A Review on Various Types of Probability Distribution

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Abstract— Need of mathematics is important in modern life in applications of physics, engineering and research field. Probability distribution is one of the most important part of statistics as well as mathematics. It is helpful in solving various problems in computer science, business and in daily life. In this paper we will discuss about the various types of probability distribution which are important in daily life, engineering and technology and in agriculture field. A probability distribution is a list of all of the possible outcomes of a random variable along with their corresponding probability values and Probability distribution comes in many shapes with different characteristics. This review paper gives a survey on various types of probability distribution. The solution on studies, allows to suggest the use of these types to model their research problem mathematically and to find the solution.

Keywords- *Probability distribution structure, service life.*

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INTRODUCTION

This paper deals with a brief overview of Probability distribution and its various types. The probability is a widely used part of mathematics with many important applications in mathematics, physics, engineering agriculture field any in daily life also. Probability distributions describe the probability of observing a particular event or chances. There are various probability distributions that are important to physicists. The binomial, while not of much practical significance, is easy to describe, and can be used to derive the other distributions used most often by experimental physicists: the Gaussian and Poisson distributions. The normal distribution is the most important as it is most often used to describe the distribution of results distribution for any measurement subject to small random error. The Poisson distribution is particularly useful in describing continuous random experiments. The fourth distribution is exponential distribution or interval distribution, describes the distribution of intervals between counting events. In this paper, you will investigate the Poisson and interval distributions.

Probability distributions and models are commonly used in m-any applied areas, such as economics, engineering, social, health, and biological sciences. Indeed, probability distributions are used to model a wide range of practical problems, from modeling the size grade distribution of onions to modeling global positioning data.

Successful applications of these probability models require a thorough understanding of the theory and familiarity with the practical situations where some distributions can be postulated. Although there are many statistical software packages available to fit a probability distribution model for a given set of data, none of the packages is comprehensive enough to provide table values and other formulas for numerous probability distributions. The main purpose of this paper is to provide users with quick and easy access to table values, important formulas, and results of the many commonly used, as well as some specialized, statistical distributions. This paper intended to serve as reference materials. With practitioners and researchers in disciplines other than statistics in mind, we have adopted a format intended to make it simple to refer the paper for reference purposes. Examples are provided mainly for this purpose.

PROBABILITY

Probability is a measure of the likelihood that an event will occur in a Random Experiment. Probability is quantified as a number between 0 and 1, where, we can say, 0 indicates uncertainty and 1 indicates certainty. The higher the probability of a trail, the more likely it is that the trail will occur. For Example: While tossing a fair (unbiased) coin, there is a possibility of occurrence of two outcomes (“heads” and

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“tails”), which are equally probable; i.e. the probability of “Heads & Tails” are equal. The probability of either “Heads” or “Tails” is half (which could also be written as 0.5, 1/2 or 50%).

Before proceeding further we should be aware of the basic terms like:

Random Experiment:

A random experiment is a physical situation whose outcome cannot be predicted with certainty.

Sample Space

The sample space (denoted S) of a random experiment is the set of all possible outcomes.

Sample space: {Head, Tail}

As we got a little understanding of Probability, we will now read about Probability Distribution and its types with the help of examples and formulas wherever required.

Distribution

In statistics theory when we use the term Distribution it usually means Probability distribution.

A Distribution is a function that shows the possible values for a variable and how often they occur.

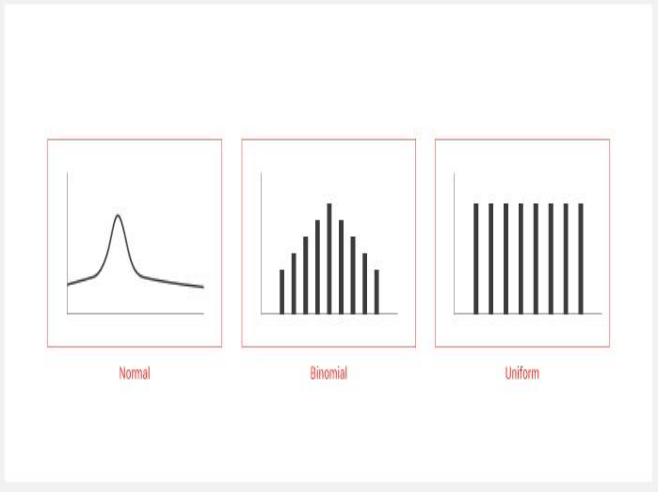
Or A Probability Distribution is a mathematical function that can be thought of as providing the probabilities of occurrence of different possible outcomes in an experiment.

