International Journal on Future Revolution in Computer Science & Communication Engineering ISSN: 2454-4248
Volume: 4 Issue: 3 275 – 279

# Optimization of Total Waiting Time of Jobs in Two Stage Specially Structured Flow Shop Scheduling Model with Transportation Time of Jobs

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***Abstract:-***A flow shop scheduling problem is one of the classical problems in production scheduling. In this paper, we study specially structured Flow Shop Scheduling Model where in transportation time from one machine to another machine is taken into account. It is assumed that maximum of the processing time on first machine is less than or equal to the minimum of processing time on second machine. The purpose of the study is to get optimal sequence of jobs in order to optimize the total waiting time of the jobs through iterative algorithm. The algorithm is made clear by numerical example.

***Key words:*** *Waiting time of jobs, Transportation time, Flow shop Scheduling, Processing time.*

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## I. INTRODUCTION

Scheduling may be defined as the problem of deciding when to execute a given set of activities, subject to chronological constraints and resources capacities, in order to optimize some function. Scheduling contains **m** different machines arranged in series on which a set of **n** jobs are to be processed. Each of the **n** jobs requires **m** operations and each operation is to be performed on a separate machine. The general **n** jobs, **m** machine scheduling is quite formidable. Consider an arbitrary sequence of jobs on each machine, there are $(n!)^m$ possible schedules which posses computational difficulties. With the aim to reduce the number of possible schedules it is reasonable to assume that all machines process jobs in the same order. A Flow shop problem exists when all the jobs share the same processing order on all the machines. In Flow shop, technological constraints demand that the jobs pass between the machines in the same order. Hence there is natural sequence of the machines characterized by the technological constraints for each and every job in flow shop. The flow of the work is unidirectional; thus every job must be processed through each machine in a given prescribed order. Efforts in the past have been made by researchers to reduce this number of feasible schedules as much as possible without compromising on optimality condition. Today's large-scale markets and instantaneous communications mean that clients expect high-quality goods and services when they require them, where they require them. Organizations, whether public or private, need to provide these products and services as effectively and efficiently as possible.

The criterion of optimality in the given problem is specified as minimization of waiting time of jobs in n job, 2 machine flow shop scheduling. Minimization of total waiting time of jobs is defined as the sum of the times of all the jobs which is consumed in waiting for their turn on the second machine. There are some papers in the literature of scheduling theory which consider the waiting time to be important for scheduling the jobs on the machines.

## II. LITERATURE REVIEW

The Johnson's algorithm [1] is especially popular among analytical approaches that are used for solving n- jobs, 2- machines sequence problem. Maggu P.L. and Dass G. [3] made attempts to extend the study by introducing the concept of equivalent job for job block in job sequencing. Solution methods of flow shop scheduling using heuristic approach was being developed by Singh T.P. [4], Rajendran C. and Chaudhuri D.[5]. Singh T.P.and et.al. [6] studied the problem related with group job restrictions in a flow shop which involves independent set- up times. Further Gupta D. [8] studied minimization of Rental Cost in Two Stage Flow Shop Scheduling Problem, in which Setup Time was separated from Processing Time and each associated with probabilities including Job Block Criteria. Aggarwal S. and et.al.[9] studied the flow shop scheduling model with separated set up time and transportation time. Gupta D. and et.al. [10] studied optimal two stage flow shop specially structured scheduling in which processing time is allied with probabilities with the intention to minimize the rental cost. Gupta D. and et.al. [11] studied n x 2 specially structured flow shop scheduling in which diverse parameters are well thought-out. Gupta D. and Goyal B.[14] with the intention of minimizing total waiting time of jobs studied two stage flow shop scheduling by associating probabilities with processing time. Gupta D. and Goyal

ISSN: 2454-4248
275 – 279

B.[15], [16], [17] in consideration with the concept of job block and separated setup time widen the study.

The problem discussed here has noteworthy use of theoretical results in process industries or in the circumstances when the objective is to lessen the total waiting time of jobs. The present paper make clear to the specially structured two stage flow shop scheduling problem in which transportation time of jobs has been well thought-out.

