# Control of Thermofluid Process using Event-based State Feedback Approach

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*Abstract*—**This paper proposes a method for approximating the behaviour of continuous state feedback using event-triggering for reducing the communication effort. The designed event-based control loop is such that the difference between the event-based loop and the continuous state feedback loop remains bounded and inter-execution time between events has a positive lower bound. The effectiveness of this approach is shown by the simulation performed on a chemical pilot plant.**

***Index Terms - Networked Control System, time-triggered control, Zeno behavior.***

## I. Introduction

In the networked control system, control loops are closed over the communication network. The exchange of control data between control system components sensor, actuator, controller takes place through a shared network. The networked control system provides advantages over traditionally used point to point configuration, where control system components are connected by wires. The advantages are reduced unnecessary wiring, cost, and weight, ease of maintenance and installation, increased reliability. Due to these advantages networked control system is gaining increased attention in many control applications [1]. At the same time, many problems are associated with the networked control system, networked control system induces delays and dropouts due to bandwidth limitation of the communication network [2].

In the digital communication network, the sampling is uniform in the time with some constant sampling period and computing and updating the control law take place for that period. This is known as time-triggered control. The reason for using this is well-established theory [3]. When no disturbances are acting on system and performance of the system is desirable, then executing the control law is wastage of computational resources. In such condition transmitting the data through the network is the waste of limited network bandwidth [4].

To decrease the waste of needless computational and communication resources, an alternative approach known as event-triggered control has been proposed. The event-triggered control takes the detailed information about the system behaviour (example, error exceeds certain prespecified level ), and uses the information to trigger the control action. The resulting sampling is aperiodic in nature. In this way, event-triggered control provides satisfactory control performance without wasting the computational and communication resources [5].

Heemels, Boasch [6] proposed an event-based control for the master-slave motor set and based on simulation showed the benefits like reducing the communication frequency while providing satisfactory performance. Tabuda [7] proposed event-triggering for a non-linear system based on input to state stability. There exists minimum inter-execution time to avoid Zeno behavior. Heemels, Sandee, Boasch [8] proposed event-based control for the perturbed linear system. They determined the region $\beta$ based on a parameter. Whenever plant states are inside that region i.e. close to the origin, control task is not executed and whenever the plant states are outside $\beta$, the control task is executed. Mazo and tabuda [9] proposed event-triggering and self-triggering for the multi-loop system, where many loops are closed via the same communication network, to reduce transmission and energy expenditure. Anta and tabuda [10] proposed self-triggering sampling for control task, it is possible to determine next event time $t_{j+1}$ at the event time $t_j$ in advance. Heemels, Donkers, Teel [11] proposed periodic event-triggering for the linear system. In periodic event-triggering, the event-triggering condition is checked periodically. The advantage of using periodic event-triggering is it provides a minimum inter-execution time of one sampling interval. Practically it is not possible to have the full state information. Laheman and Lunze [12] proposed an event-based scheme where they incorporate a state observer in event generator. For event generation instead of actual plant state, observer state is used.

In this paper, we design an event-based control loop for predesigned state feedback loop, to get the same system response as state feedback loop. Triggering rule is designed based on Lyapunov function.

The rest of the paper is organized as follows: the problem is presented in section II and we describe continuous feedback and event-based loop. The main analytical results is shown in section III, the theoretical results are verified through simulation in section IV, finally section V concludes the paper.

## II. Problem formulation

### A. Continuous state feedback

This section describes the behaviour of continuous state feedback which has to be replicated using an event-based control loop. Consider a plant

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \quad (1)$$

x(t), u(t), and d(t) are the plant state, control input, and disturbance respectively. Disturbance is assumed to be bounded with $\|d(t)\| \leq d_{max}$. Plant (1) together with state-feedback controller

$$u_c(t) = -Kx(t) \quad (2)$$

the plant (1) becomes

$$\dot{x}_{cf} = (A - BK)x_{cf} + Ed(t) \quad (3)$$

A-BK is written as $\bar{A}$. $x_{cf}$ represents the states of closed loop plant (3). The state feedback gain matrix K is chosen to get the satisfactorily closed loop system behaviour, mainly with disturbance attenuation property.

