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Three Stage Flow Shop Scheduling Model with Equipotential Machines

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Abstract: This paper deals with n-job, three machines flow shop scheduling problem with three parallel machines for the first machine. The unit rental cost of all the machines is given. The processing time of the jobs on all machines are given along with their respective probabilities. Here objective is to find the optimal sequence of jobs and allocation of machines to jobs in order to minimize the total elapsed time of jobs. The concept is made clear with the help of numerical example.

Keywords*: Flow shop, Scheduling, parallel machines, Rental policy.*

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Introduction

In our daily life there are many practical situations in which we are required to sequence the jobs as well as allocation of machines for processing in given set of jobs. In these type of problems there are more than one machine of one type. These parallel machine problems are of three types depending upon whether the parallel machines are (i) Identical (ii) uniform (iii) Unrelated.

The basic study in flow shop scheduling has been made by Johnson (1954). The work was developed by Ignall and Scharge (1965), Combell (1970). Maggu & Das (1977), Yoshida & Hitomi (1979), Singh T.P. (1985), Anup (2002) etc., by considering various parameters. The basic concept of equivalent job for a job block has been investigated by Maggu & Das (1977). Yoshida & Hitomi (1979) considered the two machine flow shop problem of minimizing the makespan whenever the set-up time are separated from processing time. Heydari (2003) dealt with a flow shop scheduling problem where in jobs are processed in two disjoint job blocks in a string consists of one job block in which order of jobs is fixed and other job block in which order of job is arbitrary. This paper deals with identical machine problem.

S.P.Bansal(1986) studied some class of parallel machines .Singh T.P. et al(2006)studied minimization of rental cost under specified rental policy in two stage flow shop and Sunita & Singh (2007) made an attempt to study the minimization of rental cost of machines in m-stage scheduling under predefined rental policy which includes job block concept. Bansal Sunita(2008) studied nx3 flow shop scheduling problems using the concept of parallel machines including transportation time under deterministic environment. Work was further extended by Gupta Deepak,Sharma S(2016) by taking the processing time of parallel machines under fuzzy environment.

Assumptions

*****Each machine of type A has different operating cost.

*Each job after being completed on first machine is then processed on the remaining two machines in the order B and C.

*No passing of jobs is allowed on machines.

*It is not necessary for a job to be processed on each of the K parallel machines of typeA.

*All the parallel machines of type A can start at the same time.

*Machine B and Machine C are hired from the market

Notations

a_{i1}=Processing time of i^{th} job on machine A

a_{i2}=Processing time of i^{th} job on machine B

a_{i3}=Processing time of i^{th} job on machine C

P_{i1}=Probability associated with the processing time of i^{th} job on machine A

P_{i2}=Probability associated with the processing time of i^{th} job on machine B

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P_{i3}=Probability associated with the processing time of i^{th} job on machine C

LB (i) = Lower bound of i^{th} job

A_j(j=1,2,3) are three parallel machines for machine A

The Mathematical model of the problem is given as:

Parallel machines/jobs	1	2	3	4	Available time(b_i)
A_1	C_{11}	C_{21}	C_{31}	C_{41}	b_1
A_2	C_{12}	C_{22}	C_{32}	C_{42}	b_2
A_3	C_{13}	C_{23}	C_{33}	C_{43}	b_3
Time required	a_{11}	a_{21}	a_{31}	a_{41}	

Machine/job	1	2	3	4
A	a_{11}	a_{11}	a_{11}	a_{11}
P_{i1}	P_{11}	P_{21}	P_{31}	P_{41}
B	a_{11}	a_{11}	a_{11}	a_{11}
P_{i2}	P_{12}	P_{22}	P_{32}	P_{42}
C	a_{11}	a_{11}	a_{11}	a_{11}
P_{i3}	P_{13}	P_{23}	P_{33}	P_{43}

c_{ij}(i=1,2..4, j=1,2,3) unit cost of i^{th} job on A_j^{th} parallel machine

b_j (j =1,2,3) is the time for which A_j(j=1,2,3) parallel machines are available for processing.

Problem formulation

Let n jobs (i = 1,2….n) are to be processed on machines A,B,C in the order A,B and C with no passing allowed. Let a_{ij} be the processing time of i^{th} job on machines A,B and C respectively. Let p_{ij} be the probabilities associated with the processing time of i^{th} job on machines A,B and C respectively.

