

On Modal Analysis of FGM Annular Plate under Thermal Effect

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Abstract: This present paper investigated the modal analysis of FGM annular plate under thermal effect. The material properties are assumed to have temperature dependent and vary in accordance with power law distribution. The finite element software COMSOL is used to obtain the natural frequencies of FGM annular plate under free-clamped boundary condition. The effect of temperature and power index on the natural frequency is also reported.

I. INTRODUCTION

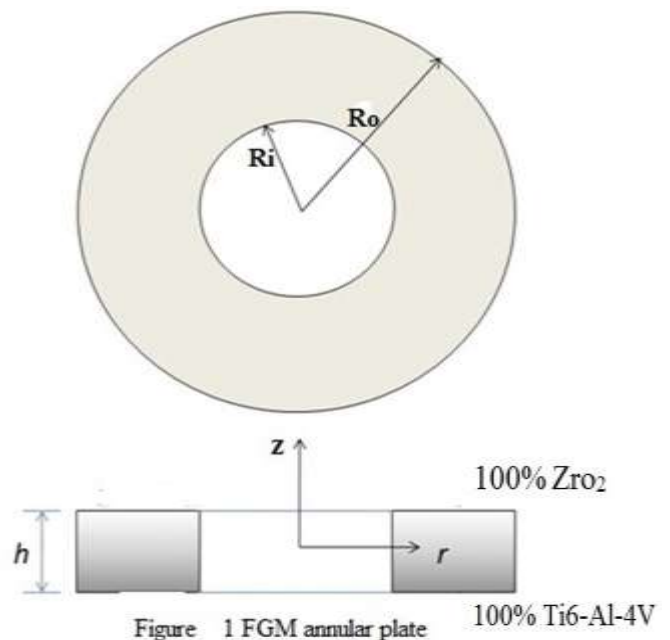
Functionally graded materials are combination of metal and ceramic, whereas metal provides high mechanical strength and low fracture and ceramic gives high thermal position to provide model designedature resistance. The investigation of FGMs annular plates are essential great tech. Various researchers are investigated vibration analysis of annular plates with different theories such as simple higher order shear deformation plate theory (HSDT), first-order shear deformation plate theory (FSDT) and other so many other theories. The different analytical and approximate methods were used to obtain the modal behaviour of FGM annular plate. But, to the best of the author's knowledge the software analysis of modal analysis of FGM annular plate under thermal effect has not been dealt before.

II. LITERATURE SURVEY

Allahverdizadeh A. et al. [2] reported nonlinear free and forced vibration analysis of thin circular FGM plates. The governing equations were solved with TMM and Kantorovich time averaging technique. Material properties were temperature - dependent and assumed to vary according to the power law. P Malekzadeh et al. [3] investigated three-dimension analysis free vibration of thick FGM annular plates in thermal environment. DQM was used to obtain the frequencies. The material properties were assumed to be temperature dependent. The equations of motion were derived from the 3D thermoelastic equations. Lee Y.Y et al. [4] investigated post - buckling of FGM cylinder shell under axial compression and thermal load using kp-Ritz method. Dong C.Y et al. [5] investigated the vibration analysis of FGM annular plates in thermal environment. 3D elasticity theory and the Chebyshev – Ritz method was used to obtain the results. Material properties are temperature dependent and vary in thickness direction according the power law distribution under clamped - clamped and free - clamped boundary conditions.

Malekzadeh P et al. [6] investigated the free vibration analysis of FGM annular plates in thermal environment. DQM was used to solve the equations of motion. Material properties are temperature - dependent and vary in thickness direction according the exponential distribution law. Reddy J.N. et al. [7] examined dynamic response of FGMs cylindrical shell. Governing equations were derived using FSDT. Material constants were varied according to power law distribution.

III. RESULTS AND DISCUSSION



The geometry of FGM annular plate is given in Fig 1. The upper surface is made of 100% metal and lower surface is made of 100% ceramic. In between, the material properties are varying in thickness direction in accordance with power law distribution. The temperature of upper surface is varies and temperature of lower surface is kept at constant room temperature. The temperature-dependent properties for

annular plate at upper and lower surface may be expressed as function of temperature [7].

$$Q = Q_0(Q_{-1}T_{-1}+1+Q_1T+Q_2T_2+Q_3T_3) \quad (1)$$

here, Q_{-1} , Q_0 , Q_1 , Q_2 , Q_3 represent coefficients of temperature. The E , P , K are representing Young's modulus, density, thermal conductivity. The material properties are assumed to have temperature – dependent and vary according to the power law distribution through the thickness, i.e. in the z direction.

