Vibration Analysis of FGM Rectangular Plate: A Review

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Abstract- This paper presents a comprehensive review of the vibration analysis of functionally graded material (FGM) rectangular plate. Functionally Graded Materials (FGMs) are the advanced materials in the field of composites, which can used for high temperatures applications and reducing the thermal stresses. The various method and theories are used to design of FGM plates. The main aim of this paper is to serve the interests of researchers and engineers already involved in the analysis and design of FGM plates.

Keywords- *Free vibration; Functionally Graded Plates*

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I. INTRODUCTION

Pure metals are use for some engineering applications because of the demand of conflicting property requirement. To solve this problem, we can use metals ally. The properties of alloys are different from the parent materials. Bronze, alloy of copper and tin, was the first alloy[1]. Bronze really impacted the word at that time. People have been experimenting with one form of alloy or the other with the sole reason for improving properties of material. There is limit to which material because of thermodynamic equilibrium limit [2] when more quantity of the alloying material is desired, then the traditional alloying cannot be used.

Powdered Metallurgy(PM) is another method of production part that cannot be produced through the convention alloying, as alloy are produced in powdered form and some of the problems associated with the conventional alloying are overcome[3]. Another method of producing materials with combination of properties is by combining materials in solid state, which is referred to as composite material. Composite materials are a class of advanced materials made up of one or more materials combined in solid states with distinct physical and chemical properties. Composite material offers an excellent combination of properties which are different from the individual parent materials and are also lighter in weight. Wood is a composite material from nature which consists of cellulose in matrix of lignin [4].

Composite materials will fail under extreme working conditions through a process called delamination (separation of fibers from the matrix) [5].This can happen for example , in high temperature application where two metals with and different coefficient of expansion are used. To solve this problem, with a novel material called functionally graded material [6,7]. Functionally graded material (FGM) a revolutionary material belongs to a class of advanced material with varying properties over a changing dimension [8, 9]. Functionally graded materials occur in nature as bones, teeth etc [10], nature designed this materials to meet their expected service requirements [5]. It replaces this sharp interface with a gradient interface which produces smooth transition from one material to the next [6, 7]. One unique characteristics of FGM is the ability to tailor a material for specific application [8]. There are different kinds of fabrication processes for producing functionally graded materials.

Functionally graded materials can be divided into two broad groups namely: thin and bulk FGM [11]. Bulk FGM is produced using powder metallurgy technique, centrifugal casting method, solid free form technology etc [10]. Functionally graded materials find their applications in aerospace, automobile, medicine, sport, energy, sensors, optoelectronic etc [12]. As the fabrication process is improved, cost of powder is reduced and the overall process cost is reduced, hence expanding the application of FGM.

