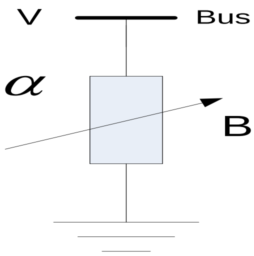

Fig. 5(b) SVC total susceptance model

Figure 6 shows the steady-state and dynamic voltage-current characteristics of the SVC. In the active control range, current/susceptance and reactive power is varied to regulate voltage according to a slope (droop) characteristic. The slope value depends on the desired voltage regulation, the desired sharing of reactive power production between various sources, and other needs of the system. The slope is typically1-5%. At the capacitive limit, the SVC becomes a shunt capacitor. At the inductive limit, the SVC becomes a shunt reactor (the current or reactive power may also be limited).

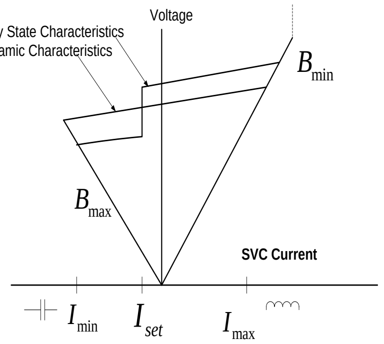

Fig.6 steady-state and dynamic voltage/current Characteristics of the SVC

SVC firing angle model is implemented in this paper. Thus, the model can be developed with respect to a sinusoidal voltage, differential and algebraic equations can be written as

$I_{SVC} = -jB_{SVC}V_k$

The fundamental frequency TCR equivalent reactance X_{TCR}

$$X_{TCR} = \frac{\pi X_L}{\sigma - \sin\sigma}$$

Where $\sigma = 2(\pi - \alpha), X_L = \omega L$

And in terms of firing angle

$$X_{TCR} = \frac{\pi X_L}{2(\pi - \alpha) + \sin 2\alpha}$$

σ and α are conduction and firing angles respectively.

At $\alpha = 90^0$, TCR conducts fully and the equivalent reactance XTCR becomes XL, while at $\alpha = 180^0$, TCR is blocked and its equivalent reactance becomes infinite.

The SVC effective reactance X_{SVC} is determined by the parallel combination of X_C and X_{TCR}

$$X_{SVC}(\alpha) = \frac{\pi X_C X_L}{X_C[2(\pi - \alpha) + \sin 2\alpha] - \pi X_L}$$

Where $X_C = 1/\omega C$

$$Q_k = -V_k^2 \left\{ \frac{X_C[2(\pi - \alpha) + \sin 2\alpha]}{\pi X_C X_L} \right\}$$

The SVC equivalent reactance is given above equation. It is shown in Fig. that the SVC equivalent susceptance $(B_{SVC} = -1/X_{SVC})$ profile, as function of firing angle, does not present discontinuities, i.e., B_{SVC} varies in a continuous, smooth fashion in both operative regions. Hence, linearization of the SVC power flow equations, based on B_{SVC} with respect to firing angle, will exhibit a better numerical behavior than the linearized model based on X_{SVC} .

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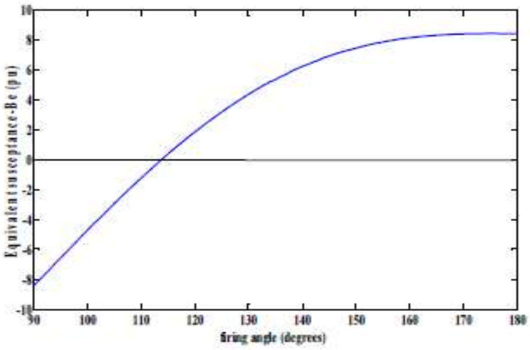

Fig.7 SVC equivalent susceptance profile

The initialization of the SVC variables based on the initial values of ac variables and the characteristic of the equivalent susceptance (Fig.7), thus the impedance is initialized at the resonance point $X_{TCR} = X_C$, i.e. Q_{SVC}=0, corresponding to firing angle $\alpha = 115^0$, for chosen parameters of L and C i.e. $X_L = 0.1134\Omega$ and $X_C = 0.2267\Omega$.