### B. Problem formulation

In this paper, we approximate the above mentioned state feedback loop using an event-based control loop such that the behaviour of event-based control loop is satisfactory as predesigned state feedback loop.

### C. Event-based control

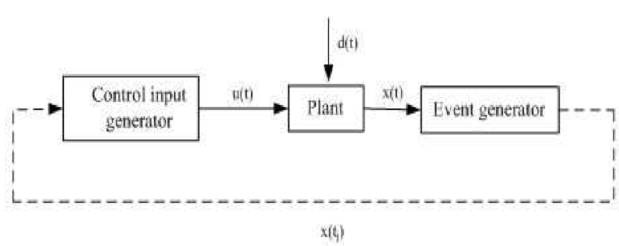


Figure 1. Block Diagram of Event-Based Control Loop

The above figure shows the structure for the event-based control loop. The control input generator provides continuous input u(t) to the plant. The solid link shows the continuous communication while the dashed link shows that the communication is done only at event time t = $t_j, t_{j+1}$...., determined by the event generator. The assumption regarding plant are plant is stable and no model uncertainty occur. The only reason for communication between event generator and control input generator is disturbance has the intolerable effect on system performance. The communication link reacts instantaneously, so no delay between event generation and data update between event generator and control input generator occur [13].

Event-based control loop consists of three components given below.

*1) Control input generator:* Control input generator uses a copy of the continuous state feedback loop. The state space model of control input generator is given by

$$\dot{x}_c = \bar{A}x_c(t) + E\hat{d}_j, \qquad x_c(t_j^+) = x(t_j) \quad (4)$$

$$u(t) = -Kx_c(t) \quad (5)$$

Since the disturbance is generally unknown so we estimate it. The estimated disturbance at event time $t_j$, is given by $\hat{d}j$. $x_c(t_j^+)$ denotes the updated state of control input generator after the event time $t_j$. The estimated disturbance is calculated as

$$\hat{d}_j = \hat{d}_{j-1} + (A^{-1}(e^{A(t_j - t_{j-1})} - I)E)^+(x(t_j) - x_c(t_j)) \quad (6)$$

Where $\hat{d}_{j-1}$ is estimated disturbance at $j-1$. $(.)^+$ is known as pseudo-inverse, that can be calculated as $(S)^+ = (S^{'}S)^{-1}S^{'}$. The inverse matrix $(S^{'}S)^{-1}$ exists if number of the state variable is greater than the number of disturbance. $x_c(t)$ is the plant state (4) but before update.

*2) Plant:* The dynamics of plant is given by (1). The input to plant is given by (5).

*3) Event generator:* Event generator uses the same model as control input generator given by (4), (6). The event is generated by comparing the plant state x(t) of (1) with input (5) is with the state $x_c(t)$ of (4).

$$\begin{aligned} \|x(t) - x_c(t)\| &= m \\ \|x_\delta(t)\| &= m \end{aligned} \quad (7)$$

Variable m is taken as event threshold. $\|.\|$ represents an Euclidean norm. We take two event threshold, one is the exponential time varying threshold and, second is based on Lyapunov function.

Whenever the above condition satisfies, the information of state $x(t_j)$ is sent from the event generator to the control input generator to reinitialize it. Since both control input generator and event generator uses (6) to estimate the disturbance. If the disturbance $d_j$ is also communicated from event generator, so no need to calculate disturbance $d_j$ by control input generator.

## III. main results

**Stability and Event-triggering Condition:** Consider plant (1) with control input (5)

$$\dot{x}(t) = Ax(t) - BKx_c(t) + Ed(t) \quad (8)$$

event threshold is given as $m = x - x_c$ so (8) can be written as

$$\dot{x}(t) = (A - BK)x(t) + BKm + Ed(t) \quad (9)$$

Let $v(x) = x^T P x$ be a Lyapunov function for the closed loop system (8). P is a positive definite solution of Lyapunov equation $(A - BK)^T P + P(A - BK) = -Q$ where Q is some positive definite matrix.

$$\dot{V} = \dot{x}^T P x + x^T P \dot{x}$$
$$= [x^T (A - BK)^T + (BK)^T m^T + (Ed)^T ]Px + x^T P[(A - BK)x + BKm + Ed]$$
$$= x^T [(A - BK)^T P + P(A - BK)]x + m^T (BK)^T Px + x^T (PBK)m + (Ed)^T Px + (Ed)Px \tag{10}$$