Let there are three machines parallel to machine A given by A_1, A_2, A_3 and the processing cost of all the jobs on all the parallel machines is given by c_{ij} and the time for which the parallel machines A_1, A_2, A_3 are available is given by b_i .

Our aim is to find the optimal or near optimal sequence of jobs in order to minimize the total elapsed time of the jobs.

Algorithm

Step 1: Find the expected processing time of all jobs on machines A, B, C and the expected available time for which the parallel machines are available for processing by using the formula

$A'_i = a_{i1} * p_{i1}$

$B'_i = a_{i2} * p_{i2}$

$C'_i = a_{i3} * p_{i3}$

Step 2: Apply the transportation method and assignment technique to find out the optimal allocation of processing time of parallel machines denoted by A_1,A_2,A_3.

Step 3: Use branch and bound technique on flow shop problem with equipotential machines to find the lower bound for one scheduled job and select the minimum value from it using the Lomnieki formula given by

Step: Branch the minimum value node to find the lower bound of two scheduled jobs and again select the minimum value from it.

Step4: Find the optimal sequence of the sequences by branching again the two schedule jobs to three schedule jobs.

Step 5: Find the elapsed time by preparing in out table.

Numerical Illustration

Optimize the total rental cost of machine B and machine C where 4-jobs and three machine (A, B, C) data is given:

Jobs	1	2	3	4	Available time(b_{i})
A_1	7	9	5	4	25
A_2	5	6	8	3	06
A_3	9	2	11	7	04
Time required(a_{i1})	40	30	50	25	
P_{i1}	0.2	0.4	0.2	0.2	

Machine/jobs	1	2	3	4
A(a_{i1})	40	30	50	25
P_{i1}	0.2	0.4	0.2	0.2
B(a_{i2})	50	20	20	30
P_{i2}	0.1	0.4	0.3	0.2
C(a_{i3})	20	20	25	40
P_{i3}	0.5	0.2	0.2	0.1

First machine A contains three parallel identical machines and rental cost of machine B is Rs 10 per unit and of machine C is Rs 9 per unit respectively

Step-1 Firstly we find the expected processing time of the jobs on machines A, B and C.

Machine/Jobs	1	2	3	4
$A'_i = a_{i1} * p_{i1}$	8	12	10	5
$B'_i = a_{i2} * P_{i2}$	5	8	6	6
$C'_i = a_{i3} * P_{i3}$	10	4	5	4

Step-2 Solve the balanced cost matrix of parallel machines by transportation method

Jobs	1	2	3	4	**Available Time**
$\mathbf{A_1}$	8	2	10	5	25
$\mathbf{A_2}$	0	6	0	0	6
$\mathbf{A_3}$	0	4	0	0	4
Time Required	8	12	10	5	

Jobs	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	**B**	**C**
1	8	0	0	5	10
2	2	6	4	8	4

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3	10	0	0	6	5
4	5	0	0	6	4

Step-3 Now we will calculate lower bound for one scheduled job as

LB (1) = Max {8+5+23,8+25+4} = Max {36,37,35} = 37.

Similarly,

LB (2) = Max {6+8+23,6+25+4,25+10} = Max {37,35,35} = 37.

LB (3) = Max {39,39,35} = 39.

LB (4) = 36.

Minimum value of lower bound is 36 that correspond to job 4. Hence we fix job 4 at the first place in the optimal sequence and proceed to fix the second job of the optimal sequence.

Step-4 Calculate lower bound for two scheduled jobs.

LB (41) = Max {28+4+5,18+14+4,25+11} = Max {37,36,36} = 37.

LB (42) = Max {38,35,36} = 38.

LB (43) = 40.

Minimum value of lower bound = 37 and it is for the subsequence 41. Hence we fix job1 at second place in the optimal sequence. Fixing the job 4 and job1 at first and second position respectively, we proceed to find the third job to be done in the optimal sequence.

Step-5 Now we branch the LB (412) = max {37,37,36} = 37.

LB (413) = 41.

Here minimum value of lower bound is 37 that correspond to 412. Hence, 4,1,2 jobs will have first, second and third places in the optimal sequence respectively and the remaining job3 will have the fourth position.

Therefore, the optimum sequences (4123).