2. *Proposed SVC power flow model:*

The proposed model takes firing angle as the state variable inpower flow formulation. From above equation the SVC linearized power flow equation can be written as

$$\begin{bmatrix} \nabla P_k \\ \nabla Q_k \end{bmatrix}^{(i)} = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{2V_k^{\ 2}}{\pi X_L}[\cos 2\alpha - 1] \end{bmatrix}^{(i)} \begin{bmatrix} \nabla \theta_k \\ \nabla \alpha \end{bmatrix}^{(i)}$$

At the end of iteration i, the variable firing angle α is updatedaccording to

$$\alpha^{(i)} = \alpha^{(i-1)} + \nabla \alpha^{(i)}$$

3. *SVC Controller Model:*

The state equations of the SVC can be written from above figure 8.

$$\dot{X}_{1SVC} = \frac{1}{T_m}[V_{SVC}(1 + KX_{3SVC}) - X_{1SVC}]$$

$$\dot{X}_{2SVC} = K_I(V_{ref,SVC} - X_{1SVC})$$

$$\dot{X}_{3SVC} = \frac{1}{T_c}\left[X_{2SVC} + K_P(V_{ref,SVC} - X_{1SVC}) - X_{3SVC}\right]$$

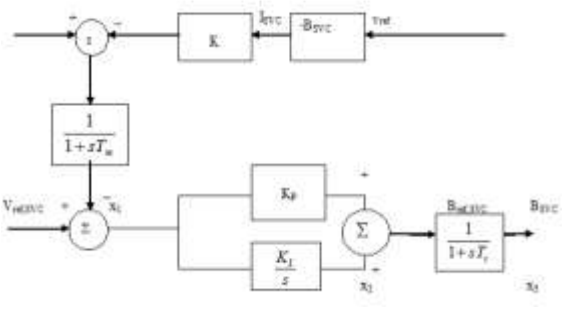

Fig.8 Block diagram of SVC

The reactive power Q_{SVC} supplied by the SVC can be written as

$$Q_{SVC} = V^2{}_{SVC} X_{3SVC}$$

Linearization of above equations ( )-( ) yields

$$\Delta\dot{X}_{1SVC} = \frac{1}{T_m}[\Delta V_{SVC}(1 + KX_{3SVCo}) + V_{SVCo}K\Delta X_{3SVC}) - \Delta X_{1SVC}]$$

$$\Delta\dot{X}_{2SVC} = K_I(\Delta V_{ref,SVC} - \Delta X_{1SVC})$$

$$\Delta\dot{X}_{3SVC} = \frac{1}{T_c}\left[\Delta X_{2SVC} + K_P(\Delta V_{ref,SVC} - \Delta X_{1SVC}) - \Delta X_{3SVC}\right]$$

$$\Delta Q_{SVC} = 2V_{SVCo}\Delta V_{SVC} X_{3SVCo} + V^2{}_{SVCo}\Delta X_{3SVC}$$

Where "Δ" denotes perturbed value and subscript "o" denotes the nominal value.The above equations are linearized, reordered and then expressed as

$$\begin{bmatrix} \nabla \dot{X}_{1SVC} \\ \nabla \dot{X}_{2SVC} \\ \nabla \dot{X}_{3SVC} \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{T_m} & 0 & \dfrac{KV_{SVCo}}{T_m} \\ -K_I & 0 & 0 \\ \dfrac{-K_P}{T_c} & \dfrac{1}{T_c} & \dfrac{-1}{T_c} \end{bmatrix} \begin{bmatrix} \nabla X_{1SVC} \\ \nabla X_{2SVC} \\ \nabla X_{3SVC} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{T_m}(1 + KX_{3SVCo)} \\ 0 \\ 0 \end{bmatrix} [\nabla V_{SVC}]$$

Above equation can be written as

$$\nabla \dot{X}_{SVC} = A_{AVC}\nabla X_{SVC} + B_{SVC}\nabla V_{SVC}$$

Where

$$A_{SVC} = \begin{bmatrix} \dfrac{-1}{T_m} & 0 & \dfrac{KV_{SVCo}}{T_m} \\ -K_I & 0 & 0 \\ \dfrac{-K_P}{T_c} & \dfrac{1}{T_c} & \dfrac{-1}{T_c} \end{bmatrix}$$

