ISSN: 2454-4248
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Optimal Lot Sizing for Perishable Products under Strict Carbon Cap Policy considering Stochastic Demand and Energy Usage cost

W. Ritha and S. Poongodisathiya
Department of Mathematics,
Holy Cross College (Autonomous),
Tiruchirappalli – 620002, India.

Abstract: In this paper, we have considered stochastic demand for perishable items under strict carbon cap policy and energy usage. Perishable foods like meat, poultry, fish, dairy products etc. which are likely to spoil if not kept refrigerated. So that we related to the energy usage for maintaining perishable items at certain climate conditions where the inventory is stocked. Due to the nature of perishable product starts to decay at certain time, so that vendor provide a discount for the product in demand rate. We model the system into two stage, On first stage holds fresh items as non-discount period and second stage as older items as discount period nearer to expiration. A mathematical model is developed to determine the optimal order quantity, reorder point and number of shipments in a two-echelon supply chain considering partial backorders. The objective is to minimize the total expected supply chain cost while satisfying the carbon emission constraint. A numerical example is given to illustrate the solution procedure.

Keywords: *Supply chain, perishable items, energy usage, strict carbon cap, carbon emission, stochastic demand*

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1. Introduction:

In this modern life, customers are more willing to buy perishable goods in marts or retail groceries. All perishable items have limited shelf life which can be stored in certain temperature, otherwise it will easily spoiled (Wang et. al(2012)). The increasing demand for perishable food leads to higher profit and more challenging of perishable food items to motivate and design of the supply chain system. (Ferguson et.al (2006), Karkkainen (2003)). Examples of perishable items are fruits, flowers, vegetables, eggs, cheese, ice- cream etc.

The development of modern technologies such as temperature and humidity sensors and RFID technology can help companies to establish food supply chain and hence improve the management of perishable food products. Since the decaying quality of perishable food leads to a demand slowdown, food retailers tend to implement promotion strategies to improve the efficiency of food supply chain.

In this paper extends the work of Arindam Ghosh, J.K.Jha, S.P.Sarmah (2017), which considered the strict carbon cap policy and partial backorder under stochastic demand. In this study, by assuming perishable items under stochastic demand. These perishable items are quickly spoil if not kept refrigerated. So that perishable items are maintaining certain climatic condition as related to energy usage cost. Also consider carbon cap policy, regulatory bodies allow organization to emit CO2 to a certain threshold limit, which helps to reduce carbon emission. We model the system into two stage of shelf life for perishable goods in demand rate. On the arrive of first stage holds fresh items as Non-discount shelf life, they may remain in the first stage upto T_0 time (i.e, $0 \leq t \leq T_0$) unit. Items that have not been requested by time upto T_0 outdate from stage one and are transferred to the second stage holds older items. On the arrive of second stage as discount shelf life period upto T time ($T_0 < t \leq T$) (Shuai Yang, Yujie Xiao, Yong-Hong Kuo (2017)). That is, perishable items come nearer to an expiry date. A mathematical model is to determine the optimal value of order quantity, reorder point and number of shipments in a incorporated stochastic demand. The objective is to minimize the total expected supply chain cost while satisfying carbon emission constraint.

The rest of the paper as follows Section 2 presents literature review, Section 3 presents notation and assumptions. Section 4 deals with the model formulation and solution technique. Section 5 presents a numerical example. Finally, conclusion are outlined in section 6.

2. Literature review:

Nowadays perishable items are major aspect in inventory control. With the modern identification and tracking technologies, advanced logistics management information system could be developed for the perishable product management. The increase in demand for perishable food brings about more profit while increased demand also makes it more difficult to manage with more quantities and varieties (Xiao, Y; Yang, S (2017)). The attrition rate of perishable food can reach 15% in retail stores and hence cause costs of billions of dollars, for example in European groceries (Hertog et. al (2014), Sciortino et. al (2016), La scalia et al.(2015)). There has been extensive work on improving the management of perishable products, where three main

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approaches have been adopted to model the characteristics of the perishability of food items. First the perishable item is assumed to have a random or fixed lifetime (Goyal, Giri (2001)). Second, by assuming that proportion of perishable products become unsalable after transhipments. While the rest retains the full value (Wee (1995), Mandal, Phaujdar (1989)). Third, by assuming the value of the perishable product decays over time due to deterioration (Zanoni, Zavanella (2012)).

