

# Review on Vibration Analysis of Functionally Graded Piezoelectric (FGPM) Actuators

Dr. Pankaj Sharma  
Assistant Professor  
Mechanical Engineering Department,  
Rajasthan Technical University, Kota, India  
*psharma@rtu.ac.in*

**Abstract-** The main objective of this work is to review the vibration behavior of FGPM actuators excited under the piezoelectric effects such as  $d_{31}$ ,  $d_{33}$  and  $d_{15}$ . The FGPM actuator incorporating different actuator geometries such as beam and annular plate is explored. The main aim of this paper is to serve the interests of researchers and engineers already involved in the analysis and design of FGPM actuators.

**Keywords-** Free vibration; Functionally Graded Piezoelectric Actuators, Beam, Annular Plates, Review

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## I. INTRODUCTION

Piezoelectric ceramics have found widespread use in actuators. They are made in the form of multilayer of either dissimilar piezoelectric materials or same piezoelectric material with different poling directions in order to get larger deflections. However, there are some drawbacks in traditional laminated smart structures such as they may crack at low temperature or peel off at higher temperature. Stress concentration may also appear at the interface due to abrupt change in the material properties which may deteriorate the performance of the system [1]. All the above shortcomings reduce the life-time and reliability of smart devices. Therefore, a new kind of piezoelectric materials with graded properties have been developed to overcome such drawbacks and are termed as functionally graded piezoelectric materials (FGPM) [2]. In the design of FGPM, volume fraction of constituents varies according to different mathematical functions such as power law, exponential law and sigmoid law. It helps to overcome the drawbacks of multilayered stacked actuators. Now days, the engineering applications of FGPMs are being explored in several areas such as ultrasonic motors, atomic force microscope (AFM) and biomedical science.

## II. FGPM BEAM EXCITED UNDER $d_{31}$ and $d_{33}$ EFFECT

The literature review of vibration analysis of functionally graded piezoelectric material (FGPM) structures is described in this section. Initially the literature is reviewed on the basis of type of structure i.e. beam and annular plate. In the later section the literature on the analysis of shear effect ( $d_{15}$ ) is discussed. Beam structures are the fundamental structure elements which are extensively used in a wide range of engineering applications such as electro-mechanical systems, control systems and aerospace systems. The vibration analysis of functionally graded piezoelectric material (FGPM) beams can be divided in two parts:

- (i) Macroscopic structures which are based on size-independent classical continuum theories
- (ii) Micro/Nano structures which are based on size-dependent classical continuum theories.

## A. ON THE BASIS OF SIZE-INDEPENDENT CLASSICAL CONTINUUM THEORIES

The static and dynamic performance of an functionally graded piezoelectric material (FGPM) monomorph structure was investigated by Li et al. [3]. The electrophoretic deposition method was adopted to make FGPM structure. Qiu et al. [4] developed the fabrication process of functionally graded piezoelectric (FGP) bending actuators and presented the vibration characteristics of FGPM actuators. Takagi et al. [5] used powder processing method to develop the PZT/Pt FGPM actuators. They studied bending displacement characteristics using classical lamination theory. Li et al. [6] reported the behavior of functionally graded piezoelectric material monomorph fabricated using electrophoretic deposition method in both static and dynamic studies.

Shindo et al. [7] presented dynamic finite element analysis of functionally graded piezoelectric actuators under alternating current electric fields. Yang and Xiang [8] employed generalized differential quadrature (GDQ) method to study the static and dynamic response of FGPM beam. In their study, Timoshenko beam theory was used. The material properties were assumed to have a power law variation across the beam thickness. The effect of slenderness ratio and boundary conditions on natural frequency was investigated in detail.

Yang and Zhifei [9] carried out the free vibration analysis of piezoelectric beam with graded properties using state-space based differential quadrature (SSDQM) method. The theory of elasticity was employed in their study. The influence of material parameters on the natural frequency of the FGPM beam was reported.

Armin et al. [10] investigated the static bending, free vibration and dynamic analysis of FGPM beam using finite element (FEM) method. In their work Euler-Bernoulli beam theory (EBT) was adopted. Material constants of the beam were assumed to vary continuously across the thickness in accordance with power law distribution.