And

$$B_{SVC} = \begin{bmatrix} \dfrac{1}{T_m}(1 + KX_{3SVCo)} \\ 0 \\ 0 \end{bmatrix}$$

4. *Incorporation of SVC in multi-machine power systems:*

In its simplest form SVC is composed of FC-TCR configuration as shown in Fig.2. The SVC is connected to a coupling transformer that is connected directly to the ac bus whose voltage is to be regulated. The effective reactance of the FC-TCR is varied by firing angle control of the thyristors. The firing angle can be controlled through a PI controller in such a way that the voltage of the bus where the SVC is connected is maintained at the desired reference value.The SVC can be connected at either the existing load bus or at a new bus that is created between two buses. As DAE model is based on power-balance, rewriting of the power-balance equations at the buses with SVC connected in the system requires modification of D_{2new}.When SVC is connected at specified load buses, and gets modified as given below

$$P_{SVCi} + P_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0$$

$$i = m+1, \ldots\ldots\ldots n$$

$$Q_{SVCi} + Q_{Li}(V_i) - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0$$

$$i = m+1, \ldots\ldots\ldots n$$

Obtained state equations after linearization of above equations

$$C_{SVC}\nabla V_l + D_{SVC}\nabla X_{SVC} + D_1\nabla V_g + D_2\nabla V_l = 0$$

$$or$$

$$D_{SVC}\nabla X_{SVC} + D_1\nabla V_g + D_{2SVC}\nabla V_l = 0$$

Where

$$D_{2SVC} = C_{SVC} + D_2$$

The incorporation of the SVC into DAE model of multi-machine power system is done on the same lines as explained in [2] given as follows:

$$\begin{bmatrix} \nabla \dot{X} \\ \nabla \dot{X}_{SVC} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{1\mathrm{mod}} & P_{1SVC} & A_{2new} & A_{3new} \\ P_{2SVC} & A_{SVC} & P_{3svc} & B_{svcnew} \\ K_2 & P_{4svc} & K_{1new} & C_{4new} \\ G_1 & D_{SVC} & D_{1new\_svc} & D_{2new\_svc} \end{bmatrix} \begin{bmatrix} \nabla X \\ \nabla X_{SVC} \\ \nabla z \\ \nabla v \end{bmatrix} + \begin{bmatrix} E \\ 0 \\ 0 \\ 0 \end{bmatrix} \nabla U$$

The state equation for the system with SVC is then given as follows:

$$\nabla \dot{X}_{sys\_svc} = A_{sys\_svc}\nabla X_{sys\_svc} + E_{SVC}\nabla U$$

The System matrix with SVC given as

$$A_{SYS\_SVC} = A_{SV1} - (A_{SV2} * (inv(A_{SV4}) * A_{SV3})$$

Where

$$A_{SV1} = \begin{bmatrix} A_{1\mathrm{mod}} & P_{1svc} \\ P_{2svc} & A_{SVC} \end{bmatrix}$$

$$A_{SV2} = \begin{bmatrix} A_{2new} & A_{3new} \\ P_{3svc} & B_{svcnew} \end{bmatrix}$$

$$A_{SV3} = \begin{bmatrix} K_2 & P_{4svc} \\ G_1 & D_{SVC} \end{bmatrix}$$

$$A_{SV4} = \begin{bmatrix} K_{1new} & C_{4new} \\ D_{1new\_svc} & D_{2new\_svc} \end{bmatrix}$$

V. CONCLUSIONS

This paper presents the development of the differential Algebraic equation (DAE) model of various FACTS controllers such as TCSC and SVC for operation, control, planning & protection of power systems.Also this paper presents the current status on development of the DAE model of various FACTS controllers such as TCSC and SVC for operation, control, planning &protection of power systems. The Proposed model of TCSC and SVC also can be used for the steady–state analysis (i.e. low frequency analysis) such as placement and coordination of FACTS controllers in power systems from different angle such as power system oscillations enhancement, voltage stability enhancement, increase the available transfer capacity, increase the load ability of power systems, and decrease the active and reactive power losses, etc.

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