

## Effect Of Break Down Interval In Three Stage Flow Shop Scheduling In Which Processing Time Associated With Probabilities Including Transportation Time And Job Block Criteria

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**Abstract**— This paper is an attempt to study n\*3 flow shop scheduling problem, where processing time are associated with their respective probabilities including the job block criteria, break down interval and transportation time. The objective of the study is to get an algorithm to minimize the total elapsed time. The method is clarified with the help of numerical illustration.

**Keywords**- Flow shop scheduling, Processing time, Transportation time, Optimal sequence, Job-block, Breakdown interval component.

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### I. INTRODUCTION

In flow-shop scheduling, the object is to obtain a sequence of jobs which when processed in a fixed order of machines, will optimize some well defined criteria. Early research on flow shop problems is based mainly on Johnson's theorem, which gives a procedure for finding an optimal solution with 2 machines, or 3 machines with certain characteristics. The research in to flow shop scheduling has drawn a great attention in the last decade with the aim to increase the effectiveness of industrial production. Now-a-days, the decision makers for the manufacturing plant must find a way to successfully manage resources in order to produce products in the most efficient way with minimum total flow time. The scheduling problem practically depends upon the important factors namely, Job Transportation which includes loading time, moving time and unloading time, Job block criteria which is due to priority of one job over the another and machine break down due to failure of a component of machine for a certain interval of time or the machines are supposed to stop their working for a certain interval of time due to some external imposed policy such as non supply of electric current to the machines may be a government policy due to shortage of electricity production. Various Researchers have done a lot of work in this direction. These concepts were separately studied by various researchers Johnson (1954), Jakson (1956), Belman (1956), Baker (1974), Bansal (1986), Maggu and Das (1981), Miyazaki & Nishiyama (1980), Parker (1995), Singh, T.P. (1985), Chandramouli (2005), Belwal & Mittal (2008), Pandian & Rajendran (2010), khodadadi (2011), Gupta & Sharma (2011). Maggu & Das (1977) introduce the concept of equivalent job blocking in the theory of scheduling. The concept is useful and significant in the sense to create a balance between the cost of providing priority in service to the customer and cost of giving services with non-priority. The decision maker may decide how much to charge extra from the priority customer. Gupta Deepak and Sehgal pooja deep(2013) proposed a heuristic algorithm for solving constrained flow shop scheduling problems with three machines in which probabilities are associated with processing time involving transportation time and job block criteria. We have extended the study made by

Singh T.P., Gupta Deepak by introducing the concept of break down interval. We have obtained an algorithm which minimizing the total elapsed time. Thus the problem discussed here is wider and practically more applicable and will have significant results in the process industry.

### II. PRACTICAL SITUATION

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. where various qualities of paper, sugar and oil are produced with relative importance i.e. weight in jobs. In many manufacturing companies different jobs are processed on various machines. These jobs are required to process in a machine shop A, B, C, ---- in a specified order. When the machines on which jobs are to be processed are planted at different places, the transportation time (which includes loading time, moving time and unloading time etc.) has a significant role in production concern. The concept of job block has many applications in the production situation where the priority of one job over the other is taken in to account as it may arise an additional cost for providing this facility in a given block. The break down of the machines (due to delay in material, changes in release and tails date, tool unavailability, failure of electric current, the shift pattern of the facility, fluctuation in processing times, some technical interruption etc.) has significant role in the production concern.

### III. NOTATIONS

S : Sequences of jobs 1,2,3,.....,n

A<sub>i</sub> : Processing time of job ith on machine A

B<sub>i</sub> : Processing time of job ith on machine B

C<sub>i</sub> : Processing time of job ith on machine C

P<sub>i</sub> : probability associated to the processing time A<sub>i</sub> of ith job on machine A

q<sub>i</sub> : probability associated to the processing time B<sub>i</sub> of ith job on machine B

$r_i$  :probability associated to the processing time  $C_i$  of  $i$ th job on machine C

$t_i$  :Transportation time of  $i$ th job from machine A to machine B

$g_i$  : Transportation time of  $i$ th job from machine B to C

$L$  : Length of break down interval

$\beta$  : Equivalent job for job block

$$\min(A_i+t_i) \geq \max(B_i+t_i)$$

if or

$$\min(C_i+g_i) \geq \max(B_i+g_i)$$

or

both

STEP 3: Take equivalent job  $\beta=(k,m)$  define processing time as follows.

$$(i) \quad G_\beta = G_k + G_m - \min(G_m, H_k)$$

$$(ii) \quad H_\beta = H_k + H_m - \min(G_m, H_k)$$

STEP 4: Define a new reduced problem with processing time A,B as defined in step1 & job(k,m) are replaced by single equivalent job  $\beta$  with processing time  $G_\beta$  &  $H_\beta$  as defined in step 3 above.