Good examples are the

Normal Distribution

Binomial Distribution

Uniform Distribution



The above image shows the three distribution respectively.

Types of Probability Distribution

There are many different types of probability distribution. Some of them that we will be covering in this blog is listed below:

(1) Normal Distribution

(2) Bernoulli’s Distribution

(3) Binomial Distribution

(4) Uniform Distribution

(5) Student’s T Distribution

(6) Poisson distribution

Each probability distribution has a visual representation. It is a graph that describes the likelihood of occurrence of every event. The graph is just a visible representation of a distribution.

1. Normal Distribution

The visual representation of Normal Distribution has already been seen above in the blog.

The Normal Distribution is a very common continuous probability distribution. This type of distributions are important in statistics and are often used to represent random variables whose distribution is not known.

The statistical term for this type of distribution is Gaussian distribution though many people call it

(1) The Normal curve is in the “Bell Shape”

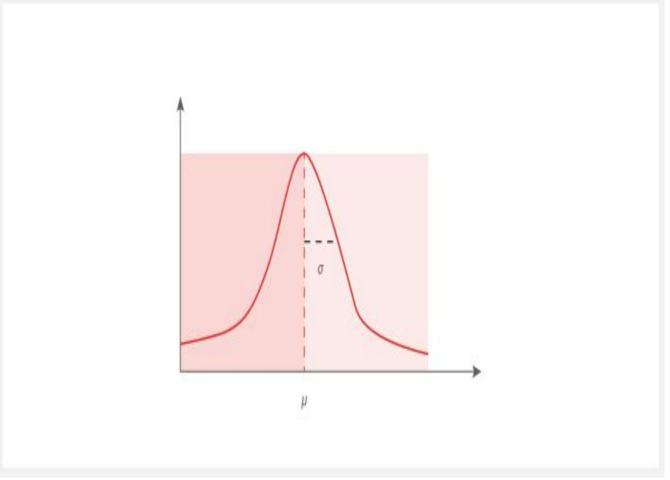
(2) This type of distribution is symmetric about highest Ordinate and its mean, median and mode are equal. Because of symmetry, the area from $z = 0 \; to \; z = \infty$ is equal to the area from $z = -\infty \; to \; z = 0$ is equal to 0.5

(3) Its total area is unity i.e. total area under the normal curve from $z = -\infty \; to \; z = \infty \; is \; 1$

Mathematically, Normal Distribution is represented as:

N~ (μ, σ²) $N \sim (\mu, \sigma^2)$

Where N stands for Normal, symbol ~ stands for distribution, symbol μ stands for mean and σ^2 stands for variance.



In the above image, we can see the highest point is located at the mean μ and the spread of the graph is determined by the standard deviation σ.

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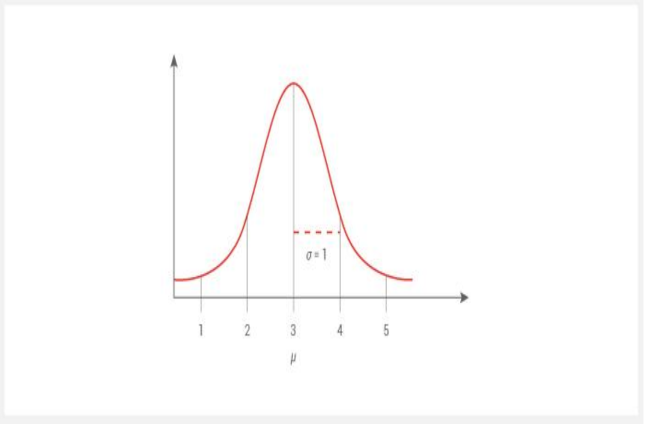
Let us understand this with the simplest example where we have a random variable X with distribution:

X = {1, 2, 3, 4, 5}

When we take the mean and standard deviation of the above data set we get mean (μ) = 3 and standard deviation (σ) = 1. When we plot it, we get some distribution like this:



This Bell curve specifies the Normal Distribution.