II. VIBRATION ANALYSIS OF FGM RECTANGULAR PLATE

Kim Young-Wann [13] developed a theoretical method to investigate vibration characteristics of initially stressed functionally graded rectangular plates. The plates were made up of metal and ceramic. The temperature was assumed to be constant in the plane of the plate and to vary in the thickness direction only. The two types of thermal conditions were considered. The first type is that one value of the temperature is imposed on the upper surface and the other (or same) value on the lower surface. The second is that the heat flows from the upper surface to the lower which was held at a prescribed temperature. Material properties were assumed to be temperature dependent, and vary continuously through the thickness according to a power law distribution. Nonlinear vibration of a functionally graded plate in a general state of non-uniform initial stress was investigated by Chen Chun-Shang [14]. Runge–Kutta and Galerkin method were used in their work. The von Karman theory was adopted by author. The variation of properties followed a simple power-law distribution in terms of the volume fractions of the constituents. Ganapathi M et al. [15] was investigated the nonlinear free vibration analysis of functionally graded material (FGM) plates subjected to thermal environment using von Karman theory. Temperature field is assumed to be a uniform distribution over the plate surface and varied in the thickness direction only. The material was assumed to be temperature dependent and graded in the thickness direction according to the power-law distribution. The effective material properties were estimated using Mori–Tanaka homogenization method. The free-vibration analysis of a simply supported functionally graded thick plate was investigated by Zenkour Ashraf M. [16]. The trigonometric shear deformation plate theory was used in their work. The material properties of plate were assumed to vary in the thickness direction only. Ferreira A.J.M et al. [17] were studied the free vibration analysis of functionally graded plate using the first and third-order shear deformation plate theories. The global collocation method was used to solve the equations of motion. The variation of the properties followed Mori–Tanaka technique. Nonlinear vibration analysis of functionally graded plate in a general state of arbitrary initial stresses was investigated by Chen Chun-Sheng [18]. The derived equations were included the effects of initial stresses and initial imperfections size. The material properties of a functionally graded plate were graded continuously in the thickness direction. The variation of the properties followed a simple power-law distribution in terms of the volume fractions of the constituents. The vibration motion of an initially stressed functionally graded plate (FGP) carried out by. Chen Chun-Sheng et al. [19]. The classical plate theory was employed in this work. The material properties of the FGM plate were assumed to vary continuously from one surface to another according to a simple power law distribution. Gale kin method was used to obtain the results. Kitipornchai. S et al. [20] were investigated the free vibration analysis of FGM plate using first-order shear deformable theory (FSDT). The mesh-free Galerkin method was used in their study. The random free vibration analysis of functionally graded laminates plate with general boundary conditions and subjected to a temperature change was investigated by Kitipornchai . S et al. [21]. The third-order shear deformation theory was used to obtain the results. A mean-centered first-order perturbation technique was adopted to obtain the second-order statistics of vibration frequencies. The mixed-type formulation and a semi-analytical approach were employed to derive the standard eigen value in terms of deflection, mid-plane rotations and stress function. The nonlinear free vibration analysis of plates made of functionally graded materials was studied by Meguid.S.A et al [22]. The material properties of the functionally graded plates were assumed to vary continuously through the thickness direction according to a power-law distribution of the volume fraction of the constituents. The von Karman theory was used to obtain the fundamental equations. The free vibration analysis of functionally graded plates was investigated by Ferreira. A.J.M et al. [23]. The multi quadric method was used in their work. This method was truly mesh less method. The higher-order shear deformation theory was adopted by the authors. They were used homogenization technique for material properties based on the Mori–Tanaka scheme. The vibration Analysis of composite rectangular plate with rectangular hole or a circular hole at different boundary condition was investigated by Patil Devidas R et al. [24]. In their work, a new methodology called Independent Coordinate Coupling Method (ICCM) was developed. The effect of the hole was taken into account by subtracting the energies of the whole domain from the total energies of the whole plate. The vibration analysis of a rectangular plate with a rectangular hole or a circular hole was investigated by Kwak Moon K. et al [25]. The effect of the hole was taken into account by subtracting the energies of the whole domain from the total energies of the whole plate. The Independent Coordinate Coupling Method (ICCM) was developed by the authors in this work. The Rayleigh–Ritz method was used to obtain the results. The in-plane vibration analysis of functionally graded with general elastically restrained boundary conditions was carried out by Du jingtao et al. [26]. The plane stress theory was adopted by the authors in this analysis. The Fourier series method was used to solve the vibration results in which the in-plane displacements were expressed as the superposition of a double Fourier cosine series and four supplementary functions in the form of the product of a polynomial function and a single cosine series expansion. The Rayleigh-Ritz energy method was used in their work. The nonlinear vibration analysis for an imperfect functionally graded plate in a general state of arbitrary initial stresses was studied by Chen Chun-Sheng et al [27] using the nonlinear partial differential equations. The perturbation technique, the Galerkin method and the Runge–Kutta method were employed in this work by the authors. The material properties of functionally graded plate were graded continuously in the thickness direction according to a simple power-law distribution in terms of the volume fractions of the constituents. The free vibrations analysis and equations for the determination of frequencies and the corresponding mode shapes of a simply supported rectangular plate were carried out by Batra.R.C. [28].The principle of virtual work to derive a higher-order shear and normal deformable theory for a plate comprised of a linear elastic incompressible anisotropic material. The theory did not use a shear correction factor and employs three components of displacement and the hydrostatic pressure as independent variables. Liew K.M. [29] was investigated free vibration analysis of metal and ceramic functionally graded plates. The element-free kp-Ritz method was used to obtain the results. The material properties of the plate were assumed to vary continuously through thickness direction according to a power-law distribution. Free vibration of functionally graded material rectangular plates with simply supported and clamped edges in the thermal environment was studied by Lu V.P [30]. The three-dimensional linear theory of elasticity was adopted by the authors. The FGM plates with temperature-dependent material properties subjected to three types of temperature distribution, (i) uniform temperature rise, (ii) linear temperature rise and (iii) nonlinear temperature rise were considered in their work. The three displacements of the plates were expanded by a series of Chebyshev polynomials multiplied by appropriate functions to satisfy the essential boundary conditions. The natural frequencies were obtained by using Ritz method. The free vibration of a functionally graded rectangular plate with in-plane material in homogeneity was investigated by Chen W.C [31]. A Levy-type solution was obtained for plates with a pair of simply supported edges that were parallel with the material gradient direction. A particular integration method was adopted to solve the fourth-order ordinary differential equation with non-constant coefficients. Malekzadeh. P et al. [32] were investigated the free vibration of functionally graded (FG) arbitrary straight-sided quadrilateral plates under thermal environment. The first order shear deformation theory (FSDT) was used in their work. The material properties were assumed to be temperature-dependent and graded in the thickness direction. The initial thermal stresses were evaluated by solving the thermo-elastic equilibrium equations. The differential quadrature method (DQM) was employed to obtain the results.