The right hand side of (10) can be bounded by

$$\dot{v} \le -x^T Qx + 2\|PBK\|\|m\|\|x\| + 2\|EdP\|\|x\| \tag{11}$$

positive definite matrix Q satisfies

$$\lambda_{min}(Q)\|x\|^2 \le x^T Qx \le \lambda_{max}(Q)\|x\|^2 \tag{12}$$

$\dot{v}$ can be bounded

$$\dot{v} \le -a\|x\|^2 + b\|x\|\|m\| + c\|x\| \tag{13}$$

where $a = \lambda_{min}(Q) > 0$, $b = \|2PBK\|$, and $c = 2\|EdP\|$

$$\dot{v} \le -a\|x\| \left\{ \|x\| - \frac{b}{a}\|m\| - \frac{c}{a} \right\} \tag{14}$$

Function $\dot{v}$ is negative if $\left\{\|x\| - \frac{b}{a}\|m\| - \frac{c}{a}\right\}$ is positive. Thus, if error is restricted to satisfy,

$$|m| \le \sigma \left\{ \frac{a\|x\| - c}{b} \right\} \tag{15}$$

where $0 < \sigma > 1$ The dynamics of v can be bounded

$$\dot{v} \le -a\|x\|(1 - \sigma) \left\{ a\|x\| - \frac{c}{a} \right\} \tag{16}$$

Control task is executed when

$$m = \sigma \left\{ \frac{a\|x\| - c}{b} \right\} \tag{17}$$

The parameter $\sigma$ influences the decrease of the Lyapunov function. The extreme case $\sigma \to 0$, leads to continuous event-triggering and another extreme case $\sigma \to 1$, allow the largest event-triggered induced threshold, but would result in a slow decay of the Lyapunov function.

**Theorem 1 :** Consider plant (1) with input (5) satisfying the event triggering condition (17) for $t > (t_j, t_{j+1} ....)$ such that inter-execution time $t_{j+1} - t_j = \tau$ remains bounded.

**Proof:**

Using (1), (5), (4)

$$\dot{m} = Ax(t) - BKx_c(t) + Ed(t) - (\bar{A}x_c(t) + E\hat{d}_j)$$
$$\dot{m} = Am(t) + Ed_\delta(t) \quad where\ d_\delta(t) = d(t) - \hat{d}_j \tag{18}$$

We assume that transformed disturbance is bounded.

$$\|d_\delta(t)\| \leqslant d_{\delta max} \qquad for\ t \ge 0 \tag{19}$$

on solving (18) we get

$$m(t) = e^{At}m(0) + \int_0^t Ed_{\delta max} e^{A(t-\tau)} d\tau \tag{20}$$

At time t = 0, the value of m is 0, since x(t) and $x_c(t)$ has same initial conditions. So

$$m = Ed_{\delta max} A^{-1}(e^{At} - 1) \tag{21}$$

$$\|m(t)\| \le \|Ed_{\delta max} A^{-1}\|(e^{At} - 1) \tag{22}$$

$\|Ed_{\delta max} A^{-1}\|$ is taken as $c_1$. The lower bound for inter-execution time $\tau$ is given by the time t, if $\|m\| = \sigma(\frac{a\|x\| - c}{b})$

$$\tau = \frac{1}{A} ln \left\{ \frac{\sigma(\frac{a\|x\| - c}{b})}{c_1} + 1 \right\} \tag{23}$$

**Remark 1 :** Theorem 1 shows that there exists a positive lower bound in inter-execution time. This avoids the Zeno behavior, which means the generation of infinite number of events in a finite time interval, to make it practically feasible.

