

Delay Reduction of Detection Algorithms for 5G Massive MIMO System

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Abstract— Multiple antenna technologies like Multiple-Input Multiple-Output (MIMO) and beamforming will thus play an important role in defining 5G system architectures. In massive MIMO there is a huge number of antenna elements, so there is a need to estimate large channel matrix which introduces much latency. The ultra-high latency and high computation complexity of massive MIMO matrices from 16 to 256 dimensions is the vital bottleneck to realizing latency for channel estimation and MIMO detection. This paper introduces a mechanism to reduce the high computational complexity that causes huge latency. Four algorithms are evaluated to measure their performance. These algorithms are Gauss-Jordan Elimination, Gaussian Elimination, RQ Decomposition and LU Decomposition. MATLAB simulation used to analyze the applied mathematical models. After that measured the BER, delay for each algorithm and evaluate the capacity and throughput, by way, found that the Gaussian Elimination has better delay about 49 percent when RQ Decomposition higher about 95 percent while LU Decomposition highest about 98 percent compared by Gauss-Jordan Elimination. In addition the result show the performance of capacity and throughput for various modulation and coding rate, while the deliverables average capacity about 10 M bit and affected by the situation of the channel, LU has the best performance than others.

Keywords-Massive MIMO; Gauss; Elimination; Decomposition; SIMD;

I. INTRODUCTION

With a development of Communication system toward 2G circuit switch is utilized, then, 3G is developed to offer high speed and data rate. Further enhancement is achieved in 4G with fulfilled application and enabling to use multimedia on the way to 5G by developing technologies over 4G LTE Advance. The cost becomes increasingly important, simultaneously with the rising user demand on the mobile operators networks. Future communication technologies need to reduce power consumption, decrease latency, increase performance, and increase computability of today different standards. Long Term Evolution (LTE) baseband system supports many techniques, such as synchronization, channel coding, interleaving, demodulation, channel estimation; multiple input multiple output (MIMO) detection.

Many redundancies introduced like Channel estimation for a multi-antenna receiver system; these redundancies lower the channels utilization, require additional processing power, and increase latency. The conventional method to address these problems is to add pilot signals and decrease the length of the cyclic prefix (CP).

In baseband processing, control, and data correlation by selecting appropriate algorithms and then optimizing these algorithms can be minimized. However, MIMO channel estimated using least square and minimum mean square error, while in detection zero forcing and minimum mean square error are used.

In massive MIMO there is a huge number of the antenna element, there is a need to estimate large channel matrix, where, the number of element rising from 8*8 to 256*256. The ultra-high latency and high computation complexity of massive MIMO matrices from 16 to 256 dimensions is the vital bottleneck to realizing latency for channel estimation and MIMO detection. For that also mechanisms for the inverse matrix to evaluate receiving signal are needed, and four algorithms are considers Gaussian elimination, LU decomposition, RQ

decomposition and Gauss-Jordan elimination Algorithms to evaluate the latency in each one and show their performance [3].

II. RELATED WORKS

LTE is a 3.9G technology according to the standard, the peak data rate of LTE is from 100 to 326.4 Mbps over the downlink and 50 to 86.4 Mbps over the uplink. LTE uses orthogonal frequency division multiple access (OFDMA) and single carrier frequency division multiple access (SC-FDMA) in downlink and uplink sequentially [4] [5].

OFDM has two defects: large peak-to-average power ratio (PAPR) and high sensitivity to carrier frequency errors [6]. The main advantage of SC-FDMA is its low PAPR [7]. The potential technologies that could use in 5G are ultra-densification, device-centric architectures, millimeter wave (mm-Wave), massive MIMO, smart devices, and native support for machine-to-machine (M2M) communication [8] [9]. Single Input Multiple Data (SIMD) instruction processing is one of the newest forms of parallel processing in Flynn's taxonomy. The basic idea of SIMD is to apply the same instruction sequence simultaneously to a huge number of discrete data streams [10].

The first viewpoint is the comparison of key technologies in baseband processing. There have been many research papers discussed the channel estimation and MIMO detection at LTE/LTE-A uplink. For channel estimation, many research works discuss how to optimize channel estimation method to gain good performance [11]. Moreover, the used method in [12] proposed to discuss further how to optimize it when a different number of resource blocks are allocated. Several research works evaluated the different algorithms in different channel model such as investigated algorithms in flat Rayleigh fading. The authors in [13] investigated the channel estimation for LTE uplink when the traveling speed of the UE is high. For MIMO detection, the authors in [14] study two low-complexity detection schemes based on MMSE for MIMO systems. In [15], the researchers evaluated the performance of different detection algorithms over Rayleigh wireless channel. Because channel

estimation and MIMO detection are two advanced procedures in LTE-A Uplink.

