_____

Fuzzy Automata: A Quantitative Review

Rahul Kumar Singh

Department of Computer Science

SLIET, Longowal, Punjab, India

rahulsinghcse25@gmail.com

Akshama Rani

Department of Computer Science

Thapar University

shama1mittal@gmai.com

Manoj Kumar Sachan

Department of Computer Science

SLIET, Longowal, Punjab, India

manojsachan@gmail.com

Abstract:- Classical automata theory cannot deal with the system uncertainty. To deal with the system uncertainty the concept of fuzzy finite automata was proposed. Fuzzy automata can be used in diverse applications such as fault detection, pattern matching, measuring the fuzziness between strings, description of natural languages, neural network, lexical analysis, image processing, scheduling problem and many more. In this paper, a methodical literature review is carried out on various research works in the field of Fuzzy automata and explained the challenging issues in the field of fuzzy automata.

Keywords:- *Deterministic fuzzy automata, Nondeterministic fuzzy automata, Fuzzy regular expression, Lattice – ordered monoids.*
_____*****_____

1. INTRODUCTION

Fuzzy automata are used for handling the system uncertainty problem because classical automata cannot deal with system uncertainty. Fuzzy automaton is a generalized version of nondeterministic finite automaton. Fuzzy automata can be used in many areas such as neural network, description of natural and programming language, artificial intelligence, learning system, controlling system, database, system modeling, Pattern matching, searching, script recognition[40,41], database, lexical analysis, string matching, and many more application [32], [3], [29], [38], [27]. The concept of fuzzy automata has been introduced by Santos in 1960's [31]. In 1969, Wee and Fu [36] introduced the mathematical representation of fuzzy finite automata. The concept of fuzzy language has been proposed in 1969 [17]. Many researches carried out the preparatory work on fuzzy automata and their properties [24], [28], [39], [7], [19]. In fuzzy automata, membership values play a pivotal role. We can determine the membership values using different types of lattices such as lattice–ordered monoid, complete lattice, complete residuated lattice and using other algebraic structure. Fuzzy automata work well where classical automata theory fail. In fuzzy automata, we can use compositional methods such as min – max automaton, max – min automaton and max – product automaton for determining the fuzzy language. In classical automata theory, either string is accepted or rejected, whereas in fuzzy automata string is accepted with certain membership value. In the next section, we explain the preliminaries of fuzzy automata.

2. PRELIMINARIES

In this section, some basic definition deterministic fuzzy automata, nondeterministic fuzzy automata, and fuzzy regular expressions are described.

2.1 Deterministic Fuzzy Automaton

Deterministic fuzzy automaton [25], [31] M consists of quintuple $M = (Q, \Sigma, \delta, q_0, F)$, Where

- Q is a finite non – empty set of states.
- Σ is finite non – empty set of input alphabets.
- $\delta : Q \times \Sigma \times Q \rightarrow [0, 1]$ is a fuzzy transition function. Where [0, 1] represent the interval.
- $q_0 \in Q$, where q_0 is fuzzy initial state.
- $F \subseteq f(Q)$ is the fuzzy subset of final states.

Fuzzy transition function δ takes input alphabet Σ on the present state and returns with membership value on states Q.

Suppose $q_0, q_1 \in Q$ and $a \in \Sigma$, then the fuzzy transition function defined as $\delta (q_0, a) (q_1)$, state q_0 takes input a and reached state q_1 with membership value and $f(Q)$ represents the set of all fuzzy subset of Q. In deterministic fuzzy automata choice (in term of states) and ε - moves are not allowed.

2.2 Nondeterministic Fuzzy Automaton

Nondeterministic fuzzy automata [7] consist of five tuple $M = (Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite non – empty set of states.
- Σ is finite non – empty set of input alphabets.
- $\delta : Q \times \Sigma \times \rightarrow 2^{f(Q)}$ is a fuzzy transition function. Where $2^{f(Q)}$ is a power set and $f(Q)$ represents the set of all fuzzy subset of Q.
- $q_0 \in Q$, where q_0 is fuzzy initial state.
- $F \subseteq f(Q)$ is the fuzzy subset of final states.