Environmental pollution is a burning issue in recent era. Continuous emission of GHG into atmosphere have raised the threat to environment and existence of human civilization (World Health Organization. Climate change and health (2014)). Production, inventory, transportation contribute a major percentage of carbon emission in supply chain. Bonney and Jaber (2011), Hua et al.(2011)., Bouchery et al (2012)., Toptal et al.(2014)., and Chen et al.(2013)., derived optimal ordering/production decision in a single stage inventory model under different carbon policies. Wahab et al. (2011) incorporated screening and holding cost of defective items in their model. Jaber et al. (2013) considered different carbon policies and possible combination of these policies to develop mathematical models. Dobos (2007) reckoned production as the source of carbon emission and expressed emission rate as a function of production rate. Absi et al. (2013) formulated by lot sizing problems considering, periodic, cumulative, global and rolling carbon emission constraints separately. Li and Gu (2012) incorporated carbon banking and borrowing option under carbon cap and trade policy. Li (2014) extended the basic Arrow – Karlin model for deteriorating items with tradable emission permit. Benjaafar et al.(2013) developed four EOQ like model under different carbon policies and also extended their work for a serial supply chain. Rosič and Jammernegg (2013) extended the dual sourcing model based on the basic newsvendor model incorporating the environmental impact of transportation. Saadany et al. (2011) confirmed with their model that incorporation of environmental related initiatives can even increase the profitability of the firms and also quality of products do influence demand.

3. Notation and assumptions

The following notation is used to develop the model.

Notation:

D average demand rate on the buyer
S the vendor's setup cost per production setup
P the vendor's production rate
A the buyer's ordering cost per order
L lead time of the buyer
b fraction of the demand during the stockout period that will be backordered, $0 \leq b \leq 1$
$\eta(r)$ expected demand shortage at the end of each cycle of the buyer
θ the buyer's shortage cost per unit
θ_0 the buyer's profit per unit
p selling price
a market scale
$q(t)$ product quality at time t
σ standard deviation of demand per unit time
σ_L standard deviation of demand during lead time, $\sigma_L = \sigma\sqrt{L}$
n_g shelf space allocated to the product
E_a activation energy
F discount rack
γ_0 fixed cost per order ($\gamma_0 > 0$)
c_e cost per unit of energy ($c_e > 0$)
E_0 energy usage per order ($E_0 > 0$)
E_1 energy usage per setup ($E_1 > 0$)
x the lead time demand
d distance between the vendor and buyer
v velocity of the vechicle
ρ production cost per unit item
h_b, h_v the buyer's and vendor's holding cost per unit item per unit time respectively
E_b, E_v the buyer's and vendor's energy usage per unit of inventory per year respectively
t_0 transportation cost per unit time when the vehicle is empty
t_Q transportation cost per unit item per unit time when the vehicle is loaded
f fixed carbon emission per production setup

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π carbon emission per unit item due to production

α_b, α_v carbon emission per unit item per unit time due to inventory at the buyer and vendor respectively

τ_0 carbon emission per unit time due to transportation when the vehicle is empty

τ_Q carbon emission per unit item per unit time due to transportation when the vehicle is loaded

$\hat{C}$ cap (maximum limit) on carbon emission per unit time

Decision variables

m number of shipments from the vendor to buyer per production cycle

r reorder point of the buyer

Q the buyer's ordering quantity per order

Assumptions:

i) There is a single vendor and single buyer and they deal with a perishable items.

ii) In first stage, Linear demand model of non-discount shelf life for perishable items,

$D = a - gp + cn_g + Iq(t) + \epsilon, 0 \le t \le T_0$ where ϵ belongs to $U[-L, L]$ and g, c, I are all positive parameters representing price elasticity, demand sensitivity to shelf space and product quality, respectively.