STEP 5: Apply johnson's(1954) technique and find an optimal schedule of given jobs.

STEP 6: Prepare in out table for the sequence in  $S_1, S_2, \dots, S_r$  obtained in step5. Let the mean flow time in minimum for the sequence  $S_k$ . Now read the effect of break down interval (a,b) different jobs on the lines of Singh T.P.(1985) for the sequence  $S_k$ .

STEP 7 : Form a modified problem with processing time  $A'_{ij}$  :  $i=1,2,3, \dots, n; j=1,2,3$ .

If the breakdown interval (a,b) has effect on job i then

$$A'_{ij} = A_{ij} + L ; \text{ where } L=b-a, \text{ the length of break down interval}$$

If the breakdown interval (a,b) has no effect on job i then

$$A'_{ij} = A_{ij} .$$

Step 8 : Repeat the procedure to get the optimal sequence for the modified scheduling problem using. Determine the total elapsed time .

#### IV. PROBLEM FORMULATION

Let n jobs are to be processed on three machines A, B and C. Let  $A_i, B_i$  and  $C_i$  ( $i=1,2,3, \dots, n$ ) be the processing time of each job on machine A,B and C respectively. Let  $t_i$  and  $g_i$  ( $i=1,2,3, \dots, n$ ) be the job-block with transportation time of job i from machine A to machine B and machine C respectively. The mathematical model of the given problem in matrix can be stated as

Table1

Jobs	Machine A			Machine B			Machine C	
	$a_i$	$p_i$	$t_i$	$b_i$	$q_i$	$g_i$	$c_i$	$r_i$
1	$a_1$	$p_1$	$t_1$	$b_1$	$q_1$	$g_1$	$c_1$	$r_1$
2	$a_2$	$p_2$	$t_2$	$b_2$	$q_2$	$g_2$	$c_2$	$r_2$
3	$a_3$	$p_3$	$t_3$	$b_3$	$q_3$	$g_3$	$c_3$	$r_3$
4	$a_4$	$p_4$	$t_4$	$b_4$	$q_4$	$g_4$	$c_4$	$r_4$
5	$a_5$	$p_5$	$t_5$	$b_5$	$q_5$	$g_5$	$c_5$	$r_5$
.	.	.	.	.	.	.	.	.
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N	$a_n$	$p_n$	$t_n$	$b_n$	$q_n$	$g_n$	$c_n$	$r_n$

#### V. ALGORITHM

STEP 1: Define expected processing time  $A_i, B_i$  and  $C_i$  on machine on machine A, B and C respectively as follows:-

$$A_i = a_i * p_i$$

$$B_i = b_i * q_i$$

$$C_i = c_i * r_i$$

Step 2: Compute, the processing time by creating two fictitious machine G & H with their processing time  $G_i$  &  $H_i$  respectively as follows:-

$$G_i = |A_i + B_i + t_i + g_i| \text{ and } H_i = |C_i + B_i + t_i + g_i|$$

Either

#### VI. NUMERICAL ILLUSTRATIONS

Let 5 jobs be processed on three machines A,B and C with processing time associated with their respective probabilities including transportation time

Table 2

Jobs	Machine A			Machine B			Machine C	
	$a_i$	$p_i$	$t_i$	$b_i$	$q_i$	$g_i$	$c_i$	$r_i$
1	12	0.2	4	3	0.3	5	11	0.3
2	14	0.3	3	7	0.2	6	13	0.3
3	11	0.1	4	5	0.1	4	14	0.1
4	13	0.1	4	3	0.2	3	10	0.1
5	16	0.3	1	2	0.2	7	12	0.2

Find optimal or near optimal sequence when the break down interval is  $(a,b) = (13,15)$  and jobs 2&5 are to be processed as an equivalent group job. Also calculate the total elapsed time.