Q, Σ, q_0, F are similar to the deterministic fuzzy automata and fuzzy transition function of nondeterministic fuzzy automata is different from deterministic fuzzy automata. The main deference between nondeterministic fuzzy automata and deterministic fuzzy automata is in fuzzy transition function. In nondeterministic fuzzy automata choice (in term of states) are allowed whereas in deterministic fuzzy automata choice are not allowed.

2.3 Fuzzy Regular Expression

Fuzzy regular expressions (*FRE*) [32], [18] play vital role in the string matching, pattern matching, and searching. The family of *FRE* over input alphabet Σ is defined as:

- $r \in FRE, \forall r \in \Sigma;$

_____

_____

- $(r^*) \in FRE$, $\forall$ $r \in RE$ (star operation);
- $(r_1 + r_2) \in FRE$, $\forall$ $r_1, r_2 \in FRE$ (addition);
- $(r_1\, r_2) \in FRE$, $\forall$ $r_1, r_2 \in FRE$ (concatenation);
- $(\lambda\, r) \in FRE$, $\forall$ $\lambda \in$ L and $r \in FRE$ (scalar multiplication);
- $\varepsilon \in FRE$;
- $\phi \in FRE$;

In the family of fuzzy regular expression, there are only seven steps that are defined above and star operation has a higher priority than concatenation and addition.

The language of fuzzy regular expression is defined as $L(r)$.

 i. $L(\emptyset)(w) = 0$, $\forall$ $w \in \Sigma^*$;
 ii.   For $r \in \Sigma \cup \{\varepsilon\}$, $L(r) = f_r$.

 Where f_r is the characteristic function of fuzzy regular expression r that is explained as:

$$f_r(w) = \begin{cases} 1 & if\ w = r, \\ 0 & else; \end{cases}$$

 iii. $L(\lambda r) = \lambda \otimes L(r)$ $\forall$ $\lambda \in$ L and $r \in FRE$; where L $\rightarrow$ [0, 1].
 iv. $L(r_1 + r_2) = L(r_1) \vee (r_2)$, $\forall$ $r_1, r_2 \in FRE$;
 v. $L(r_1 r_2) = L(r_1)\, L(r_2)$, $\forall$ $r_1, r_2 \in FRE$;
 vi. $L(r^*) = L(r)^*$ $\forall$ $r_1, r_2 \in FRE$;

These are fuzzy language of fuzzy regular expression. The fuzzy language of fuzzy automaton and fuzzy regular expression should be same.

3. LITRETURE REVIEW

    In this section, a systematic review is carried out on various researches in the field of Fuzzy Automata based on various parameters such as algebraic aspects, minimization of fuzzy automata, closure properties of fuzzy automata, application of fuzzy automata, conversion of fuzzy regular expressions to fuzzy automata and the various terminologies and researches in the field of fuzzy automata. We also explain the challenging issues in various fields of fuzzy automata.

3.1 Algebraic Aspects

Membership values in the fuzzy automata are determined using different types of lattice such as lattice – ordered monoid, complete residuated lattice, complete lattice and many other type of lattice. Li and Pedrycz [18] suggested the closure property of nondeterministic and deterministic fuzzy automata. They proved that the nondeterministic fuzzy automata are not closed under the Complement operation where as deterministic fuzzy automata are not closed under the Kleene closure operation. Further, they proved that the nondeterministic fuzzy automata are more powerful as compared with the deterministic fuzzy automata. They used the concept of lattice – ordered monoid for constructing the fuzzy automata with membership value in lattice – ordered monoid. Ignjatovic *et al.* [13] introduced complete residuated lattice for determinization of fuzzy finite automata with membership degree. Tiwari and Sharan [33] explained the concept of lattice – ordered monoid for fuzzy finite automata with membership value. For fuzzy

finite automata, they proposed some topological and algebraic concept. Lattice – ordered monoid, complete residuated lattice, complete lattice and other algebraic structures can be used in fuzzy automata. Li and Pedrycz [18] used the concept of lattice – ordered monoid. Ignjatovic *et al.* [13] used the concept of complete residuated lattice, whereas Tiwari and Sharan [33] studied some algebraic and topological concept of fuzzy finite automata with membership value in lattice – ordered monoid. In fuzzy automata, membership values play a pivotal role. Different algebraic aspects are used for determining these membership values.