## III. PRACTICAL SITUATION

Manufacturing units/industries play an important role in the economic development of a country. Flow shop scheduling occurs in various offices, service stations, banks, airports etc. In our day to day working in factories and industrial units different jobs are processed on various machines. In textile industry different types of fabric is produced using different types of yarn. Here, the maximum equivalent time taken in dying of yarn on first machine is always less than or equal to the minimum equivalent time taken in weaving of yarn on the second machine. The idea of minimizing the waiting time may be an economical aspect from Factory /Industry manager's view point when he has minimum time contract with a commercial party to complete the jobs.

## IV. NOTATIONS

$A_k$: Sequence obtained by applying the algorithm proposed.
$X_k$: Time for processing of $k^{th}$ job on machine X.
$Y_k$: Time for processing of $k^{th}$ job on machine Y.
$X_k{}'$: Equivalent time for processing of $k^{th}$ job on machine X.
$Y_k{}'$: Equivalent time for processing of $k^{th}$ job on machine Y.
$t_k$ : Transportation time of $k^{th}$ job from machine X to machine Y.
$T_{aY}$: The completion time of job a on machine Y.
$W_\mu$: Waiting time of job $\mu$.
W: Total waiting time of all the jobs.

## V. PROBLEM FORMULATION

The machines X and Y are dealing out n jobs in the sort X Y. $X_k$ and $Y_k$ are the relevant processing times of the $k^{th}$ job correspondingly, $t_k$ is the transportation time of $k^{th}$ job from machine X to machine Y. Our goal is to come across a optimal sequence $\{A_k\}$ of jobs with lessen total waiting time of jobs. Dealing out times of $k^{th}$ job on machine X & Y are defined as $X'_k = X_k + t_k$, $Y'_k = Y_k + t_k$ satisfying structural relationship Max $X'_k \le$ Min $Y'_k$

TABLE 1**:** Matrix form of the Mathematical Model of the problem

| **Job** | **Machine X** | **Transportation Time** | **Machine Y** |
|---|---|---|---|
| J | $X_k$ | $t_k$ | $Y_k$ |
| 1. | $X_1$ | $t_1$ | $Y_1$ |
| 2. | $X_2$ | $t_2$ | $Y_2$ |
| 3. | $X_3$ | $t_3$ | $Y_3$ |
| . | . . | . . | . . |
| n. | $X_n$ | $t_n$ | $Y_n$ |

## VI. ASSUMPTIONS

In the given flow shop scheduling the following assumptions are made

1) There are n number of jobs (J) and two machines (X & Y).
2) The order of sequence of operations in all machines is the same.
3) Jobs are independent to each other.
4) Machines break down interval, set up times are not considered for calculating waiting time.
5) Pre- emption is not allowed i.e. jobs are not being split clearly, once a job is started on a machine, the process on that machine can't be stopped unless the job is completed.

**Lemma 1.** Two machines X, Y are handing out n jobs in sort X Y among no passing permissible. $X_k$ and $Y_k$ are the processing times of job $k$ ( $k = 1,2,3, \ldots\ldots, n$) on both machines, $t_k$ is the transportation time of $k^{th}$ job from machine X to machine Y. Dealing out times of $k^{th}$ job on machines X and Y are defined as $X'_k = X_k + t_k$, $Y'_k = Y_k + t_k$ satisfying structural relationship Max $X'_k \le$ Min $Y'_k$ then for the n job sequence $C: \mu_1, \mu_2, \mu_3, \ldots \ldots \ldots \mu_n$
$T_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} \ldots + Y'_{\mu_n}$
**Proof.** Using mathematical Induction hypothesis on n:
Consider $S(n)$: $T_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} \ldots + Y'_{\mu_n}$
$T_{\mu_1 X} = X'_{\mu_1}$ , $T_{\mu_1 Y} = X'_{\mu_1} + Y'_{\mu_1}$
$S(1)$ is true.
Assume the result holds for less than n jobs, $T_{\mu_n Y} = \text{Max}\left(T_{\mu_n X}, T_{\mu_{n-1} Y}\right) + Y'_{\mu_n}$
As Max $X'_k \le$ Min $Y'_k$
Consequently, $T_{\mu_n Y} = X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} \ldots + Y'_{\mu_n}$
$S(n)$ is true for all $n \in N$