In second stage, Linear demand model of discount shelf life for perishable items, $D = a - glp + Iq(t) + F + \epsilon$, $T_0 < t \le T$ where g, I are the same as in first stage, $1 - l$ is the discount rate.

iii) Energy usage cost for maintaining perishable items at certain climate conditions, where the inventory is stocked.

iv) The buyer orders a lot of size Q and the vendor produces mQ units with a finite production rate P (P>D) in one production setup but ships in quantity Q to the buyer over m times. The buyer reviews inventory continuously, and an order is placed whenever the inventory level drops to the reorder point r and there are no orders outstanding.

v) Any demand not met from stock is partially backordered during the stock out period.

vi) Production, inventory and transportation are the sources of emissions. Transportation emission is assumed inversely proportional to truck velocity and the truck will not speed.

4. Model formulation

In this paper extends the work of Arindam Ghosh, J.K.Jha, S.P.Sarmah (2017). In this paper incorporated stochastic demand into the perishable food supply chain considering carbon cap policy and energy usage. At this mathematical model is to find the optimal order quantity, reorder point and the number of shipments.

Quality degradation:

Quality degradation is a major issue for perishable items. The quality degradation of perishable food is affected by several factors such as storage time, temperature and ambient atmosphere condition . Quality degradation can be expressed by

$$\frac{dq}{dt} = -kq^n$$

where q as quality of a perishable product, k as rate of degradation, n as chemical order of the reaction. In above equation, n could be equal to 0 or 1, When $n = 0$, the quality decays at a constant rate. When $n = 1$, the quality decays exponentially. For this reason, we assumed that $n = 1$.In this above equation, k can be expressed as

$$k = k_0 e^{-\left(E_a/RT_0\right)}$$

where k_0 is constant, R is gas constant, T_0 is absolute temperature.

The quality of the perishable product at time t can be expressed as

$$q(t) = q_0 e^{-k_0 t e^{-\left(E_a/RT_0\right)}}$$

where q_0 as initial quality

We introduce λ as, $\lambda = k_0 e^{-\left(E_a/RT_0\right)}$

Hence, the quality at time t, becomes $q(t) = q_0 e^{-\lambda t}$

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Energy usage cost:

To keep the model as simple as possible, the focus will be limited to the environmental component of sustainability (viz., green sourcing) and using one of the measure as "Energy Usage". This is motivated by the fact that energy consumption reduction initiatives. As related to energy usage cost for perishable items to maintaining certain temperature at the warehouse where the inventory is stocked.

Now consider the situation where both the ordering and holding cost are partly determined by the use of energy.
Ordering cost per order A, $A = \gamma_0 + c_e E_o$
Setup cost per order S, $S = \gamma_0 + c_e E_1$
The buyer's and vendor's Holding cost are determined by the use of energy respectively,
i.e, $h_b = \gamma_H + c_e E_b$ and $h_v = \gamma_H + c_e E_v$ where γ_H per unit annual inventory holding cost not related to the use of energy
Consider α be relative weight assigned to cost objective ($0 \le \alpha \le 1$) when compared to environmental objective.
The model is derived based on Hadley and Whitin (1963), By assuming that the buyer orders a lot of size Q, the vendor produces the item in a lot of size mQ, with a constant production rate P in each production cycle and ships to the buyer in m lots each of size Q. The first lot of size Q is ready for shipment after Q/P units of time after the start of the production, and then the vendor continues the delivery on average every Q/D units of time.
When the lead time demand follows normal distribution, the expected shortage per cycle can be obtained as,

$$\eta(r) = \int_r^\infty (x-r)f(x)dx = \sigma_L f\left(\frac{r-DL}{\sigma_L}\right) + (DL-r)G\left(\frac{r-DL}{\sigma_L}\right),$$

where $f(x)$ and $G(x)$ are the standard normal density function and the complimentary cumulative distribution function of the standard normal distribution, respectively.
We model the system into two stages, on the first stage holds fresh items as non-discount shelf life upto T_0 time unit (i.e, $0 \le t \le T_0$) and on the second stage holds older items as discount shelf life after T_0 unit of time (i.e, $T_0 < t \le T$).