3.1.1 Challenging Issues in Algebraic Aspects

Fuzzy automata are fully dependent on membership value. Many researchers have explained the concept of complete residuated lattice, lattice – ordered monoid, and complete lattice for finding the membership value. There are many challenging issues in this context such as other types of algebraic structures like different type of lattices, weighted automata over the semiring for finding the membership value can be used.

3.2 Minimization of fuzzy Automata

Malik *et al.* [24] proposed the concept of minimization of fuzzy finite automata. They described that the procedure for the construction of equivalent minimal fuzzy finite automata M_2 from M_1 using the homomorphism image property. Topencharov and Peeva [35] suggested a new approach for the problem of minimization and reduction of fuzzy finite automata. They introduced an algorithm for the reduction and minimization of fuzzy automata. Ignjatovic *et al.* [13] introduced the complete residuated lattice for determinization of the fuzzy finite automata with their membership values. Further, they proposed an efficient approach for the construction of smaller automaton using the fuzzy right congruence. Malik *et al.* [33] used a classical approach and theorem for the minimization procedure, whereas Topencharov and Peeva [35] used the algebraic approach for minimization of fuzzy automata. Using the fuzzy right congruence, we can able to reduce the size of fuzzy automata. Ciric *et. al.* [9] described that bi-simulation can be used in various areas of computer science. Using bi-simulation, we can reduce the number of states in a system. They defined the uniform fuzzy relation, state reduction, equivalence of automata, backward and forward simulation. Ciric *et al.* [8] explained the alternating reduction by using two approaches *i.e.* left invariant fuzzy finite equivalence and right invariant fuzzy finite equivalence. They explained efficient steps for finding the fuzzy equivalence. They explained complete residutaed lattice, factor fuzzy automaton, alternate reduction, fuzzy relation equation, and state reduction. Cheng and Mo [11] proposed minimization algorithm, mealy type of fuzzy automata, minimization, and fuzzy automata. Using equivalence relations, we can reduce the number of states. Many such approaches are proposed in the literature for the minimization of fuzzy automata.

3.2.1 Challenging Issues for Minimization of Fuzzy Automata

Many researchers have explained different methods for minimization of fuzzy automata such as using fuzzy right congruence, bi-simulation, left invariant fuzzy finite

_____

_____

equivalence and right invariant fuzzy finite equivalence etc. The equivalence class approach can also used for minimization of fuzzy automata.

3.3   Closure Properties

Mizumoto *et al.* [22] worked on the closure properties of fuzzy automata. They explained the fuzzy language, optimistic fuzzy automata, fuzzy transition matrix and fundamental properties of fuzzy automata. Li and Pedrycz [18] introduced the concept of closure properties for nondeterministic and deterministic fuzzy automata. They proved that the nondeterministic fuzzy automata are not comes under the Complement operation and deterministic fuzzy automata are not comes under the Kleene closure operation. In the term of power, they proved that nondeterministic fuzzy automata are more powerful than the deterministic fuzzy automata. Li *et al.* [20] described relationship between several types of fuzzy finite automata. In relationship they include nondeterministic lattice ordered monoid fuzzy automata with ε - move, nondeterministic lattice ordered monoid fuzzy automata without ε - move, deterministic fuzzy automata using lattice ordered monoid, and lattice ordered monoid fuzzy finite state machine. They also compared the power of all the fuzzy automata. Bedregal and Figueira [5] proposed that the power fuzzy Turing machine is mare than the classical Turing machine and fuzzy Turing machine are more efficient also. Bedregal and Figueira, explained universal machine, recursively enumerable set, and fuzzy function. Research is still going on for studying various properties of fuzzy automata.

3.3.1 Challenging Issues in Closure Properties

The closure properties in the field of fuzzy automata are intersection, union, set-deference, kleene closure etc. The researchers are still working on the closure properties of fuzzy automata.