Dynamic analysis of FGPM beam using hybrid Fourier-Laplace transformation method was investigated by Doroushi et al. [11]. The governing equations of motion were derived using higher-order shear deformation theory. The effects of material composition, aspect ratio and shear deformation on

the transient response were examined. Free and forced vibration response of FGPM beam using finite element method was reported by Doroushi et al. [12]. In their work Reddy's higher deformation theory was incorporated to account the effect of rotary inertia and transverse shear deformation across the beam thickness. They examined the effect of power index and slenderness ratio on natural frequency under different end conditions. In both studies, the material properties were assumed to vary continuously in thickness direction in accordance with power law distribution.

Yao and Shi [13] studied forced vibration of FGPM beam under different cases (i.e. electrical, mechanical and electro-mechanical) using state-space based differential quadrature (SSDQM) method. They observed that the performance of the FGPM beam is greatly depend on volume fraction index. Power law rule was used to characteristics the material constants.

The free vibration analysis of FGPM beam using finite element technique which was based on refined sinus beam mode was presented by Lezgy-Nazargah et al. [14]. This beam mode does not require shear correction factor. The material properties were assumed to vary continuously according with exponential law distribution.

Komijani et al. studied the small free vibration of FGPM beams in pre-buckling and post buckling regimes [15]. The equations of motion were obtained using Ritz-based finite element method. The governing equations were solved using Newton-Raphson method. The non-linear thermal stability analysis of FGPM beam using Ritz-based finite element technique was adopted by Komijani et al. [16]. In their study Timoshenko beam theory was used and von-Karman type non-linearity was assumed. The Newton-Raphson technique was used to solve the governing equations.

Design study of PZT-Pt functionally graded piezoelectric material (FGPM) bimorph actuator was carried out by Patel and Vaish [17]. In their work, the FGPM's parameters were optimized based on their first resonance frequency. Yang et al. [18] presented non-linear vibration analysis of FGPM beam with graded properties. Their formulation was based on von-Karman kinematic relationship and Timoshenko beam theory was used to consider the transverse shear deformation effect. The material constants were assumed to vary continuously in thickness direction in accordance with power law rule.

Jin and Su [19] gave exact solution for free vibration analysis of FGPM beam under arbitrary end conditions. The Timoshenko beam theory was used and material constants were varied according to power law. Pandey and Parashar [20] investigated the static bending and free vibration analysis of FGPM beam under electro-mechanical loading. The material properties were assumed to have a sigmoid law variation in thickness direction. The results were obtained through finite element modeling using COMSOL multiphysics software. A unified solution for vibration analysis and transient response of a FGP curved beam using Timoshenko beam theory is reported by Su et al. [21]. The material properties are assumed to vary across the beam thickness according to voigt's rule of mixture.

### B. ON THE BASIS OF SIZE-DEPENDENT CLASSICAL CONTINUUM THEORIES

In recent years, piezoelectric materials are being considered in micro-electro-mechanical and nano-electro-mechanical systems such as nanogenerators [22], field effect transistors

[23], piezoelectric gated diodes [24], gas sensors [25], nanowire resonators and oscillators [26]. These materials have also size dependent properties [27]. The mechanical and electrical properties of piezoelectric materials have been investigated by experimental techniques. In addition to the experimental studies, atomistic simulations are employed to predict the size-dependent properties. Although atomistic simulation are computationally expensive so size dependent continuum theories have been needed to study the properties of piezoelectric materials used in MEMS and NEMS. The commonly-used size dependent continuum theories are couple stress theory [28], nonlocal elasticity theory [29], strain gradient theory [30] and surface elasticity model [31].

Komijani et al. [32] reported the stability and free vibration of microstructure dependent FGPM beam in pre/post-buckling regimes. The Timoshenko beam theory with von-Karman non-linearity was incorporated. The modified couple stress theory was employed to include the size effect. The nonlinear vibration analysis of FGP beams with or without microstructure-dependency is presented by Komijani and Gracie [33]. The modified couple stress theory is used to account for size effect. The finite element modeling is employed to obtain the equations of motion. The available governing equations are solved by Newton's method. The material properties are assumed to have continuous variation in thickness direction according with power law distribution. Li et al. [34] modeled size-dependent functionally graded piezoelectric beam used in NEMS and MEMS. The theoretical formulation was based on the modified strain gradient theory and Timoshenko beam theory. A power law distribution across the beam thickness was assumed.