All of these research works only focused on one scheme of channel estimation or MIMO detection. Meanwhile, there are some researches on channel estimation or MIMO detection algorithm for multi-antennas 2*2 or maximum 4*4 MIMO system. Therefore, all the above mentioned channel estimation works did not consider the future massive MIMO-system. So, it is required to find a suitable algorithm for matrix inverse for these systems. From a deep literature review and investigation, several conventional algorithms were selected that could be used to compute the matrix inverse for the complex matrix. The conventional methods used to perform matrix inverse are Gauss-Jordan Elimination [16], Gaussian Elimination [1], LU Decomposition [17], and QR Decomposition [18]. The research work conducted by Xin, in [19] is a most related to this research work. Both focus on investigating and analyzing key technologies such as Channel estimation and MIMO detection in large-Scale MIMO. The work in [20] and [21] explored matrix computation on matrices larger than 512*512 using LU decomposition and Gauss-Jordan-Floyd-War-shall method respectively. In [22] [23], the design and implementation of a parallel algorithm utilize multi-core task-level parallelism, another form of coarse-grained parallelism.

The 8192 number of antennas is extended by massive MIMO systems there by enhancing the user efficiency. Services to users with 2048 antennas in simple MIMO schemes were classically adopted where both 5 and 50 percentile of full efficiency is reached. The MIMO systems with 5096 are common with optimal service provided and it can make huge number of data transmission with advanced digital signal processing tools [24].

In order to analyze this process two conventional algorithms performance and complexity for channel estimation and MIMO detection are compared. The key features, which affects the algorithms' speed, it identified as the need for "massive complex matrix inversion".

A parallel coding scheme it suggested to implement a matrix inversion kernel algorithm on SIMD vector processor in [2] [3].

Fig. 1 explains the detection algorithms for massive MIMO as shown in blue color beginning with the physical layer in uplink for PUSCH channel and then work on massive MIMO technology and focus on the algorithm for equalizing the channel to extract transmitted signal.

The fifth generation going to increase the frequency and trying to decrease cell area and this enable to usage huge number of antenna especially in massive MIMO can arrive at 4096 antennas [19]. Therefore, this needs very high computational complexity for that we look for which method able to reduce the delay happen in computation thereby comparison between two methods using only Gauss-Jordan Elimination, and the work with SIMD and Normal calculation founded that the SIMD has less delay than normal calculation[3]. Our general orientation is to develop wireless system's performance in large-Scale MIMO. The chosen algorithm is Gaussian-Jordan Elimination with the sizes of matrices ranging from 8*8 to 256*256, [3].

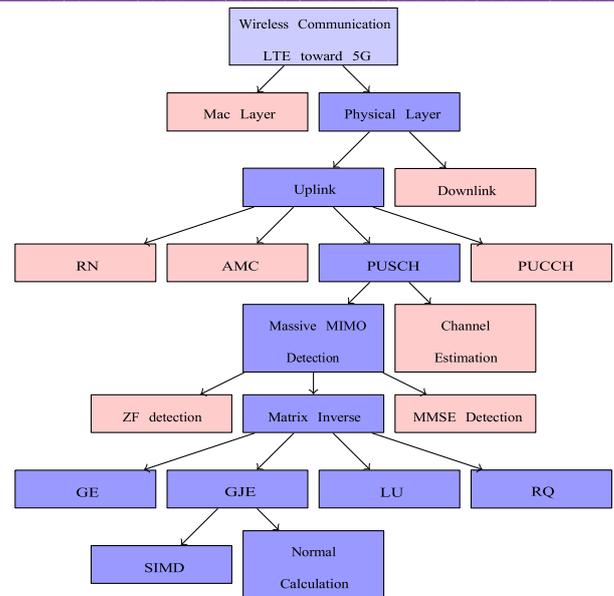


Figure 1. Detective Algorithms for Massive MIMO [3]

III. FIFTH GENERATION KERNEL ALGORITHMS

Four algorithms are considered in this paper, in this section we will define the four algorithms with flow charts beginning with Gauss-Jordan, followed by Gaussian Elimination, LU decomposition and RQ Decomposition.