**Lemma 2.** Following the similar notations as used in Lemma1, n job sequence $C: \mu_1, \mu_2, \mu_3, \ldots \ldots \ldots \mu_n$
$W_{\mu_1} = 0$

International Journal on Future Revolution in Computer Science & Communication Engineering
Volume: 4 Issue: 3
ISSN: 2454-4248
275 – 279

$$W_{\mu_n} = X'_{\mu_1} + \sum_{r=1}^{n-1} x_{\mu_r} - X'_{\mu_n}$$

$x_{\mu_r}$ is defined as $x_{\mu_r} = Y'_{\mu_r} - X'_{\mu_r}$, $\mu_r \epsilon (1, 2, 3, \dots, n)$

Proof. $W_{\mu_1} = 0$

$$W_{\mu_n} = \mathrm{Max}\left(T_{\mu_n X}, T_{\mu_{n-1} Y}\right) - T_{\mu_n X}$$

$$= X'_{\mu_1} + Y'_{\mu_1} + Y'_{\mu_2} \dots + Y'_{\mu_{n-1}} - X'_{\mu_1} - X'_{\mu_2} \dots - X'_{\mu_n}$$

$$= X'_{\mu_1} + \sum_{r=1}^{n-1} (Y'_{\mu_r} - X'_{\mu_r}) - X'_{\mu_n}$$

$$= X'_{\mu_1} + \sum_{r=1}^{n-1} (x_{\mu_r}) - X'_{\mu_n}$$

**Theorem 1**. Following the similar notations as used in Lemma1, for the n job sequence

$C: \mu_1, \mu_2, \mu_3, \dots \dots \dots \mu_n$, the total waiting time (W) is given by the following formula

$$W = nX'_{\mu_1} + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{k=1}^{n} X'_k$$

$$y_{\mu_r} = (n - r)x_{\mu_r}\ ;\ \mu_r \epsilon (1, 2, 3, \dots, n)$$

**Proof**. From Lemma 2 we have

$W_{\mu_1} = 0$

For $n = 2$,

$$W_{\mu_2} = X'_{\mu_1} + \sum_{r=1}^{1} x_{\mu_r} - X'_{\mu_2}$$

For $n = 3$,

$$W_{\mu_3} = X'_{\mu_1} + \sum_{r=1}^{2} x_{\mu_r} - X'_{\mu_3}$$

Continuing in this way

$$W_{\mu_n} = X'_{\mu_1} + \sum_{r=1}^{n-1} x_{\mu_r} - X'_{\mu_n}$$

Hence total waiting time

$$W = \sum_{i=1}^{n} W_{\mu_i}$$

$$W = nX'_{\mu_1} + \sum_{r=1}^{n-1} y_{\mu_r} - \sum_{k=1}^{n} X'_k$$

Where $y_{\mu_r} = (n - r)x_{\mu_r}$

**Theorem 2:** For a natural number 'n' and real numbers $y_1, y_2, y_3, \dots \dots y_n$ if $y_{\mu_1} \le y_{\mu_2} \le y_{\mu_3} \le \dots \dots \le y_{\mu_n} \Rightarrow ny_{\mu_1} + (n-1)y_{\mu_2} + (n-2)y_{\mu_3} + \cdots \dots + y_{\mu_n}$ is minimum for $\mu \in S_n$, permutation group of n-symbols.