## IV. Example

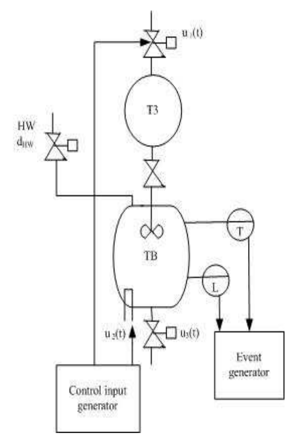


Figure 2. Part of Chemical Plant VERA for Realising Thermofluid Process

The simulation is performed on chemical pilot plant VERA, is used to relise the thermofluid process [14]. The main component of the process is batch reactor TB where level and temperature of fluid are held on prescribed operating points. $u_1(t)$ is the valve angle which controls the inflow into TB and $u_2(t)$ is the heating power induced by heater. The additional inflow into TB is disturbance d(t). $u_3(t)$ is the continuous outflow of the reactor TB. State $x_1$ is level of fluid and $x_2$ is the temperature of the fluid. The state space model for the system is given by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = 10^{-3} \begin{bmatrix} -0.8 & 0 \\ -10^{-7} & -1.7 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} .211 & 0 \\ .108 & .020 \end{bmatrix} u(t) + \begin{bmatrix} .070 \\ -.040 \end{bmatrix} d \tag{24}$$

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state feedback is designed as

$$K = \begin{bmatrix} -0.08 & -0.02 \\ 0.17 & 0.72 \end{bmatrix} \quad (25)$$

The system is subjected to constant disturbance d(t) = 0.05. We simulate the system for 1000 seconds. Initial conditions are $x(0) = x_c(0) = x_{cf}(0) = [1 \quad 1]^T$

## V. SIMULATION RESULT

We simulate the system with time-dependent event threshold $m = 0.25 + .1 * exp(-0.7t)$ and $m = 0.1 * (\frac{a\|x\|-c}{b})$. We compare the responses and inter-execution time with both event threshold.