A. Gauss Jordan Elimination Algorithm

Fig. 2 illustrated the operation of Gauss Jordan Elimination algorithm, in which the channel response (H) estimated first and then multiplied by the received signal (Y) to get the transmitted signal (X).

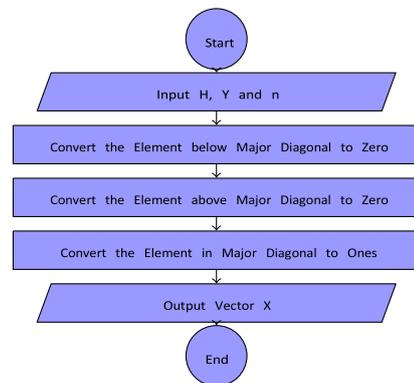


Figure 2. Flow Chart for Gauss Jordan Algorithm

The equations for this algorithms are illustrated below:

$$HxX = Y \tag{1}$$

$$H = \begin{pmatrix} h_{11} & \dots & h_{1j} \\ \vdots & \ddots & \vdots \\ h_{i1} & \dots & h_{ij} \end{pmatrix} \tag{2}$$

Where $i=1, 2 \dots n$ and $j = 1, 2 \dots n$

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_i \end{pmatrix} \tag{3}$$

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_n \end{pmatrix} \quad (4)$$

By applying Gauss Jordan Elimination for N*N matrix:

$$H = \sum_{k=1}^{n-1} \sum_{i=2}^n \sum_{j=1}^n h_{ij} - h_{kj} * \frac{h_{ik}}{h_{kk}} \quad (5)$$

The result is shown in equations below:

$$H = \begin{pmatrix} h_{11}' & h_{12}' & \dots & h_{1j}' & | & y_1' \\ 0 & h_{22}' & \dots & h_{2j}' & | & y_2' \\ \vdots & \vdots & \ddots & \vdots & | & \vdots \\ 0 & 0 & \dots & h_{ij}' & | & y_i' \end{pmatrix} \quad (6)$$

After that doing back substitution:

$$x_i = 1/h_{ii} * (y_i - \sum_{k=i+1}^n (h'_{ik} * x_k)) \quad (7)$$

B. Gaussian Elimination Algorithm

Fig. 3 illustrates the operation of Gaussian Elimination algorithm, in which the channel response (H) estimated first and then multiplied by the received signal (Y) to get the transmitted signal (X).

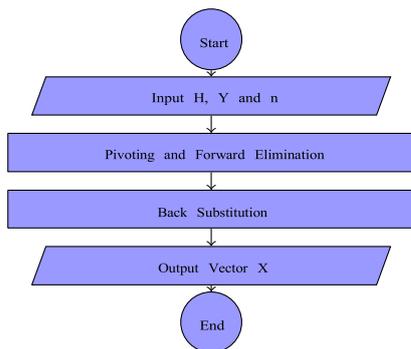


Figure 3. Flow Chart for Gaussian Elimination

The equations for this algorithms are illustrated bellow:

$$HxX = Y \quad (8)$$

$$H = \begin{pmatrix} h_{11} & \dots & h_{1j} \\ \vdots & \ddots & \vdots \\ h_{i1} & \dots & h_{ij} \end{pmatrix} \quad (9)$$

Where i=1, 2 ... n and j = 1, 2 ... n

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_i \end{pmatrix} \quad (10)$$

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_i \end{pmatrix} \quad (11)$$

By applying Gaussian Elimination for N*N matrix:

$$H = \sum_{k=1}^{n-1} \sum_{i=1}^n \sum_{j=1}^n h_{ij} - h_{kj} * \frac{h_{ik}}{h_{kk}} \quad (12)$$

The result is shown in equations below:

$$H = \begin{pmatrix} h_{11}' & h_{12}' & \dots & h_{1j}' & | & y_1' \\ 0 & h_{22}' & \dots & h_{2j}' & | & y_2' \\ \vdots & \vdots & \ddots & \vdots & | & \vdots \\ 0 & 0 & \dots & h_{ij}' & | & y_i' \end{pmatrix} \quad (13)$$