Step: Total elapsed time

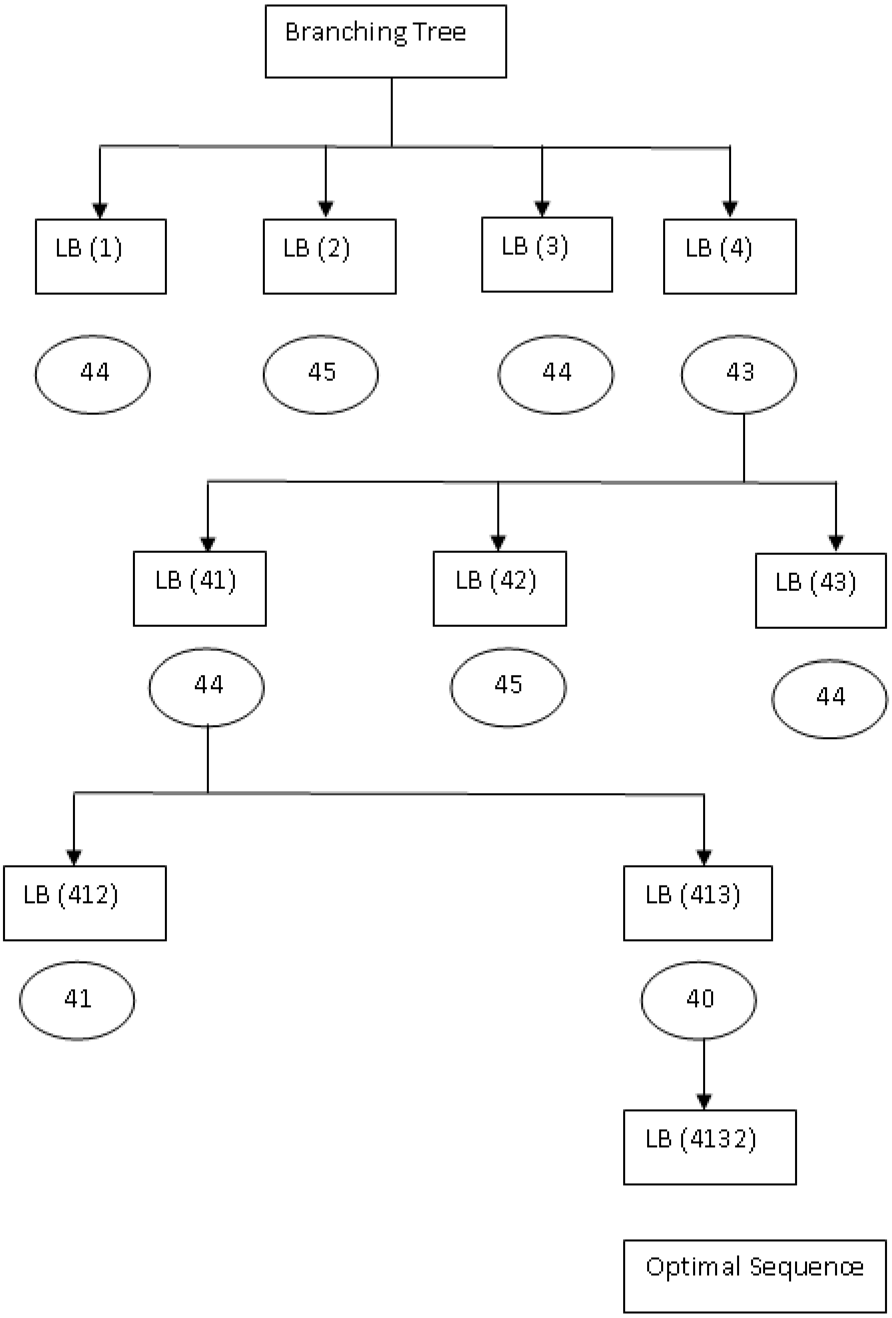
Total elapsed time =46 hours

Remarks

Heuristic methods do not guarantee the optimality of solution found. While the branch and bound method is the exact method to find optimal solution of the problem and confirms its optimality. Work can be further extended by considering transportation time, job weightage etc.

The work may further be extended for n jobs m machine problems.

Jobs	1	2	3	4
A	0-6	1-9	5-10	3-8
B	6-12	12-20	20-28	28-37
C	12-18	20-27	28-38	38-46



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