**Proof:** Using mathematical Induction hypothesis on n:
For n=1, the result holds trivially

Assume the result holds for less than n real numbers

For $y_{\mu_1} \le y_{\mu_2} \le y_{\mu_3} \le \dots \dots \le y_{\mu_n}$

$$ny_{\mu_1} + (n-1)y_{\mu_2} + (n-2)y_{\mu_3} + \cdots \dots + y_{\mu_n}$$
$$= (n-1)y_{\mu_1} + (n-2)y_{\mu_2} + (n-3)y_{\mu_3} + \cdots \dots + y_{\mu_{n-1}} + \sum_{k=1}^{n} y_{\mu_k}$$

By induction hypothesis, $ny_{\mu_1} + (n-1)y_{\mu_2} + (n-2)y_{\mu_3} + \cdots \dots + y_{\mu_n}$ is minimum for $\mu \in S_n$

## VII. ALGORITHM

**Step 1:** Equivalent dealing out times of $k^{th}$ job on machines X & $Y$ is calculated as defined.

**Step 2:** Calculate the entries for the following table

TABLE 2

| Job | Machine X | Machine Y | | $y_{kr} = (n-r)x_k$ | | | | |
|---|---|---|---|---|---|---|---|---|
| **J** | $X'_k$ | $Y'_k$ | $x_k$ | $r = 1$ | $r = 2$ | $r = 3$ | ... | $r =$ n-1 |
| 1. | $X'_1$ | $Y'_1$ | $x_1$ | $y_{11}$ | $y_{12}$ | $y_{13}$ | $\cdots$ | $y_{1\,n-1}$ |
| 2. | $X'_2$ | $Y'_2$ | $x_2$ | $y_{21}$ | $y_{22}$ | $y_{23}$ | $\cdots$ | $y_{2\,n-1}$ |
| 3. | $X'_3$ | $Y'_3$ | $x_3$ | $y_{31}$ | $y_{32}$ | $y_{33}$ | $\cdots$ | $y_{3\,n-1}$ |
| . | . | . | . | . | . | . | . | . |
| n. | $X'_n$ | $Y'_n$ | $x_n$ | $x_{n1}$ | $y_{n2}$ | $y_{n3}$ | $\cdots$ | $y_{n\,n-1}$ |

**Step 3:** Assemble the jobs in increasing order of $x_k$.

Assuming the schedule found be $(\mu_1, \mu_2, \mu_3, \dots \dots \dots \mu_n)$

**Step 4:** Locate $a = \min\{X'_k\}$

$X'_{\mu_1} = a$, Schedule noticed previously is the requisite favorable schedule.

$X'_{\mu_1} \ne a$, Move on to step 5

**Step 5:** Regarding the different sequence of jobs $A_1, A_2, A_3, \dots \dots, A_n$. Where $A_1$ is the schedule described in step 3, Schedule $A_j (j = 2,3, \dots \dots, n)$ can be achieved by placing $j^{th}$ job in the schedule $A_1$ to the initial position and rest of the schedule remaining same.

**Step 6:** Calculate the total waiting time W for each and every of the schedules $A_1, A_2, A_3, \dots \dots, A_n$ using the formula derived in Theorem 1:

$$W = nX'_b + \sum_{r=1}^{n-1} y_{ar} - \sum_{k=1}^{n} X'_k$$

 ISSN: 2454-4248
275 – 279

$X'_b$ = Equivalent processing time of the first job on machine X in schedule $A_j$

$y_{ar} = (n-r)x_{ar}$ ; $a = \mu_1, \mu_2, \mu_3, \dots \dots \dots \mu_n$

As $\sum_{k=1}^{n} X'_k$ is constant for the problem and $x_{\mu_1} \le x_{\mu_2} \le x_{\mu_3} \le \dots \dots \le x_{\mu_n}$

Theorem 2 justifies that the schedule with minimum (W) is the required optimal schedule.