Now the part of matrix under diagonal equal zero also they need to make the part upper diagonal zero by using equation below

$$H = \sum_{k=n}^1 \sum_{i=n}^2 \sum_{j=n}^2 h_{ij} - h_{kj} * \frac{h_{ik}}{h_{kk}} \quad (14)$$

$$H = \begin{pmatrix} h_{11}' & 0 & \dots & 0 & | & y_1' \\ 0 & h_{22}' & \dots & 0 & | & y_2' \\ \vdots & \vdots & \ddots & \vdots & | & \vdots \\ 0 & 0 & \dots & h_{ij}' & | & y_i' \end{pmatrix} \quad (15)$$

After that divide each row by h_{kk} :

$$H = \begin{pmatrix} 1 & 0 & \dots & 0 & | & y_1'/h_{11}' \\ 0 & 1 & \dots & 0 & | & y_2'/h_{22}' \\ \vdots & \vdots & \ddots & \vdots & | & \vdots \\ 0 & 0 & \dots & 1 & | & y_i'/h_{ii}' \end{pmatrix} \quad (16)$$

The solution of the above matrix will be as:

$$x_i = y_i/h_{ii} \quad (17)$$

C. LU Decomposition Algorithm

Fig. 4 explains the operation of LU Decomposition Algorithm, in which the channel response (H) estimated first and then multiplied by the received signal (Y) to get the transmitted signal (X).

The equations for this algorithms are illustrated below:

$$H = \begin{pmatrix} h_{11} & \dots & h_{1j} \\ \vdots & \ddots & \vdots \\ h_{i1} & \dots & h_{ij} \end{pmatrix} \quad (18)$$

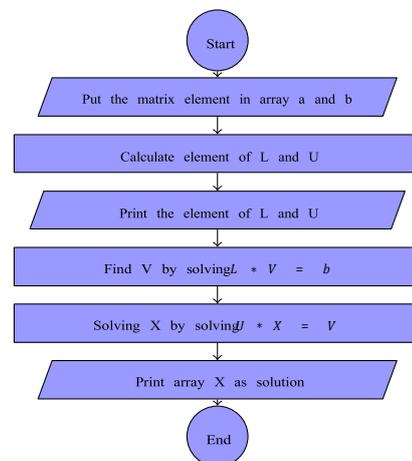


Figure 4. Flow Chart for LU Decomposition

By applying LU Decomposition for N*N matrix:

$$l = \begin{pmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ l_{i1} & l_{i2} & \dots & l'_{ij} \end{pmatrix} \quad (19)$$

$$H = \begin{pmatrix} u_{11} & u_{12} & \dots & u_{1j} \\ 0 & u_{22} & \dots & u_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u_{ij} \end{pmatrix} \quad (20)$$

The result is shown in equation below:

$$h_{11} = l_{11} * u_{11} \quad (21)$$

$$h_{12} = l_{21} * u_{12} + l_{22} * u_{22} \quad (22)$$

$$h_{ij} = \sum_{k=1}^i l_{ik} * u_{kj} \quad i > j \quad (23)$$

$$h_{ij} = \sum_{k=1}^j l_{ik} * u_{kj} \quad i < j \quad (24)$$

$$HxX = Y \quad (25)$$

$$LxUxX = Y \quad (26)$$

$$LxV = Y \quad (27)$$

$$UxX = V \quad (28)$$

The solution using:

1-Forward Substitution

$$v_i = 1/l_{ii} * \left(y_i - \sum_{j=1}^{i-1} (l'_{ij} * v_j) \right) \quad (29)$$

1-Back Substitutions

$$x_i = 1/u_{ii} * \left(v_i - \sum_{j=i+1}^n (u_{ij} * x_j) \right) \quad (30)$$

D. RQ Decomposition Algorithm

Fig. 5 illustrated the operation of RQ Decomposition Algorithm, in which the channel response (H) estimated first and then multiplied by the received signal (Y) to get the transmitted signal (X).

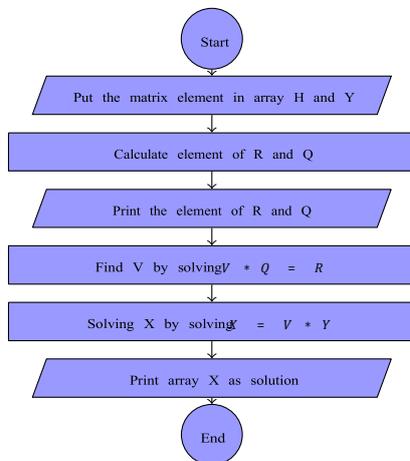


Figure 5. Flow Chart for RQ Decomposition

The equations for RQ Decomposition are shown below:

$$H = \begin{pmatrix} h_{11} & \dots & h_{1j} \\ \vdots & \ddots & \vdots \\ h_{i1} & \dots & h_{ij} \end{pmatrix} \quad (31)$$

$$H1 = \begin{pmatrix} h_{11} \\ \vdots \\ h_{i1} \end{pmatrix} \quad H2 = \begin{pmatrix} h_{12} \\ \vdots \\ h_{i2} \end{pmatrix} \quad Hj = \begin{pmatrix} h_{1j} \\ \vdots \\ h_{ij} \end{pmatrix} \quad (32)$$

$$(H1 \quad H2 \quad \dots \quad Hj) \quad (33)$$

By applying RQ Decomposition for N*N matrix:

$$R = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1j} \\ 0 & r_{22} & \dots & r_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & r_{ij} \end{pmatrix} \quad (34)$$

$$r_{11} = \|H1\| = \sqrt{h_{11}^2 + h_{12}^2 + \dots + h_{ij}^2} \quad (35)$$

$$q_1 = H1/\|H1\| = \begin{pmatrix} h_{11}/\|H1\| \\ \vdots \\ h_{i1}/\|H1\| \end{pmatrix} \quad (36)$$

$$s_2 = (1 - q_1 * q_1^T) * H2 \quad (37)$$

$$q_2 = s_2 / \|s_2\| \quad (38)$$

$$q_n = s_n / \|s_n\| \quad (39)$$

$$s_n = (1 - q_1 * q_1^T) \dots (1 - q_n * q_n^T) * H2 \quad (40)$$

$$r_{22} = s_2 \quad (41)$$

$$r_{nn} = s_n \quad (42)$$

$$r_{ij} = q_i^T * Hj \quad (43)$$

$$Q = (q_1 \quad q_2 \quad \dots \quad q_j) \quad (44)$$

$$R = \begin{pmatrix} r_{11} & r_{12} & \dots & r_{1j} \\ 0 & r_{22} & \dots & r_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & r_{ij} \end{pmatrix} \quad (45)$$

Solve these equations to get the transmitted signal:

$$H = QxR \quad (46)$$

$$Q * Q^T = 1 \quad (47)$$

$$H * X = y \quad (48)$$

E. Processing Time for Four Algorithms

In this section, the delay happen for each algorithm it computed by using TIC TOC function by using MATLAB Simulation as illustrated in Fig. 6.

The function TIC reset the timer, while TOC is beginning count after the reset. However, these functions used to evaluate the processing time happen in each algorithm for different number of antenna and the result it put in variable Elapsed Time. The number of antennas with Elapsed Time it plotted as shown in next section, to determine the optimal algorithm. In addition, there are delay time adding from computer processor and MATLAB software it's very small compared to delay caused by massive MIMO can be negligible.