Note: Not all but more than 70% of the data distribution usually follows this pattern.

When we talk about Normal Distribution we have often heard the term Empirical Formula. What exactly does this formula states, well is what we will be covering next.

1.1 Empirical Formula

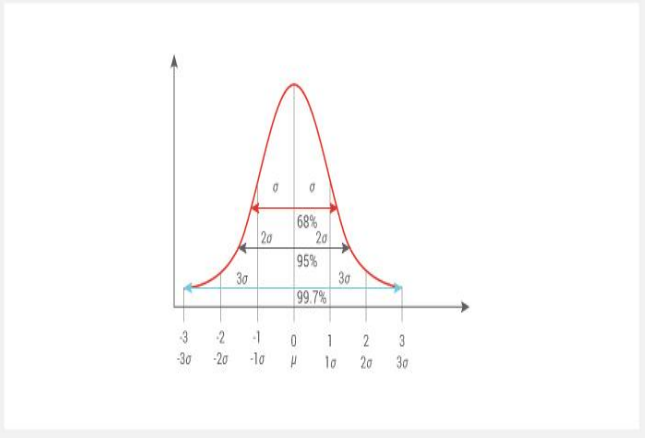
The empirical rule states that for a Normal Distribution, nearly all of the data will fall within three range of standard deviations of the mean. The empirical rule can be understood when broken down into three parts:

68% of the data falls within the first standard deviation from the mean.

95% fall within two standard deviations.

99.7% fall within three standard deviations.

We can understand this with the help of the below image.



Approximately 68% of the data falls within one standard deviation of the mean (i.e., between the mean minus (-) one times the standard deviation, and the mean + 1 times the standard deviation). In mathematical notation, this is represented as $\mu \pm 1\sigma$

Approximately 95% of the data falls within two standard deviations of the mean (i.e., between the mean (–) 2 times the standard deviation, and the mean + 2 times the standard deviation). The mathematical notation for this is: $\mu \pm 2\sigma$

Approximately 99.7% of the data falls within three standard deviations of the mean (i.e. between the mean (–) three times the standard deviation and the mean + three times the standard deviation). The following notation is used to represent this fact: $\mu \pm 3\sigma$

The rule is also called the **68-95-99.7** Rule or the Three Sigma Rule.

The Empirical Rule is often used in statistics for forecasting, especially when obtaining the right data is difficult or impossible to get. The rule can give you a rough estimate of what your data collection might look like.

1.2 Standard Normal Distribution

When a Normal Distribution is standardized, the result is called a Standard Normal Distribution.

Understanding Standardization in the context of statistics. Every distribution can be standardized. Let say if the mean and the variance of a variable are μ and σ^2 respectively.

Standardization is the process of transforming a variable to one with a mean of 0 and a standard deviation of 1.

i.e., **$\sim (\mu, \sigma^2) \rightarrow \sim (0, 1)$**

When a Normal Distribution is standardized, the result is called a Standard Normal Distribution.

i.e., **$N \sim (\mu, \sigma^2) \rightarrow \sim N(0, 1)$**

We use the following formula for standardization:

$$y = \frac{1}{\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \text{where } z = \frac{x-\mu}{\sigma}$$

Where x is data element or abscissa, μ is mean and σ is the standard deviation

We use the letter Z to denote standardization. The standardized value i.e., Z is known as the z-score.

These Z scores are important because they tell you how far a value is from the mean. When you standardize a random variable, its ‘mean’ becomes 0 and its standard deviation becomes 1.

If the Z score of x is zero, then the value of x is equal to the mean.