$$E(z,T) = [E_m(T) - E_c(T)]\left(\frac{z+0.5h}{h}\right)^g + E_c(T) \quad (2)$$

Here, E_m and E_c are the Young's modulus of metal and ceramic respectively. g is power index and T is the temperature. z is the thickness coordinate and h is the thickness of the plate. Here power index value varies from 0 to infinity. At $g = 0$, the FGM plate is behaves similar to the pure ceramic plate and at g is tends to infinite, the FGM plate behaves as pure metallic plate

A. Validation Study

Initially to validate the results, the modal analysis of FGM annular plate without thermal consideration (Inner radius (R_i) = 0.5 m, outer radius (R_o) = 2 m, $E_m = 70$ GPa, $\nu = 0.28$, $P_m = 2707$ (kg/m³), $E_c = 168$ GPa, $h = 0.1$ m, ($\Delta T = 0$) [3] is presented. The results are obtained using three-dimensional finite element software COMSOL version 4.2.

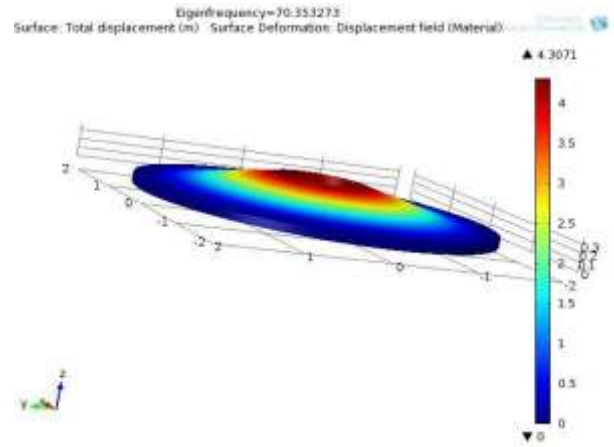
Table 1 Comparison of natural frequency (Hz) of an FGM annular plate under free-clamped boundary condition without consideration of thermal effect for different modes

g	Mode	COMSOL	Ref.[8]	%Diff
0.6	(0,0)	68.871	67.130	2.5
	(0,1)	284.676	284.618	1.0
1	(0,0)	67.588	66.783	1.2
	(0,1)	286.731	283.124	1.2
5	(0,0)	68.316	69.639	1.8

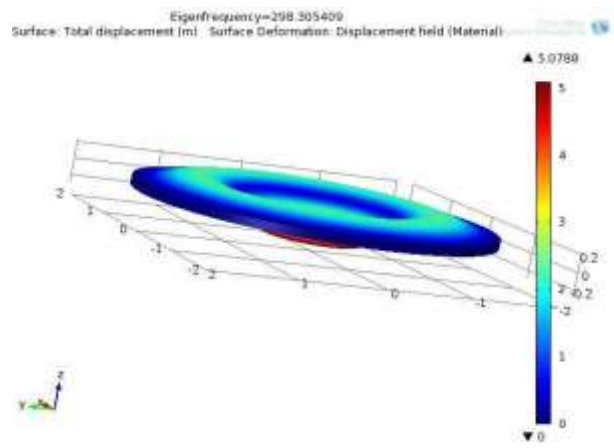
Table 1 shows the comparison of natural frequency (Hz) of an FGM annular plate under free-clamped boundary condition without consideration of thermal effect for modes (0,0) and (0,1) for range of power index values. Here, the first digit represents number of nodal diameter and second term denotes the number of nodal circle. It may be seen from the Table 1 is that the results obtained by present work (COMSOL) are agree well with the results obtained by previous published work. Some of the mode shapes are also given in Fig.2.

The percentage difference is given by.

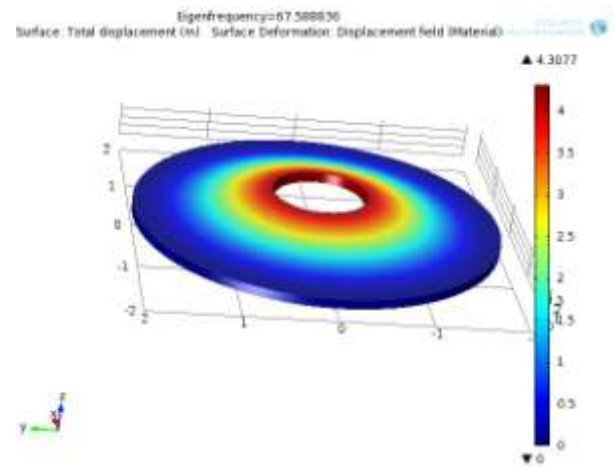
$$\% \text{ Diff} = \frac{[(\text{Ref.}) - (\text{COMSOL.})]}{(\text{Ref.})} \times 100$$