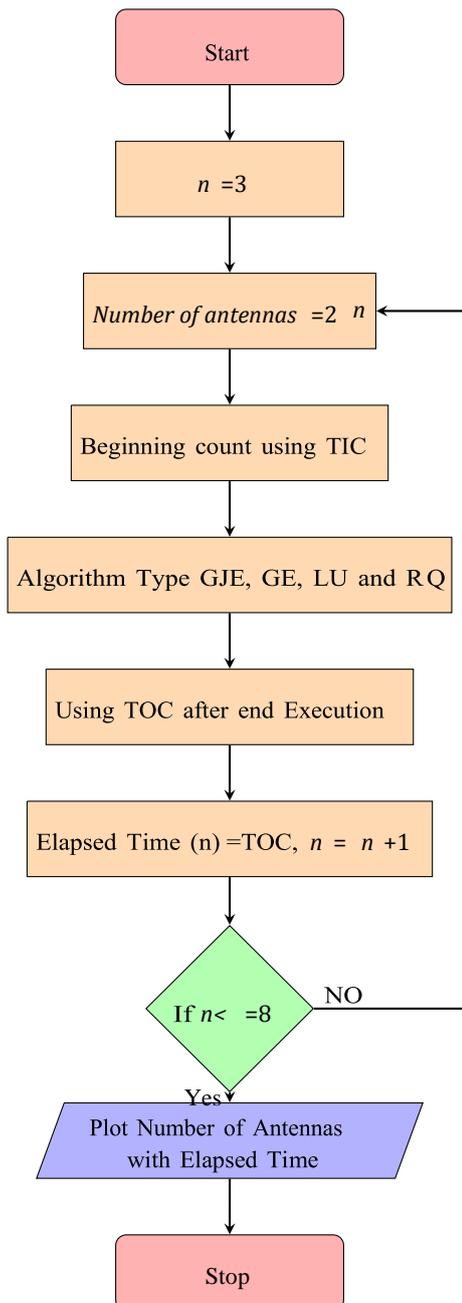


Figure 6. Flow Chart of Processing Time for Each Algorithm

IV. RESULTS AND DISCUSSION

MATLAB simulation is used to evaluate the performance of four algorithms, Gauss Jordan Elimination, Gaussian Elimination, LU Decomposition and RQ Decomposition. The result are presented in term of BER, Capacity and Throughput considering the number of antennas.

A. Processing Delay for Four Algorithms and BER

In this section, the elapsed delay caused by algorithms is calculated and plotted versus a number of antennas as shown in Fig. 7. In which the less delay is for LU decomposition that distinction by delay Timeless than 1m second followed by RQ and the third one Gaussian Elimination while Gauss-Jordan elimination had the highest delay as compared to other algorithms.

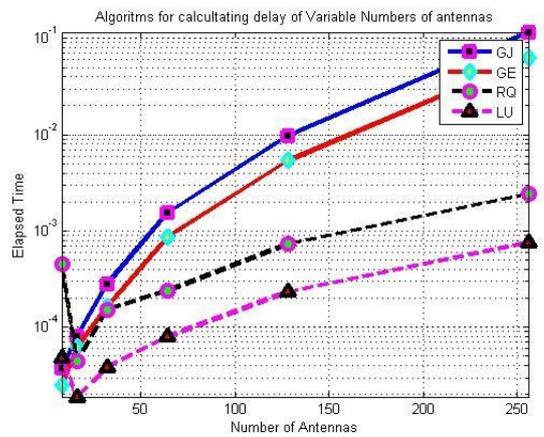


Figure 7. Processing delay for GE, GJE, LU and RQ in Massive MIMO

From the result above can be noticed that as the number of antenna in massive MIMO increase, the processing delay increased. However LU and RQ decomposition algorithms have less delay because they use decomposition methods, while Gauss Jordan and Gaussian Elimination algorithms use eliminations methods. The percentage of the processing delay of Gaussian Elimination, LU Decomposition and RQ Decomposition as compared to Gauss Jordan Elimination are illustrated in TABLE I.

TABLE I COMPARISON BETWEEN GJE AND OTHER ALGORITHMS

NO.	Name of Algorithm	Percentage compare to GJE (%)
1	GE	49
2	RQ	94
3	LU	98

Here we compared the percentage of delay compared to Gauss-Jordan Elimination for three other algorithms Gaussian Elimination, LU and RQ decomposition. These percentages give the value of reducing delay in each algorithm compared by Gauss-Jordan Elimination. From TABLE I the LU had better performance than other algorithms secondly RQ and the last one is Gaussian Elimination.

The BER happen for four algorithms are shown and in that the Gauss-Jordan Elimination have the highest BER than Gaussian Elimination but there are no BER happen for LU and RQ or can be negligible. Furthermore at 128 both algorithms had maximum BER which reach 10^{-9} for Gauss Jordan Elimination and 10^{-11} for Gaussian Elimination as shown in the Fig. 8.

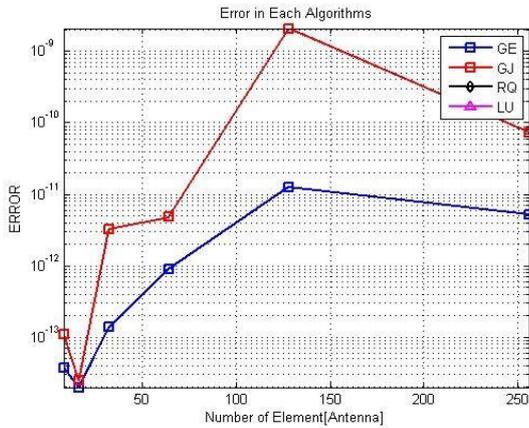


Figure 8. BER for GE, GJE, RQ and LU decomposition

B. Capacity and Throughput for Kernal Algorithms

This section, mentioned the performance evaluation of these algorithms for capacity and throughput using BPSK modulations and code rate 2/3.