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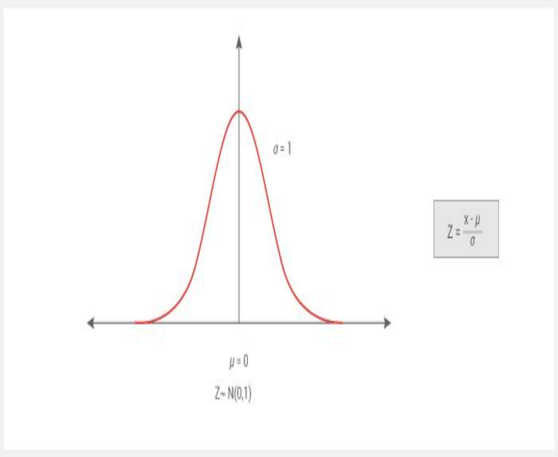
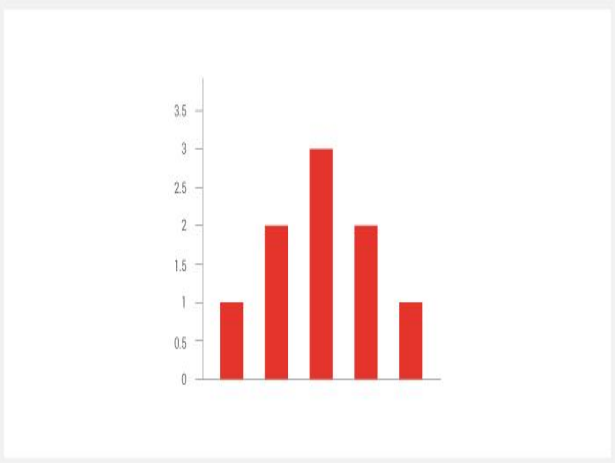


Let us understand the steps involved in Standardization with the help of a simple example.

Suppose we have a dataset with elements

X = {1, 2, 2, 3, 3, 3, 4, 4, 5}

And Uniformly Distributed as:



We get mean as 3, variance as 1.49 and std dev as 1.22 i.e., N ~ (3, 1.49).

Now we will subtract the mean from all data points, i.e., **x – μ.**

We will get a new data set as below:

X1 = {-2, -1, -1, 0, 0, 0, 1, 1, 2}

Now we get mean as 0, but variance and std dev still as 1.49 and 1.22 respectively i.e., N ~ (0, 1.49)

So far we have a new distribution but it is still normal and needs to standardize.

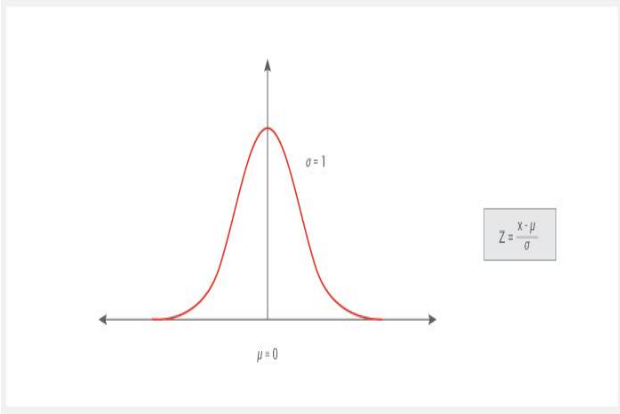
So the next step of standardization is to divide all the data points by the standard deviation, i.e., (x – μ)/**σ.**

Dividing each data point by 1.22(std dev) we get a new data set as:

X2 = {-1.6, -0.82, -0.82, 0, 0, 0, 0.82, 0.82, and 1.63.}

Now if we calculate the mean we get as 0 and standard deviation as 1 i.e., N ~ (0, 1)

Plotting it on a graph we get something like this



This is how we can obtain Standard Normal Distribution from any normally distributed dataset.

Using this standardized normal distribution makes inferences and predictions much easier.