(0,0)g=0, Natural frequency=70.353



(0,0) g = 0.6, Natural frequency = 68.87



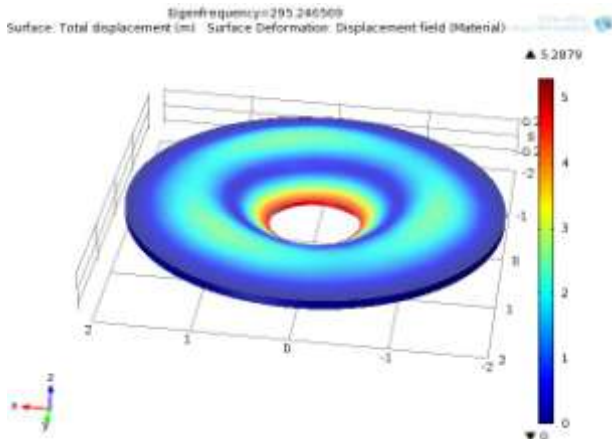
(0,0) g = 1, Natural frequency = 67.58

B. Parametric Study

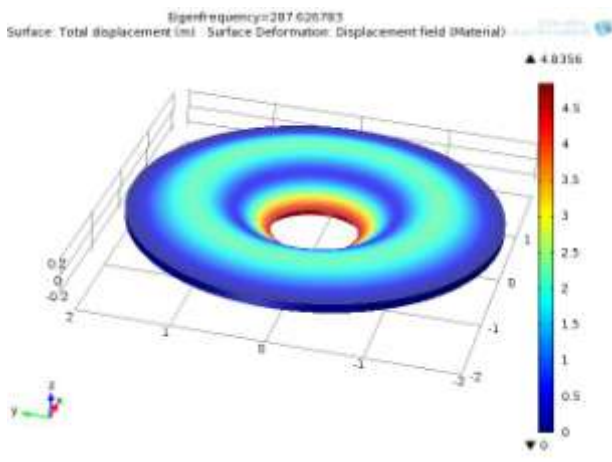
In this study, three dimensional FGM annular plate with inner radius $R_i = 0.5m$, outer radius $R_o = 2m$, and thickness $h=0.1m$, is using to obtain the modal analysis under thermal effect.

Table 2 Natural frequencies (Hz) for an FGM annular plate for free - clamped boundary condition under thermal effect at power index $g = 0.6$

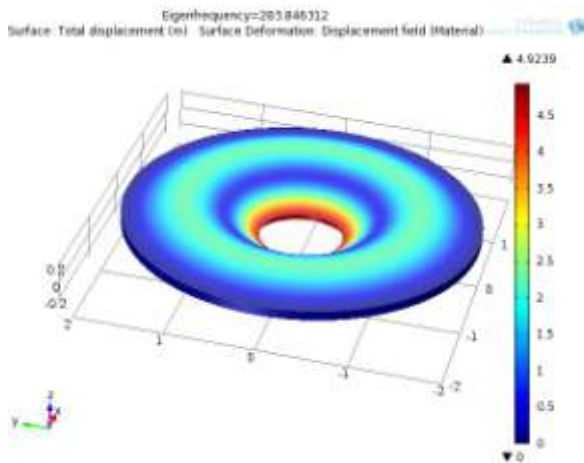
Tupper (K)	(0,0) Mode Frequency (Hz)	(1,0) Mode Frequency(Hz)	(0,1) Mode Frequency (Hz)
300	63.099	107.609	308.821
400	59.577	105.716	303.387
500	56.452	103.788	297.853
600	55.383	101.823	292.215