Fig. 9 shows the capacity of different algorithms using BPSK with code rate 2/3. In which the better capacity is gained by LU decomposition algorithm followed by RQ decomposition and all of them has capacity that increased with the number of antennas. However Gaussian Elimination and Gauss Jordan Elimination algorithms had gained less capacity a compared to LU and RQ Decomposition, and also then have decreased capacity when the number of antennas become more than 64 because of large elapsed time.

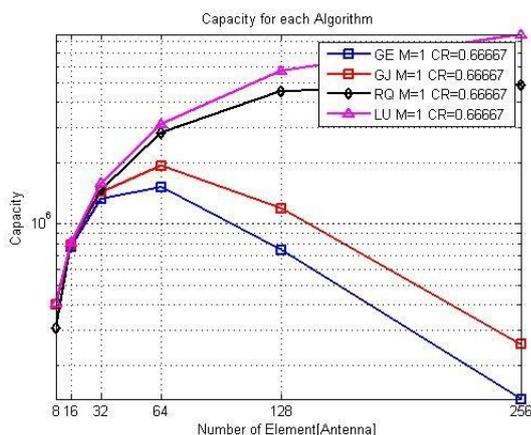


Figure 9. Capacity for Algorithms through using BPSK and CR=2/3 versus number of Antenna

Fig. 10 shows the throughput of different algorithms using BPSK with code rate 2/3. In which the better throughput is gained by LU decomposition algorithm followed by RQ decomposition and all of them has throughput that increased

with the number of antennas. However Gaussian Elimination and Gauss Jordan Elimination algorithms had gained less throughput a compared to LU and RQ Decomposition, and also then have decreased throughput when the number of antennas become more than 64 because of large elapsed time.

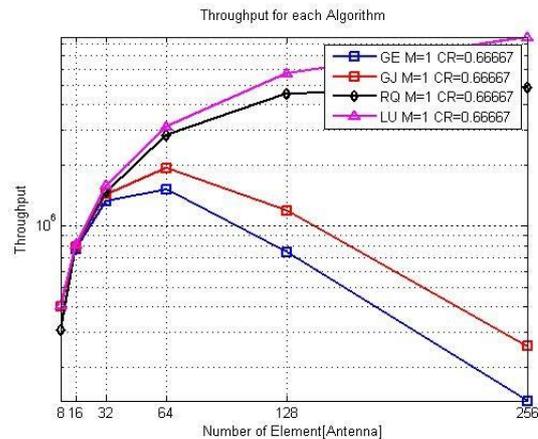


Figure 10. Throughput for Algorithms through using BPSK and CR=2/3 versus number of Antenna

While in Gaussian Elimination and Gauss Jordan Elimination algorithms had highest throughput at 64 elements for this BPSK modulation and coding rate 2/3. Delay is constrained that effect on channel bandwidth and hence throughput is affected as illustrated in TABLE II for capacity and throughput.

TABLE II THROUGHPUT FOR ALGORITHMS THROUGH USING BPSK AND CR=2/3 VERSUS NUMBER OF ANTENNAS

NO.	Algorithm	Capacity (b/s)	Throughput (b/s)	Percentage (%)
2	GE	7.2087e+06	7.1943e+06	43.7756
3	RQ	1.8173e+07	1.8136e+07	96.7278
4	LU	2.8008e+07	2.7952e+07	98.9848

V. CONCOLUSION

This paper introduced and evaluates a mechanism to reduce the elapsed time in massive MIMO detection considering four algorithms Gaussian Jordan Elimination, Gaussian Elimination, LU Decomposition and RQ Decomposition.

We found that the LU Decomposition had the lowest delay with percentage about (98%) compared Gauss-Jordan Elimination algorithms while RQ had (95%) delay element and Gaussian Elimination about (45%). in addition the best capacity for LU Decomposition is about 3 to 25 Mbs, RQ Decomposition from 2 to 16 Mbs, Gaussian Elimination from 1 to 8 Mbs and Gauss-Jordan Elimination from 0.7 to 6 Mbs and this depend on modulation and coding rate. In conclusion we can say LU Decomposition Algorithm has the best performance than another algorithms.

VI. RECOMMENDATIONS

After finishing this research work there are some other issues can be considering for future research these include:

It is requires work at channel rather than AWGN channel such as Rayleigh fading channel or different communication environment to be realistic. In addition there is a need to Study Massive MIMO in Downlink rather than Uplink in order to evaluate affect and performance on various channels.

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