3.4   Application of Fuzzy Automata

Fuzzy automata can be used in various areas such as a learning system, system modeling, neural network, pattern matching, image processing etc. Wee and Fu [36] proposed the model of learning system using the fuzzy automata. They explained that using the concept of Zadeh's fuzzy sets into mealy finite state machine, we can formulate the class of fuzzy finite automata, and the behaviour of fuzzy automaton is similar to the deterministic fuzzy automaton. They also described that fuzzy automata and stochastic automata have many similar properties. Rigatos [29] suggested that a fuzzy automaton can be used in the system modeling. He proposed two approaches named as the syntactic analysis and fuzzy automata for the fault detection. They explained how fuzzy automata are used for the pattern matching, fault detection and system modeling. Blanco *et al.* [3] suggested that how we can infer fuzzy regular grammar using two layers neural – network architecture from fuzzy examples. They explained that the fuzzy grammatical inference, fuzzy recurrent neural network and fuzzy regular grammar. Fuzzy automata can be used in the artificial intelligence. Astrain *et al.* [1] suggested how to measure the fuzziness between the strings. They defined fuzzy edit operations and string alignments approaches for measuring the fuzziness between strings. They explained the fuzzy automata with ε - move and their relationships with fuzzy automata. The main contribution of this paper is to measure the fuzziness between strings. Wu *et al.* [37] explained the fuzzy automata system to achieve improved image processing and target recognition. Using fuzzy finite automata system, firstly good image processing achieved and then carried out for the target recognition. The fuzzy finite automata system comprises four factors *i.e.* experiment, factor extraction, target machine and image processing. They used two features *i.e.* local and global of target image and using fuzzy automata system, target recognition accomplished. They defined image processing, target recognition, fuzzy automata and proved that results of recognizing the target is more than 94.59%. Bailador and Trivino [2] introduced temporal fuzzy automata for pattern recognition. They explained pattern recognition, fuzzy models, hidden Markov model. Pattern recognition using temporal fuzzy automata is more efficient then pattern recognition using hidden Markov model. Ciric *et al.* [10] explained four type of bi-simulation *i.e.* (backward, forward, backward – forward and forward – backward bi-simulation) and two type of simulation *i.e.* (backward, forward) in fuzzy finite automata. They explained complete residuated lattice, fuzzy relation, post – fixed point, fuzzy relation inequality, bi-simulation, simulation, and fuzzy automata. They introduced an efficient approach for finding whether there is a bi-simulation/simulation between fuzzy automata and if there exists bi-simulation/ simulation between fuzzy automata then find out the greatest bi-simulation/simulation. Research is still going on for studying various applications of fuzzy automata.

3.4.1 Challenging Issues

Fuzzy automata can be used in many applications such as expert system, pattern recognition, vending machine, neural network, and game theory etc.

3.5   Conversion of Fuzzy Regular Expression to Fuzzy Automata

Stamenkovic and Ciric [32] introduced the new approach for the conversion of fuzzy regular expressions to fuzzy finite automata. They used the concept of position automata for the conversion of fuzzy regular expressions to fuzzy finite automata and proposed that the other methodologies for the conversion of regular expressions to non-deterministic finite automata can be extended for the same purpose. They introduced the concept of lattice–ordered monoids, fuzzy automata, fuzzy regular expressions and applications of the fuzzy automata. Li and Pedrycz [18] suggested the conversion of fuzzy regular expressions to fuzzy automata, but there is a scope of improvement in the efficiency of the conversion. Ilie and Yu [15] suggested that a novel approach for the construction of non-deterministic finite automata (NFA) from regular expressions known as follow automata. Initially they convert the regular expressions into smaller NFA, and then eliminate the ε - transitions. Using the follow automata, the number of states will be less than or equal to n + 1 (where n = total count of input alphabets that are presented in the regular expression), whereas in position automata number of states are always n + 1. Stamenkovic and Ciric [32] introduced an efficient approach for the conversion of fuzzy regular expressions to fuzzy automata using position automata.

_____

_____

3.5.1 Challenging Issues

The construction of efficient fuzzy automata from fuzzy regular expressions is also a main area of research.