(0,1) $g = 5$, Natural frequency = 295.24

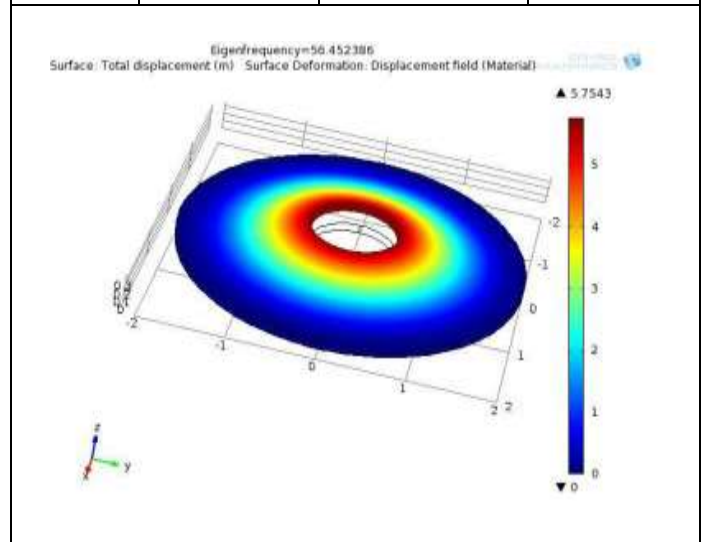


(0,1) $g = 20$, Natural frequency = 287.62

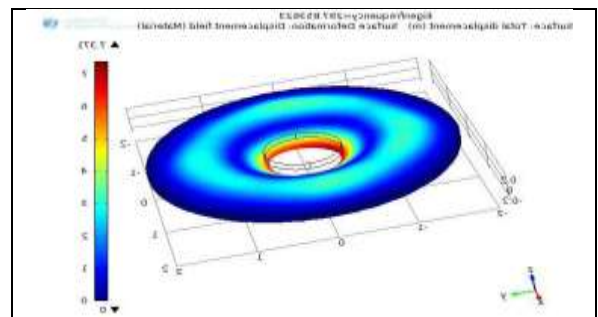


(0,1) $g = 50$, Natural frequency = 284.84

Fig 2 Mode shapes of FGM annular plate under free-clamped boundary condition

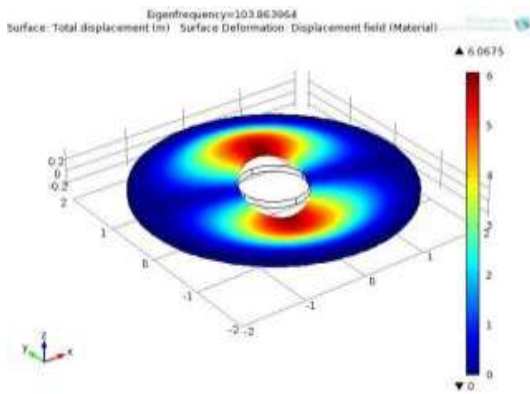


(0,0) Mode Natural frequency = 56.452



(0,1) Mode Natural frequency = 297.853

Fig 3: Mode shapes of FGM annular plate under thermal effect at different values of g



(1,0) Mode Natural frequency =103.788

Table 2 shows the variation of natural frequency (Hz) with different temperature under free - clamped boundary condition. Results are obtained for modes (0,0), (1,0) and (0,1). It can be seen that the natural frequency decreases with increase in temperature for modes (0,0), (1,0) and (0,1). Some of the mode shapes of FGM annular plate are shown in Figure 3.

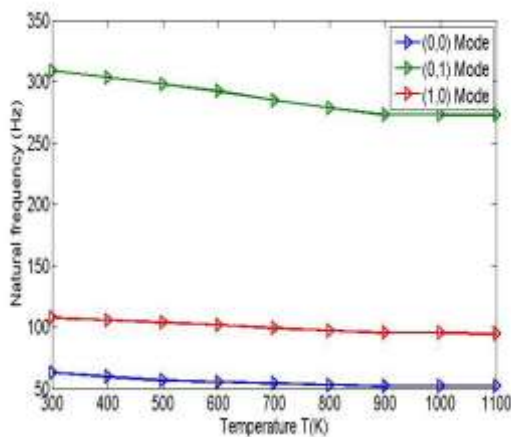


Figure 4 Effect of temperature on the natural frequencies (Hz) of an FGM annular plate for free - clamped boundary conditions under power law ($g = 0.6$)

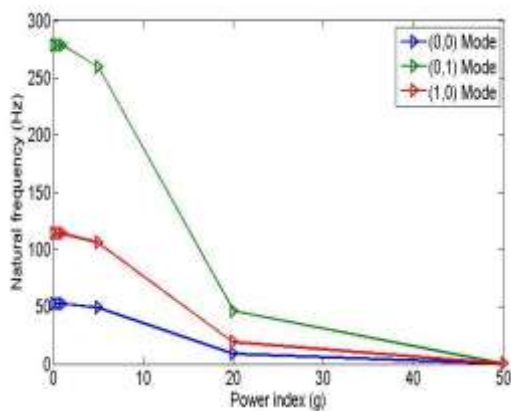


Figure 5 Effect of gradient index on the natural frequencies of FGM annular plate for free - clamped boundary condition

Figure 4 displays the effect of temperature on the natural frequencies (Hz) of an FGM annular plate for free - clamped boundary conditions under power law ($g = 0.6$). It can be seen that with the increase in temperature, the natural frequencies decrease for all given modes.

The effect of gradient index on the natural frequencies of FGM annular plate for free - clamped boundary condition is shown in Figure 5. It may be seen that with the increase in power index, the natural frequencies decrease for all given modes.

IV. Conclusion

Modal analysis of FGM annular plate under thermal effect is reported. Material properties are temperature dependent and vary according to the power law distribution in the thickness direction. The thermal effect on FGMs annular plate is done by imposing different temperature on upper surface and keeps constant temperature (ambient) at lower surface. The conclusions of this study can be summarized are as follows:

1. With the increase in temperature of upper surface of plate, the natural frequencies decrease for modes (0,0) (0,1) and (1,0).
2. With the increase in power index, the natural frequency decreases for modes (0,0) (0,1) and (1,0).

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