1.3 Probability Density Function and Probability Mass Function

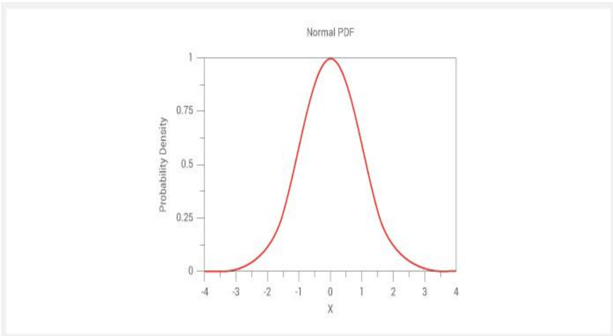
Probability density function and Probability mass function is a statistical expression that defines a Probability Distribution for a random variable.

Do not get confused between the two terms. Probability density function (PDF) is used to determine the probability distribution for a Continuous Random Variable. When the PDF is graphically plotted the area under the curve indicates the interval in which the variable will fall.

Whereas the Probability Mass Function (PMF) is used to determine the probability distribution for a Discrete Random Variable.

As we know Continuous Random Variables are the one which takes an infinite number of possible values e.g.: the weight of a person can be 50.2, 44.5, 60.7, etc. and Discrete Random Variables are the one which may take on only a countable number of distinct values such as 0, 1, 2, 3, 4….

If we know the mean and variance of our dataset we can compute the PDF and PMF. PDF and PMF tell how well our data has been distributed with respect to the mean and standard deviation within a particular curve.



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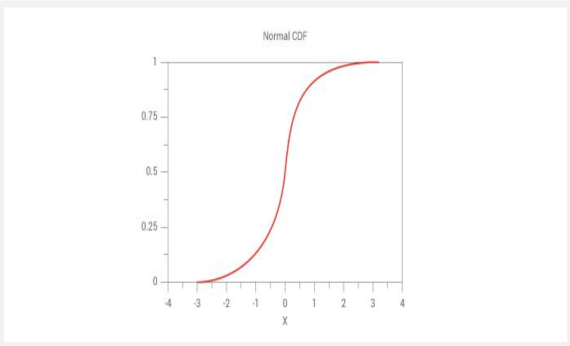
1.4 Cumulative Density Function

The cumulative distribution function (CDF) of a random variable is another method to describe the distribution of random variables.

The cumulative frequency is the sum of the relative frequencies. It starts at the frequency of the first brand, then we add the second, the third and so on until it finishes at 100%.

The advantage of the CDF is that it can be defined for any kind of random variable (discrete, continuous, and mixed).



2. Binomial Distribution

This type of distribution is used when there are exactly two outcomes of a trial. These outcomes are labeled as "happening" and "not happening".

Here the probability of both the outcomes is the same for all the trials.

Each trial is independent since the outcome of the previous toss doesn't determine or affect the outcome of the current toss. An experiment with only two possible outcomes repeated n number of times is called binomial. The parameters of a binomial distribution are n, p and q where n is the total number of trials and p is the probability of success in each trial and q is probability of failure in each trial.

We have already seen the graph representing Binomial Distribution above.

We define Binomial Distribution with the below formula:

Probability of r successes in n trials

$$\mathbf{P_{(x=r)} = {}^{n}C_{r}\, p^{r}\, q^{n-r}}$$

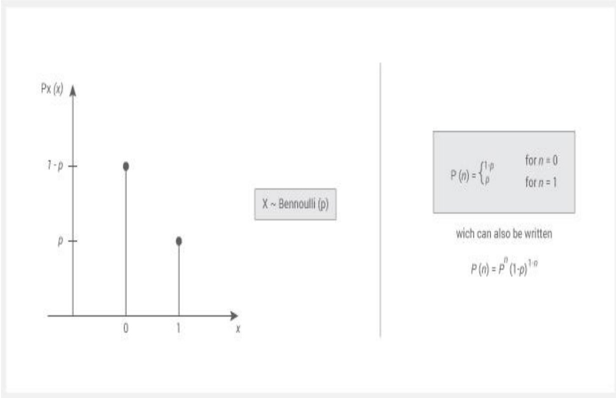
3. Bernoulli's Distribution

Binomial Distribution is closely related to Bernoulli's Distribution.

Bernoulli distribution is a special case of Binomial Distribution with a single trial.

The Bernoulli distribution is a discrete distribution having two possible outcomes that is, 0 and 1, where n = 1 (usually called a "success") occurs with probability p and n = 0 (usually called a "failure") occurs with probability q = 1 – p, where $0 < p < 1$.