The free vibration and static analysis of functionally graded material (FGM) plates was studied by Singh B.N et al. [33]. The higher order shear deformation theory was adopted by the authors. The finite element model was used in their work. The mechanical properties of the plate were assumed to vary continuously in the thickness direction by a simple power-law distribution. The free vibration analysis of plates made of functionally graded materials with an arbitrary gradient was carried out by Tounsi Abdelouahed et al. [34]. The four variable refined plate theory was used in their work. Material properties of the plate were assumed to be graded in the thickness direction according to a simple power-law distribution with an arbitrary gradient. The equations of motion for FG rectangular plates were obtained through Hamilton's principle. Hosseini-Hashemi.Sh et al. [35] were investigated free vibration analysis of moderately thick rectangular plates having two simply supported opposite edges. The Reissner–Mindlin plate theory was used to obtain equations of motion. The material properties were change continuously through the thickness of the plate vary according to a power law distribution. The free vibration of functionally graded (FG) arbitrary straight-sided quadrilateral plates rested on two-parameter elastic foundation and in thermal environment was carried out by BENI A. ALIBEYGI [36]. The first-order shear deformation theory (FSDT) was used by the authors. The material properties were assumed to be temperature-dependent and graded in the thickness direction. The large amplitude free flexural vibration analysis of shear deformable functionally graded material (FGM) plates was studied by Talha Mohammad et al. [37]. The higher order shear deformation theory was employed to obtain the equations of motion. Li. Q et al. [38] were investigated free vibration analysis of rectangular functionally graded material plates on different support conditions. The three dimensional linear theory of elasticity was adopted by the authors. The Ritz method was used to obtain the natural frequencies of functionally graded material plates. The free vibration analysis of functionally graded material (FGM) sandwich rectangular plates was studied by HADJI L et al. [39]. The four-variable refined plate theory (RPT) was used in their work. The plate properties were assumed to be varied through the thickness direction according to power law distribution. The free vibration analysis of thick functionally graded sandwich plates was carried out by Tounsi Abdelouahed et al. [40]. A new hyperbolic shear deformation theory was adopted by the authors in their work. The plate properties were assumed to be varied through the thickness direction according to the power law distribution. A n-order model for functionally graded and composite sandwich plate was developed by Xiang Song et al [41]. This model was used the n-order polynomial term to represent the displacement field. In their work, zero transverse shear stress boundary conditions at the top and bottom of the plate were satisfied by the authors. Liew K.M et al [42] were investigated free vibration analysis of metal and ceramic functionally graded plates. The local Kriging mesh less method was used to obtain the results. The first-order shear deformation theory and the local Petrov–Galerkin formulation were employed by the authors in their work. The free vibration analysis of functionally graded rectangular thick plates based on the Reddy's third-order shear deformation plate theory was carried out by Hosseini-Hashemi. Sh et al [43]. The elasticity modulus and mass density of the plate were assumed to vary according to a power-law distribution in terms of the volume fractions of the constituents. The frequency responses of functionally graded plate with surface-bonded piezoelectric layers under thermal, electrical and mechanical loads were investigated by Abdolreza Ohadi et al. [44]. The higher order shear deformation plate theory was employed in their work. The von Karman nonlinear strain–displacement relationship was used to account for the large deflection of the plate. The material properties of functionally graded material (FGM) were assumed temperature-dependent. The temperature field had uniform distribution over the plate surface and varied in the thickness direction only. The nonlinear vibrations analysis of simply supported FGM rectangular plates in thermal environments was investigated by Alijani F et al. [45]. The first-order shear deformation theory and von Karman theory were used to model simply supported FGM plates with movable edges. Thai Huu-Tai et al. [46] were used an efficient shear deformation theory for vibration analysis of functionally graded plates. The theory was accounted for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. The mechanical properties of functionally graded plate were assumed to vary according to a power law distribution of the volume fraction of the constituents. Misra R.K. [47] was used multiquadric radial basis function for free vibration analysis of isotropic plate. The least square error norm used this work. The spatial discretization of the differential equations generates greater number of algebraic equations than the unknown coefficients. The vibration analysis of functionally graded plate was carried out by Uymaz Bahar et al. [48]. The analysis was based on a five-degree-of-freedom shear deformable plate theory with different boundary conditions. The material properties were assumed to vary as a power form of the in-plane direction. The Ritz method and Chebyshev polynomials were used to obtain the solutions. The small- and large-amplitude vibrations of functionally graded rectangular plate resting on a two-parameter (Pasternak-type) elastic foundation in thermal environments were investigated by Hui-Shen Shen et al [49]. Two kinds of micromechanics models, namely, Voigt (V) model and Mori–Tanaka (M–T) model, were considered by the authors in their work. The