Ebrahimi et al. [35] reported vibration behavior of functionally graded piezoelectric nanobeams under electrical and thermal loading. In their work, Eringen's nonlocal elasticity theory with Euler-Bernoulli beam theory was employed. The equations of motion were obtained by using Hamilton principle. The governing equations were solved by Navier technique. The material properties were assumed to vary according to power law and sigmoid law distribution. Beni [36] developed nonlinear size-dependent formulation for FGPM Euler-Bernoulli beam. The non-linearity arised due to mid-plane stretching was taken into consideration.

### III. FGPM BEAM EXCITED UNDER $d_{15}$ EFFECT

It has been observed from literature survey that in most of the FGPM structures (beam), the direction of polarization and electric field are taken along the thickness direction for actuation (i.e.  $d_{31}$  effect). In piezoceramics, when the electric field is applied perpendicular to the direction of initial polarization, it induces the vibrations of the piezoceramic due to shear effect (i.e.  $d_{15}$  effect). As discussed earlier that in piezoelectric actuators, the shear effect is more attractive for engineering applications since piezoelectric coupling coefficient for shear  $d_{15}$  is much higher than the other piezoelectric coupling coefficients. Parashar and Sharma [37] investigated modal analysis of shear induced flexural vibration of FGPM beam. In their work, modified Timoshenko beam theory was employed. The Hamilton principle was used to obtain the equations of motion and available partial differentiation governing equations were converted into linear algebraic equations using Generalized Differential Quadrature

(GDQ) method. The effects of aspect ratio, boundary conditions and power index on natural frequency were also examined. The exact solution of vibration analysis of FGPM beam excited under the shear effect was reported by Sharma et al. [38].

#### IV. FGPM CIRCULAR/ANNULAR PLATE EXCITED UNDER $d_{31}$ and $d_{33}$ EFFECT

An exact three-dimensional solution for free vibration analysis of functionally graded piezoelectric circular plate was investigated by Zhang and Zhong [39]. In their work, the finite Hankel transformation was used to obtain the state-space equations of functionally graded piezoelectric (FGP) circular plate. Material properties were varied continuously along the thickness of plate in accordance with exponential law distribution.

Ebrahimi et al. [40] presented nonlinear vibration analysis of piezoelectric circular plate with graded properties. The Kirchhoff-Love theory with von-Karman nonlinearity was employed by authors. Based on three-dimensional theory of elasticity, Jam and Nia [41] reported the free vibration analysis of functionally graded piezoelectric annular plate using state-space method.

Yas et al. [42] employed state-space based differential quadrature (SSDQM) method to study the free vibration analysis of FGP annular plate resting on elastic foundation under clamped-clamped and simply supported-clamped end conditions. Jodaei et al. [43] carried out the free vibration analysis of functionally graded piezoelectric (FGPM) annular plate using three-dimensional theory of elasticity under different end conditions. A state space based differential quadrature method and an artificial neural network (ANN) method was employed by authors. The material properties were varied with exponential law rule across the thickness of plate [42, 43].

Dai et al. [44] used state space method to investigate free vibration analysis of FGPM circular plate placed in a uniform magnetic field. The electrical and mechanical properties were function of thickness of plate and were varied by exponential law distribution.

Yas and Moloudi [45] employed state-space based differential quadrature method to study the free vibration analysis of a multi-directional functionally graded piezoelectric (FGP) annular plate. The exponential law rule was used to characterize the material parameters.

#### V. FGPM CIRCULAR/ANNULAR PLATE EXCITED UNDER $d_{15}$ EFFECT

Free vibration analysis of FGPM annular plate excited under the shear effect is presented by Sharma et al. [46]. The modified Mindlin plate theory was employed and material properties are assumed to vary across the thickness in accordance with power law distribution. The Hamilton principle is used to obtain the equations of motion. The GDQ method is used to obtain the natural frequencies of the plate.

#### VI. CONCLUSION

The paper shows the comprehensive review on vibration analysis of FGPM actuators. The two types of actuator geometries i.e. beam and annular/circular plate are discussed. The main aim of this paper is to serve the interests of

researchers and engineers already involved in the analysis and design of FGPM actuators.

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