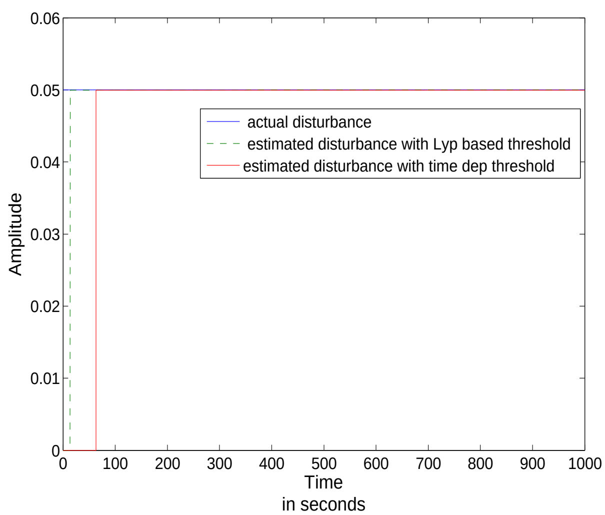


Figure 3. Actual and Estimated Disturbances for the System

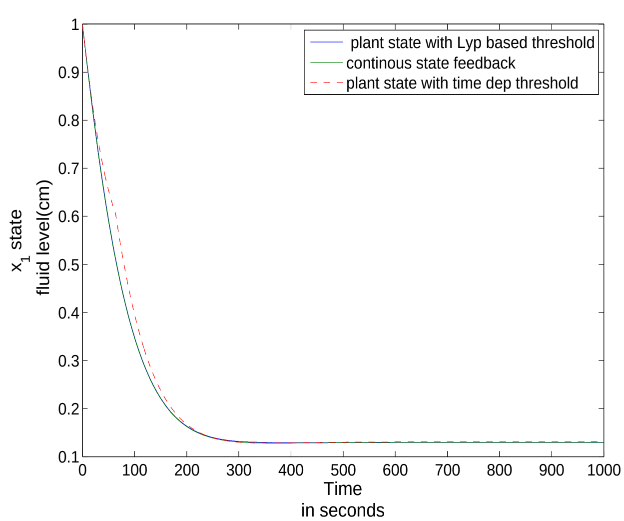


Figure 4. Performance Evaluation of $x_1$ State of System

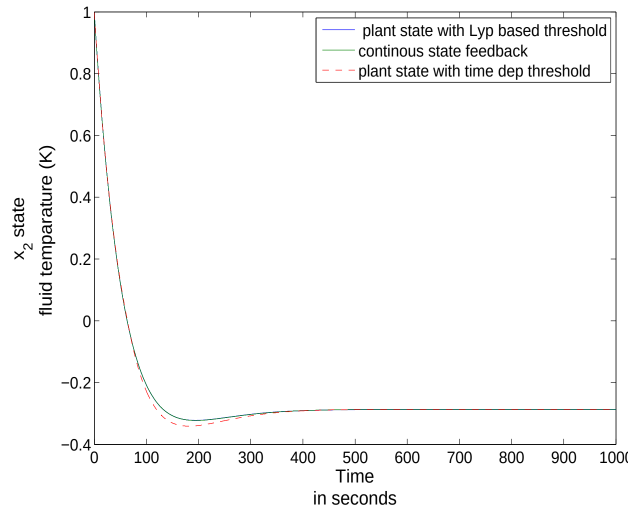


Figure 5. Performance Evaluation of $x_2$ State of System

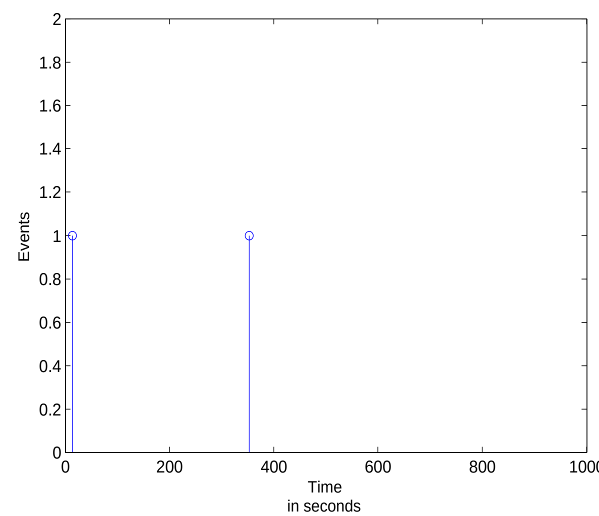


Figure 6. Events Generated by Lyapunov based Threshold

The figure 3 shows that the given disturbance and estimated disturbances with both the event thresholds. An event takes place when $\|x(t) - x_c(t)\| = m$ holds. At this event time the disturbance is estimated correctly by disturbance estimator. There is restriction for the maximum value of disturbance for the Lyapunov based event threshold. If we increase the disturbance then negative term $\frac{c}{b}$ becomes dominating and event threshold become negative this leads to satisfy event condition for every time. This generate continuous events. For this system maximum value of disturbance is 0.1 for which no accumulation of events occur. There is no such restriction with time-dependent event threshold. Figure 4, 5 show that the plant states $x_1, x_2$ mimic the continuous state feedback.

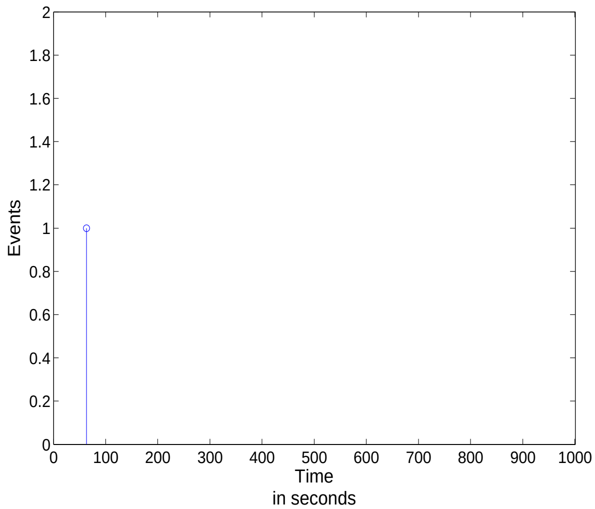


Figure 7. Events Generated by Time-dependent Threshold

2 events are generated while applying Lyapunov based event threshold and 1 event is generated while applying time-dependent event threshold.

## VI. Conclusion

This paper proposes event-based loop for approximating the behaviour of continuous state feedback loop. It has been shown that event-based loop remains in surrounding of continuous state feedback. The inter-execution time has a positive lower bound and depends on disturbance. The control input generator uses a continuous closed loop to adapt the continuous input thus, it is no longer a zero-order hold. With the both event-triggering threshold the plant state mimics the continuous state feedback loop and performance is almost same. One event is more generated in the case of designed Lyapunov function based event threshold.

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