## VIII. NUMERICAL ILLUSTRATION

Assume 5 jobs 1, 2, 3, 4, 5 has to be processed on two machines X & Y with processing times $X_k$ and $Y_k$, $t_k$ is the transportation time of k[th] job from machine X to machine Y

TABLE 3: PROCESSING TIME MATRIX

| Job | Machine X | Transportation time | Machine Y |
|---|---|---|---|
| **J** | $X_k$ | $t_k$ | $Y_k$ |
| 1. | 5 | 4 | 15 |
| 2. | 8 | 3 | 21 |
| 3. | 12 | 2 | 24 |
| 4. | 11 | 4 | 19 |
| 5. | 14 | 2 | 23 |

Intention is to achieve a most favorable schedule, lessening the total waiting time for the jobs.

**Solution**

**As per step 1-** Equivalent processing time $X'_k$ & $Y'_k$ on machine X & Y given in the following table

TABLE 4: EQUIVALENT PROCESSING TIME MATRIX

| ob | Machine X | Machine Y |
|---|---|---|
| **J** | $X'_k$ | $Y'_k$ |
| 1. | 9 | 19 |
| 2. | 11 | 24 |
| 3. | 14 | 26 |
| 4. | 15 | 23 |
| 5. | 16 | 25 |

Max $X'_k = 16 <$ Min $Y'_k = 19$

**As per step 2-** Obtaining the values for TABLE 2

TABLE 5

| Job | Machine X | Machine Y | | $y_{kr} = (n-r)x_k$ | | | |
|---|---|---|---|---|---|---|---|
| I | $X'_k$ | $Y'_k$ | $x_k$ | r = 1 | r=2 | r=3 | r=4 |
| 1. | 9 | 19 | 10 | 40 | 30 | 20 | 10 |
| 2. | 11 | 24 | 13 | 52 | 39 | 26 | 13 |
| 3. | 14 | 26 | 12 | 48 | 36 | 24 | 12 |
| 4. | 15 | 23 | 8 | 32 | 24 | 16 | 8 |
| 5. | 16 | 25 | 9 | 36 | 27 | 18 | 9 |

**As per step 3-**.The schedule thus found be 4, 5, 1, 3, 2.

**As per step 4-** $\text{Min}\{X'_k\} = 9 \neq X'_4$

**As per step 5-** Different schedule of jobs can be considered as:

$A_1$: 4, 5, 1, 3, 2; $A_2$: 5,4,1,3,2 ; $A_3$: 1,4, 5, 3, 2; $A_4$: 3,4, 5,1,2; $A_5$: 2,4, 5, 1, 3

**As per step 6-** The total waiting time for the schedules obtained in step 5 can be calculated$A_1$

Here, $\sum_{k=1}^{5} X'_k = 65$
For the schedule $A_1$: 4, 5, 1, 3, 2
Total waiting time W =101
For the schedule $A_2$: 5,4,1,3,2
Total waiting time W =107
For the schedule $A_3$: 1,4, 5, 3, 2
Total waiting time W =74
For the schedule $A_4$: 3,4, 5,1,2
Total waiting time W =105
For the schedule $A_5$: 2,4, 5, 1, 3
Total waiting time W =94
Hence schedule $A_3$: 1,4, 5, 3, 2 is the required schedule with minimum total waiting time.

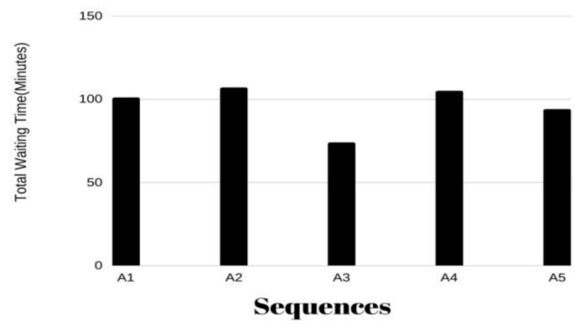


**Figure 1**

## IX. CONCLUSION

The current study deal with the flow shop scheduling problem with the foremost thought to minimize the total waiting time of jobs. However it may increase the other costs like machine idle cost or penalty cost of the jobs. The study can further be extended with the consideration of a variety of parameters like job block, break down interval etc.

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