Solution:

Step 1: The expected processing time  $A_i, B_i & C_i$  on machine A, B and C respectively in table 3. Table 3

Jobs	$A_i$	$B_i$	$C_i$
1	2.4	0.9	3.3
2	4.2	1.4	3.9
3	1.1	0.5	1.4
4	1.3	0.6	1.0
5	4.8	0.4	2.4

Step 2: Here  $\min(A_i + t_i) > \max(B_i + t_i)$  is satisfies therefore. Let us create two fictitious machine G and H with their processing time  $G_i & H_i$  using  $G_i = |A_i + B_i + t_i + g_i|$  and

$H_i = |C_i + B_i + t_i + g_i|$  as follows

Table 4

Jobs	$G_i$	$H_i$
1	12.3	13.2
2	14.6	14.3
3	9.6	9.9
4	8.6	8.6
5	13.2	10.8

Step 3: The corresponding processing time for  $\beta = (2,5)$  on machine  $G_i$  &  $H_i$  follows:

$$G_\beta = G_k + G_m - \min(G_m, H_k)$$

$$= 14.6 + 13.2 - \min(13.2, 14.3)$$

$$= 14.6 + 13.2 - 13.2$$

$$= 14.6$$

$$H_\beta = H_k + H_m - \min(G_m, H_k)$$

$$= 14.3 + 10.8 - \min(13.2, 14.3)$$

$$= 14.3 + 10.8 - 13.2$$

$$= 11.9$$

Step 4: Represent new reduced problem in table 4 with processing time as per step 2 & 3

Table 5

Job	$G_i$	$H_i$
1	12.3	13.2
$\beta$	14.6	11.9
3	9.6	9.9
4	8.9	8.6

Step 5: on applying jhonson (1954) , technique the optimal schedule  $(3, 1, \beta, 4)$

i.e. 3, 1, 2, 5, 4.

Step 6: The optimal sequence (3, 1, 2, 5, 4) is the optimal schedule for the original problem.

Table 6

Jobs	Machine A	Machine B	Machine C
I	In-out	In-out	In-out
3	0-1.1	5.1-5.6	9.6-11.0
1	1.1-3.5	7.5-8.4	<b>13.4-16.7</b>
2	3.5-7.7	10.7-12.1	18.1-22.0
5	7.7-12.5	<b>13.5-13.9</b>	22.0-24.4
4	<b>12.5-13.8</b>	17.8-18.4	24.4-25.4

Step 7 : On considering the effect of break down interval  $(13,15)$  on sequence S

Table 7

Jobs	$A_i$	$t_i$	$B_i$	$g_i$	$C_i$
1	2.4	4	0.9	5	5.3
2	4.2	3	1.4	6	3.9
3	1.1	4	0.5	4	1.4
4	3.3	4	0.6	3	1.0
5	4.8	1	2.4	7	2.4

Now, On repeating the procedure to get the optimal sequence for the modified scheduling problem.

Step 2: Here  $\min(A_i+t_i) > \max(B_i+t_i)$  is satisfies therefore. Let us create two fictitious machine G and H with their processing time  $G_i$  &  $H_i$  using  $G_i=|A_i+B_i+t_i+g_i|$  and

$H_i=|C_i+B_i+t_i+g_i|$  as follows

Table 8

Jobs	$G_i$	$H_i$
1	12.3	15.2
2	14.6	14.3
3	9.6	9.9
4	10.9	8.6
5	15.2	12.8

Step 3: The corresponding processing time for  $\beta=(2,5)$  on machine  $G_i$  &  $H_i$  follows:

$$G_\beta = G_k + G_m - \min(G_m, H_k)$$

$$= 14.6 + 15.2 - \min(15.2, 14.3)$$

$$= 14.6 + 15.2 - 14.3$$

$$= 15.5$$

$$H_\beta = H_k + H_m - \min(G_m, H_k)$$

$$= 14.3 + 12.8 - \min(15.2, 14.3)$$

$$= 14.3 + 12.8 - 14.3$$

$$= 12.8$$

Step 4: Represent new reduced problem in table 4 with processing time as per step 2 & 3

Table 9

Job	$G_i$	$H_i$
1	12.3	15.2
$\beta$	15.5	12.8
3	9.6	9.9
4	10.9	8.6

Step 5: on applying Johnson (1954), technique the optimal schedule (3,1, $\beta$ ,4)

i.e. 3,1,2,5,4.

Step 8: The optimal sequence (3, 1, 2, 5, 4) is the optimal schedule for the original problem.

The in out table for the modified scheduling problem

Table 10

Jobs	Machine A	Machine B	Machine C
I	In-out	In-out	In-out
3	0-1.1	5.1-5.6	9.6-11.0
1	1.1-3.5	7.6-8.4	13.5-18.8
2	3.5-7.7	10.7-12.1	18.8-22.7
5	7.7-12.5	13.5-15.9	22.9-25.3
4	12.5-15.8	19.8-20.4	25.3-26.3

Total elapsed time=26.3 hrs.

## VII. CONCLUSION

The study may further be extended by introducing different parameters such as setup time separated, weightage of jobs etc.

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