motion equations were based on a higher order shear deformation plate theory that included plate foundation interaction. The thermal effects were also included and the material properties of functionally graded materials (FGMs) are assumed to be temperature-dependent. The vibration of functionally graded plates was carried out by Thai Huu-Tai et al [50]. The shear deformation theory was adopted by the authors in their work. The theory accounts for parabolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. The properties of functionally graded plate were assumed to vary according to a power law distribution of the volume fraction of the constituents. Atashipour S.R et al. [51] were studied both in-plane and out-of-plane free vibrations for thick functionally graded simply supported rectangular plates. The 3-D elasticity theory was used in their analysis. The 3-D finite element method was used to obtain the results. Wattanasakulpong Nuttawit et al.52] were investigated Thermo-elastic vibration response of functionally graded (FG) plates carrying distributed patch mass. The analysis was based on third order shear deformation theory. The solutions were obtained by using energy method. It was assumed that the volume fraction of material constituents varied across the plate thickness direction according to the power law distribution. Natarajan Sundararajan et al. [53] study the free vibration and the mechanical buckling of plates using a three dimensional consistent approach based on the scaled boundary finite element method. The in-plane dimensions of the plate were modeled by two-dimensional higher order spectral element. The material properties are assumed to be temperature independent and graded only in the in plane direction by a simple power law. Chakraverty .S et al. [54] were investigated free vibration of functionally graded (FG) rectangular plates subject to different sets of boundary conditions. The Classical or Kirchhoff's plate theories were used in their work. The Rayleigh–Ritz method was used to obtain the Eigen value equations. Material properties of FG plate were assumed to vary along thickness direction of the constituents according to exponential law. The objective was to study the effects of constituent volume fractions and aspect ratios on the natural frequencies. Natarajan . S et al [55] were studied the static bending, free vibration, and mechanical and thermal buckling behavior of functionally graded material (FGM) plates. The first-order shear deformation theory was adopted by the authors. The discrete shear gap method was used in this work. The energy equivalence principle was used to evaluate shear correction factors .The material property was assumed to be temperature dependent and graded only in the thickness direction. The effective properties were computed by using the Mori-Tanaka homogenization method. The nonlinear dynamic response and vibration of thick functionally graded material (FGM) plates were investigated by Duc Nguyen Dinh et al. [56]. The first-order shear deformation plate theory and stress function with full motion equations (not using Volmir's assumptions). The FGM plate was adopted by authors in this work. They were assumed to rest on elastic foundation and subjected to mechanical, thermal, and damping loads. The Runge–Kutta method was used to obtain numerical results for dynamic response of the FGM plate. The free vibration analysis of simply supported functionally graded double-layer plate was investigated by M.Mohammadimehr et al. [57]. The Hamilton's principle was used to obtain the governing equations of motion by applying a modified couple stress and von Karman nonlinear strain theories. The vibration analysis of FGM plate using different shear deformation theories was presented by Kumar S. [58]. The Hamilton principle was used to obtain governing equations of motion. Free vibration and thermal stability analyzes of functionally graded (FG) sandwich plates were carried out by using the advanced Hierarchical Trigonometric Ritz Formulation (HTRF). The Refined higher-order kinematics plate model used by Fazzolari Fiorenzo a [59]. The non-linear temperature distribution was given in different forms: (i) power law through-the-thickness variation; (ii) solution of the one-dimensional Fourier heat conduction equation; and (iii) sinusoidal. The effect of initial thermal stresses on the free vibration behavior of the FG sandwich plates was investigated. The free vibration analysis of functionally graded cracked plate was studied by Joshi P.V et al. [60].The Classical plate theory was used to obtain equations of motion. The moments due to thermal environment was neglected in the results and only uniform heating of the cracked plate was considered. The Galerkin's method was used to obtain solution for natural frequencies of cracked plate. The vibration analysis of orthotropic double-layered grapheme sheets under hygrothermal conditions was investigated by Sobhy Mohammed [61].The trigonometric shear deformation plate theory was adopted by the author. The nonlinear thermal stability analysis of perforated Ni/Al2O3 FGM plate was studied by Sharma Kanishk et al. [62]. The finite element approach was used in their work. The first-order shear deformation theory and von-Karman's nonlinear kinematics theories were used to obtain the displacement field and nonlinear frequencies for the system. The thermoelastic material properties of FGM plate were varied in the thickness direction according to the Mori-Tanaka homogenization scheme.

CONCLUSION

The vibration analysis of FGM rectangular plate is described in their article. The various 2D plate theories such as CPT, FSDT, TSDT and SSDT are employed for the vibration analysis of FGM plates. The gradients in material properties play an important role to determining the response of FGM plates. The main objective of this paper is to study the vibration analysis of FGM plate used in the design of them.

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