3.6 Various Terminologies and Researches in the field of Fuzzy Automata

Cao and Ezawa [7] described that the fuzzy automata can handle the uncertainty in any system model. The concept of fuzzy finite automata has been arising from nondeterministic finite automata. To handle non-determinism in automata, the concept of non-deterministic fuzzy automata has been introduced. Nondeterministic automata have two properties such as nondeterministic fuzzy automata with the null string and nondeterministic fuzzy automata without null string. They explained all the automata such as nondeterministic fuzzy finite automata with or without empty string and deterministic fuzzy automata are comparable in the impression that they observe the same class of fuzzy language. Santos [30] suggested the concept of max-min automata. They proposed the different classes of automata, behaviour of max-min automata, homomorphism and equivalence of max-min automata. Ignjatovic *et al.* [14] introduced myhill – Nerode theorem for the fuzzy automata and language. They explained the relationship between fuzzy language, deterministic automata and fuzzy automata. Qiu [28] suggested the characterizations of fuzzy finite automata. They explained essential relationships and some fundamental concepts in fuzzy automata with bi-fuzzy property. They explained some basic concepts, properties of fuzzy finite automata and bi-fuzzy property such as bi-fuzzy topological characterization, source and successor operators, sub-automata etc.

Malik *et al.* [23] introduced two concepts for determining fuzzy languages from fuzzy automata *i.e*. max – min automaton and min – max automaton. Using max – min and min – max fuzzy language, we can able to find out the membership value of a particular string in fuzzy automata. They proposed the fuzzy pumping lemma as a necessary and sufficient condition for max – min fuzzy language.

Xing [39] described the concepts of fuzzy context – free grammars, fuzzy pushdown automata, and fuzzy context – free language. Xing described the behaviour and movement of multi-stack and one – stack fuzzy pushdown automata (FPDA) for recognizing the fuzzy languages.

Tiwari and Srivastava [34] suggested the decomposition of fuzzy automata in a unique form. Tiwari and Srivastava explained the concept of source – splitting sub-automaton and decomposable sub-automaton. Krithivasav and Sharda [16] proposed the concept of fuzzy ω – automata for the fuzzy ω – languages. They explained the concept of distributed fuzzy ω –

finite automata and fuzzy ω – finite automata respectively for accepting the different mode such as t – mode, * - mode etc and fuzzy ω – regular language. They proved that languages accepted by distributed fuzzy ω – finite automata and the fuzzy ω – finite automata are same.

Li [21] introduced the fuzzy Turing machine using distributive lattice with membership value that is also called lattice ordered fuzzy Turing machine. He explained deterministic fuzzy Turing machine, nondeterministic Turing machine, recursively enumerable language, universal machine and fuzzy system model. He also introduced the approach of lattice – ordered nondeterministic fuzzy polynomial time computations, lattice – ordered deterministic fuzzy polynomial time computations. Belohlavek [4] proposed the concept of deterministic fuzzy finite automata. They compared the deterministic fuzzy automata and deterministic finite automata. They explained that the power of nested system of deterministic finite automata and deterministic fuzzy automata are equal, and the power of deterministic fuzzy automata and fuzzy automata is equal.

Petkovic [26] suggested the concept of homomorphism and congruence of fuzzy automata and proved the theorem of homomorphism. He described lattice of congruence of fuzzy automata and algorithm for finding the maximum congruence.

Doostfatemeh and Kremer [12] described that classical automata theory deals with discrete space, whereas fuzzy automata theory deals with continuous space. Classical automata cannot manage the fuzziness in system modeling. In order to manage fuzziness in any system, fuzzy logic is incorporated with the automata theory, and the resulted fuzzy automata can deal with fuzziness in any of the system. They proposed that the deterministic finite automata (*DFA*) are widely used in many applications, but they have some issues such as they do not deal with fuzziness etc. In particular, they emphasized on the multi membership resolution.

Benlahcen and Lamotte [6] defined fuzzy automata and recalled some functional properties and theorem of fuzzy logic that are used in fuzzy automata. They proposed the synthesis method of fuzzy automata and use numerical for the illustration of the efficiency of synthesis method.

Santos [31] proposed the concept of fuzzy automata and language. They described that how fuzzy automata theory is similar to classical automata theory. They used max – product composition in fuzzy automata for generating the fuzzy language.

Various researches carried out in the field of Fuzzy automata are summarized in Table 1.