Therefore the probability density function (pdf) and the graph for Bernoulli's Distribution is shown in the figure below:



In the above graph, 1 refers to success and 0 specifies the failure.

The head and tail distribution in coin tossing is an example of Bernoulli's Distribution with p = q = ½.

4. Uniform Distribution

A uniform distribution, sometimes known as rectangular distribution, is a distribution that has a constant probability. The notation for uniform distribution is unif (a, b) or U (a, b) where the parameters $-\infty \leq a \leq b \leq \infty$.

We have already seen the graphical representation of uniform distribution above. Let us understand this with the help of an example.

EXAMPLE: If we roll a die (numbered from 1 to 6), then the probability of getting 1 is one out of six i.e., 1/6

Similarly, the probability of getting 2, 3, 4, 5 and 6 also is ⅙. There is an equal chance of getting each of the 6 outcomes.

Now, if we check for the probability of getting 7, then it is 0 since it is impossible to get a 0 when rolling a die.

For the probability of outcomes for 1 to 6, we have an equal chance of occurrence and this is what we call a Discrete Uniform Distribution.

Remember that the sum of their probabilities is equal to 1 or 100%.

5. Student's T-Distribution

T Distribution or Student's T Distribution is a probability distribution that is used to estimate population parameters when the sample size is small and/or when the population variance is unknown. The *t*-distribution plays a role in a number of widely used statistical analyses, including Student's *t*-test for assessing the statistical significance of the difference between two sample means, the construction of confidence intervals for the difference between two population means, and in linear regression analysis. The Student's *t*-distribution also arises in the Bayesian analysis of data from a normal family.

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Visually, the Student's T distribution is symmetric and bell shaped much like a Normal distribution but generally has fatter tails. Fatter tails, allow for a higher dispersion of variables, as there is more uncertainty.

As the z-statistic is related to the standard Normal distribution, the t-statistic is related to the Student's T distribution.

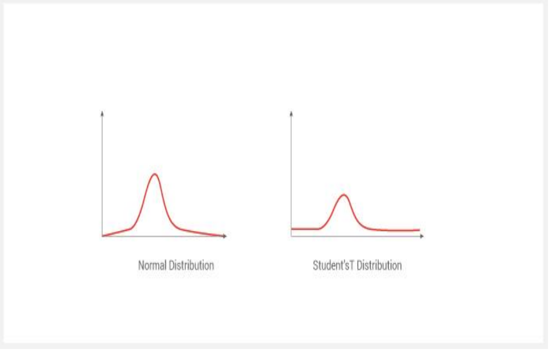
The formula that allows us to calculate it is:

$$t = [\bar{x} - \mu] / (s / \sqrt{n})$$

t with n-1 degrees of freedom equals the sample mean minus the population means, divided by the standard deviation of the sample by n which refers to the sample size.

The degrees of freedom refers to the number of independent observations in a set of data**.**

Now we will see the graph for Student's T Distribution and will also see how it is different from Normal Distribution.



Why use T-Distribution?

According to the Central Limit Theorem, the distribution follows Normal Distribution when the sample size is sufficiently large. Here we know the standard deviation and can calculate the z-score and can plot the Normal Distribution.

But sometimes the sample sizes are small and also we do not know the standard deviation of the population. This is where statistician prefer on the distribution of T-Distribution (also known as t-score).

6. Poisson Distribution

The Poisson distribution is a discrete probability distribution which states that the number of events occurring in a fixed interval of time or space conditionally that the value of an average number of occurrence of the event is known.

For instance, if the average number of diners for seven days is 500, we can predict the probability of a certain day having more customers.

Another example is if a call center gets 30 calls in an hour, we can predict the probability that he received no calls in the first 3 minutes. Among patients admitted to the intensive care unit of a hospital, the number of days that the patients spend in the ICU is not Poisson distributed because the number of days cannot be zero and so many more examples.

The Poisson distribution results from a Poisson's Experiments which states that for a series of discrete event where the average time between events is known, but the exact timing of events is random.