_____

_____

Table 1: Summary of various Researches in the field of Fuzzy Automata

Parameters	Researchers	Descriptions of the Research
Algebraic Aspects	Ignjatovic et al. [13]	Introduced the new approach to construct smaller fuzzy automaton with the help of fuzzy right congruence.
	Tiwari and Sharan [33]	Explained lattice – ordered monoid for membership value in fuzzy automata and other algebraic concepts of fuzzy automata.
	Li and Pedrycz [18]	Explained some closure property of deterministic and nondeterministic fuzzy automata. Introduced the concept of fuzzy regular expressions and lattice – ordered monoids.
Minimization of Fuzzy Automata	Malik et al. [24]	Introduced how to minimiz]e fuzzy finite automata M1 to equivalent fuzzy finite automata M2.
	Topencharov and Peeva [35]	Explained new approach for minimization and reduction of fuzzy finite automata.
	Ignjatovic et al. [13]	Introduced the new approach to construct smaller fuzzy automaton with the help of fuzzy right congruence.
	Ciric et al. [9]	Explained the benefits of bi-simulation in fuzzy automata.
	Ciric _et al._ [8]	Explained alternating reduction using right invariant and left invariant fuzzy equivalence.
	Cheng and Mo[11]	Explained the minimization algorithm. Using equivalence relation, minimize the fuzzy automata.
Closure Properties	Mizumoto et al. [22]	Suggested some closure properties of fuzzy automata.
	Li and Pedrycz [18]	Explained some closure property of deterministic and nondeterministic fuzzy automata. Introduced the concept of fuzzy regular expressions and lattice – ordered monoids.
	Li _et al._ [20]	Explained the relationship among the several types of fuzzy automata.
	Bedregal and Figueira [5]	Explained some property of fuzzy Turing machine.
Application of Fuzzy Automata	Wee and Fu [36]	Proposed model of learning system that is one of the application of fuzzy automata.
	Rigatos [29]	Explained fault detection with the help of fuzzy automata.
	Blanco et al. [3]	Explained the concept of fuzzy regular grammar and fuzzy grammatical inference.
	Astrain et al.[1]	They introduced the fuzzy automata with ε - move and measure the fuzziness between strings.
	Wu _et al._ [37]	Introduced the fuzzy automata system to achieve improved image processing and target recognition.
	Bailador and Trivino [2]	Proposed temporal fuzzy automata for pattern recognition.
	Ciric _et al._ [10]	Explained different type of bi-simulation and simulation.
Conversion of Fuzzy Regular	Li and Pedrycz [18]	Explained some closure property of deterministic and nondeterministic fuzzy automata. Introduced the concept of fuzzy regular expressions and lattice – ordered monoids.
	Stamenkovic and Ciric [32]	Constructed the fuzzy automata from fuzzy regular expressions using position automata.

15

_____

_____

	Ilie and Yu [15]	Introduced follow automata for the conversion of regular expression to nondeterministic finite automata.
Various Terminologies	Cao and Ezawa [7]	Proposed nondeterministic fuzzy finite automata with or without ε – move and fuzzy language recognized by deterministic and nondeterministic with and without ε – moves are same.
	Santos [30]	Introduced the concept of max-min automata and behaviour of max-min automata.
	Ignjatovic et al. [14]	Introduced the myhill–nerode theorem for fuzzy automata and language.
	Qiu[28]	Explained fuzzy finite automata with bi-fuzzy property.
	Malik et al. [23]	Introduced the concept of $min – max$ and $max – min$ fuzzy language for fuzzy automata.
	Xing [39]	Described the working of fuzzy pushdown automata in a single stack and multi-stack.
	Tiwari and Srivastava [34]	Obtained the decomposition of fuzzy automata.
	Krithivasav and Sharda [16]	Introduced the concept of fuzzy ω - automata and define the concept of distributed fuzzy ω – automata.
	Li [21]	Explained deterministic fuzzy Turing machine, nondeterministic fuzzy automata and recursively enumerable language.
	Belohlavek [4]	Proposed the new definition of deterministic fuzzy automata.
	Petkovic [26]	Introduced the concept and proof of homomorphism and congruence of fuzzy automata.
	Doostfatemeh and Kremer [12]	Compared fuzzy automata with classical automata theory and explain various issues of fuzzy automata.
	Benlahcen and Lamotte [6]	Suggested the synthesis method of fuzzy automata.
	Santos [31]	Proposed the concept of fuzzy language and automata.

4. Challenging Issues in the field of Fuzzy Automata
- To construct fuzzy regular expression from fuzzy automata.
- To design efficient fuzzy pushdown automata from fuzzy context free grammar.
- Conversion of nondeterministic fuzzy automata to deterministic fuzzy automata.
- Closure properties for nondeterministic and deterministic fuzzy automata.
- Fuzzy automata can be used in many real time applications such as game theory, learning system, etc.

5. Conclusion and Future Investigations
Fuzzy automata can be used in various applications and there is a scope of future research work can be carried out in the field of Fuzzy automata. In this paper, a systematic review is carried out of the researches in the field of fuzzy automata. Future work will concentrate on the following areas:

- Design of an efficient approach for the conversion of Fuzzy regular expressions to Fuzzy automata.
- More application of fuzzy automata can be explored.

- Design of an approach for the conversion of fuzzy automata to fuzzy regular expressions.

6. References
[1] J. J. Astrain, J. R. G Mendívil, and J. R Garitagoitia "Fuzzy automata with ε-moves compute fuzzy measures between strings", *Fuzzy Sets and Systems*, 157, (2006), 1550 – 1559.

[2] G. Bailador and G. Trivino "Pattern Recognition Using Temporal Fuzzy Automata", 161, (2010), 37 – 55.

[3] Blanco, M. Delgado and Pegalajar "Fuzzy automaton induction using neural network", *International Journal of Approximate Reasoning,* 27, (2001) 1 – 26.

[4] R. Belohlavel, "Determinism and fuzzy automata", *Information Sciences*, 143, (2002), 205 – 209.

[5] B. C. Bedregal and S. Figueira, "On the Computing Power of fuzzy Turing machine", *Fuzzy Set and System*, 159, (2008), 1072 – 1083.

[6] D. Benlahcen, and M. Lamotte, "A Fuzzy Automaton Synthesis Method", *International Federation of Automatic Control*, 17, (1981), 299 – 306.

[7] Y. Cao, and Y. Ezawa, "Nondeterministic fuzzy automata", *Information Sciences*, 191, (2012), 86-97.

[8] M. Ciric, A. Stamenkovic, J. Ignjatovic, and T. Petkovic, "Fuzzy relation equation and reduction of fuzzy automata",

16

_____

_____

Journal of Computer and System Sciences, 69, (2010), 609 – 633.

[9] M. Ciric, J. Ignjatovic, N. Damljanovic, and M. Basic "Bisimulations for fuzzy automata," *Fuzzy Sets and Systems*, 86, (2012), 100 – 139.

[10] M. Ciric, J. Ignjatovic, I. Jancic, and N. Damljanovic, "Computation of the greatest simulations and bisimulations between fuzzy automata", *Fuzzy Sets and Systems*, 208, (2012), 22 – 42.

[11] W. Cheng and Z – W Mo. "Minimization algorithm of fuzzy finite automata," *Fuzzy Set and System*, 141, (2004), 439 – 448.

[12] M. Doostfatemeh, and S.C. Kremer "New directions in fuzzy automata", *International Journal of Approximate Reasoning*, 38, (2005), 175 – 214.

[13] J. Ignjatovic, M. Ciric, and S. Bogdanovic, "Determinization of fuzzy automata with membership values in complete residuated lattices", *Information Sciences*, 178, (2008), 164 – 180.

[14] J. Ignjatovic, M. Ciric, S. Bogdanovic, and T. Petkovic "Myhill–Nerode type theory for fuzzy languages and automata", *Fuzzy Sets and Systems*, 161, (2010), 1288 – 1324.

[15] L. Ilie, and S. Yu, "Follow automata", *Information and computation,* 186, (2003), 140-162.

[16] K. Krithivasan, and K. Sharda, "Fuzzy ω - automata", *Information Science*, 138, (2001), 257 – 281.

[17] E. T. Lee and L. A. Zadeh, "Note on fuzzy languages", *Information Sciences*, 1, (1969), 421- 434.