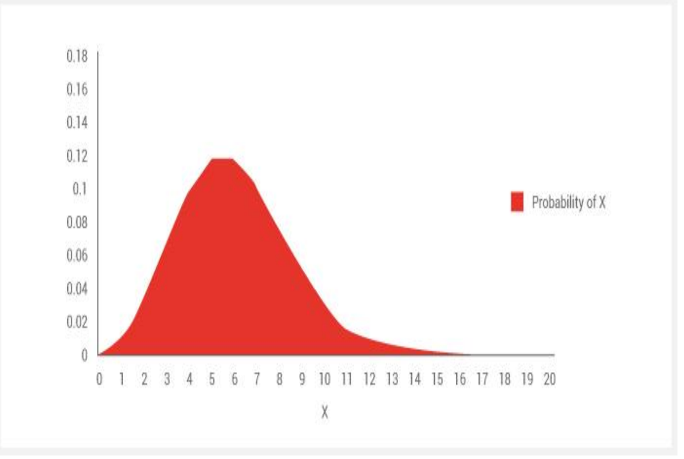
If we conduct a Poisson experiment, in which the average number of successes within a given region is μ.

Then, the Poisson probability is:

$$P(x;\mu) = (e^{-\mu}) (\mu^{x}) / x!$$

Where x is the result of actual number of successes in the experiment, and e is approximately equal to 2.71828.

The graph of Poisson distribution is as shown below:



The mean and variance of x follows Poisson distribution:

Mean → E (x) = μ Variance → Var(x) = μ

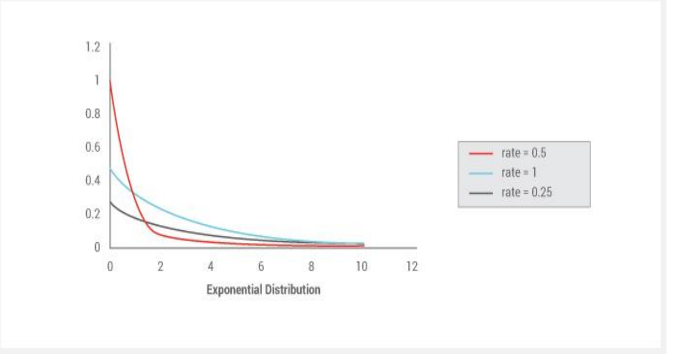
7. Exponential Distribution

Exponential Distribution is one of the most widely used continuous distributions. It measures the expected time of an event to occur.

The exponential distribution is very useful for survival analysis purposes. An example of an exponential distribution is the lifespan of a machine.

It basically answers our query as to how much time do we need to wait before a given event occurs.

The graph of Exponential Distribution is shown below:



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CONCLUSIONS

This paper has presented the basic concepts of probability distribution by emphasizing motivation, use, and interpretation rather than formal arguments. Probability distribution is important in the social sciences because it gives specified observation in terms of an underlying sample space, which is the set of all possible outcomes of the random phenomenon being observed.

In probability theory, a probability distribution is a mathematical function that provides the probabilities of occurrence of different possible outcomes in an experiment. We hope this paper helped you in learning Probability Distribution.

REFERENCE

[1] Cooke, R. M. (1991). Experts in Uncertainty: Opinion and Subjective Probability in Science. New York: Oxford University Press.

[2] Field Guide to Continuous Probability Distributions, Gavin E. Crooks

[3] B. S. Everitt: The Cambridge Dictionary of Statistics, Cambridge University Press, Cambridge (3rd edition, 2006). ISBN 0-521-69027-7

[4] Barbour, A.D. and Eagleson, G.K. (1983), Poisson approximation for some statistics based on exchangeable trials. Adv. Appl. Prob. 15, 585-600.

[5] Bhakar S. R., Iqbal M., Mukesh Devanda, Neeraj Chhajed and Bansal A. K. (2008) Journal of Agricultural Research, 42, 201-206

[6] Upadhaya A. and Singh S. R. (1997) Indian Journal of Soil Conservation, 26, 193-201

[7] Billingsley, P. (1995), Probability and Measure, 3rd edition. Wiley, New York.