[18] Y. M. Li and W. Pedrycz, "Fuzzy finite automata and fuzzy regular expressions with membership values in lattice-ordered monoids", *Fuzzy Sets System*, 156, (2005), 68–92.

[19] Y. Li, and C. Liang "Algebraic properties on the cuts of lattice-valued regular languages", *Soft Computing*, 12, (2008), 1049 – 1057.

[20] Z. Li, P. Li, and Y. Li, "The relationships among several type of fuzzy automata", *Information Sciences*, 176, (2006), 2208 – 2226.

[21] Y. Li, "Lattice valued fuzzy Turing machine computing power universality and efficiently", *Fuzzy Set and System*, 160, (2009), 3453 – 3474.

[22] Mizumoto, M., Toyoda, J. and Tanaka, K. "Some Considerations on Fuzzy Automata", *Journal of Computer and System Science*, 3, (1969), 409 – 422.

[23] D. S. Malik, J. N. Mordeson, and M. K. Sen, "On fuzzy regular language", *Information Science*, 88, (1969), 263 – 273.

[24] D. S. Malik, J. N. Mordeson and M. K. Sen "Minimization of fuzzy finite automata", *Information Science*, 113, (1999), 323 – 330.

[25] J. N. Mordeson and D.S. Malik, "Fuzzy Automata and Languages: Theory and Applications", *Chapman & Hall*, *CRC*, *Boca Raton*, London, (2002).

[26] T. Petkovic "Congruences and homomorphisms of fuzzy automata", *Fuzzy Sets and Systems*, 157, (2006), 444 – 458.

[27] K. Peeva, and Z. Zahariev, "Computing behavior of finite fuzzy machines—algorithm and its application to reduction and minimization", *Information Science* 178, (2008), 4152–4165.

[28] D.W. Qiu "Characterizations of fuzzy finite automata", *Fuzzy Sets System*, 141, (2004), 391–414.

[29] G. G. Rigatos "Fault detection and isolation based on fuzzy automata", *Information Science*, 179, (2009), 1893 – 1902.

[30] E. S. Santos, "Maximin automata", *Information Control*, 12, (1968), 367–377.

[31] E. S. Santos "Fuzzy automata and languages", *Information Sciences*, 10, (1976), 193–197.

[32] A. Stamenkovic, and M. Ciric, "Construction of fuzzy automata from fuzzy regular Expressions", *Fuzzy Sets and Systems,* 199, (2012), 1-27.

[33] S. P. Tiwari and S. Sharan "Fuzzy Automata Based on Lattice-ordered Monoid with Algebraic and Topological Aspects", *Fuzzy Information Engineering*, 2, (2012), 155 – 164.

[34] S. P. Tiwari, and A. K. Srivastava, "On a decomposition of fuzzy automata", *Fuzzy Sets and Systems*, 151, (2005), 503 – 511.

[35] V. V. Topencharov, and K. G. Peeva, "Equivalence, Reduction and Minimization of Finite Fuzzy-Automata", *Journal of Mathematical Analysis and Applications*, 84, (1981), 270 – 281.

[36] W.G. Wee, and K.S. Fu, "A formulation of fuzzy automata and its application as a model of learning systems", IEEE Transactions Systems Man cybernetics, 5, (1969), 215–223.

[37] Q – E. Wu, v Pang, and Z – Y. Han, "Fuzzy automata system with application to target recognition based on image processing", *Computer and Mathematic with application*, 61, (2011) 1267 – 1277.

[38] W. G. Wee, "On Generalizations of Adaptive Algorithm and Application of the Fuzzy Sets Concept to Pattern Classification", Ph.D. Thesis, Purdue University, (1967).

[39] H. Xing, "Fuzzy pushdown automata," *Fuzzy Sets and Systems*, 158, (2007), 1437 – 1449.

[40] Singh, Gurpreet, and Manoj Sachan. "Multi-layer perceptron (MLP) neural network technique for offline handwritten Gurmukhi character recognition." Computational Intelligence and Computing Research (ICCIC), 2014 IEEE International Conference on. IEEE, 2014.

[41] Singh, G., and M. Sachan. "A framework of online handwritten gurmukhi script recognition." International Journal of Computer Science and Technology (IJCST) 6 (2015): 52-56.

_____