Stage 1: Demand model for non - discount shelf life at $0 \le t \le T_0$

The linear model of demand can be characterized by

$$D = a - gp + cn_g + Iq(t) + \epsilon, 0 \le t \le \mathrm{T}_0$$

The total demand on non-discount shelf life is

$$\int_0^{\mathrm{T}_0} D(t)dt = (a - gp + cn_g)\mathrm{T}_0 + \mathrm{Iq}_0\left(1 - e^{-\lambda \mathrm{T}_0}\right)/\lambda + \epsilon\mathrm{T}_0,$$

where ϵ belongs to $U[-L, L]$

Let D be the expected demand on non-discount shelf life is

$$D = (a - gp + cn_g)\mathrm{T}_0 + \mathrm{Iq}_0\left(1 - e^{-\lambda \mathrm{T}_0}\right)/\lambda$$

The total expected cost per unit time for the supply chain can be expressed as

$$\boldsymbol{TC}\,(\boldsymbol{Q}, \boldsymbol{m}, \boldsymbol{r}) = \alpha\left(\frac{AD}{Q} + \frac{SD}{mQ}\right) + \rho D + \alpha h_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right)$$
$$+\alpha h_v \frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + (1-\alpha)E_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right)$$
$$+(1-\alpha)\left(\frac{E_0 D}{Q} + \frac{E_1 D}{mQ}\right) + (1-\alpha)E_v \frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + \frac{t_0 dD}{vQ} + \frac{t_Q dD}{v}$$
$$+\frac{\theta D\eta(r)}{Q} + \frac{\theta_0(1-b)D\eta(r)}{Q} \tag{1}$$

where $D = (a - gp + cn_g)\mathrm{T}_0 + \mathrm{Iq}_0\left(1 - e^{-\lambda \mathrm{T}_0}\right)/\lambda$

The total expected carbon emission from unit time as

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$$TE(Q,m,r) = \frac{fD}{mQ} + \pi D + \alpha_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right)$$

$$+\alpha_v \frac{Q}{2}\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + \frac{\tau_0 dD}{vQ} + \frac{\tau_Q dD}{v} \qquad (2)$$

As we considering strict carbon cap policy, The carbon emission constraint can be written as

$$\frac{fD}{mQ} + \pi D + \alpha_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right) + \alpha_v \frac{Q}{2}\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + \frac{\tau_0 dD}{vQ} + \frac{\tau_Q dD}{v} \leq \hat{C}$$
(3)

where $D = \left(a - gp + cn_g\right)\mathrm{T}_0 + \mathrm{Iq}_0\left(1 - e^{-\lambda \mathrm{T}_0}\right)/\lambda$

Solution Methodology:

To find the optimal value of order quantity Q, reorder point r and number of shipments m.

The convexity of the TC$(Q,m.r)$ with respect to m for fixed (Q,r)

i.e, $\frac{\partial \mathrm{TC}(Q,m.r)}{\partial m} = 0$ then

$$m = \sqrt{\frac{2D(S\alpha + (1-\alpha)E_1)}{Q^2\left(1-\frac{D}{P}\right)[\alpha h_v + (1-\alpha)E_v]}}$$

where $D = \left(a - gp + cn_g\right)\mathrm{T}_0 + \mathrm{Iq}_0\left(1 - e^{-\lambda \mathrm{T}_0}\right)/\lambda$

To derive the optimal value of Q and r for fixed m

$$\frac{\partial \mathrm{TC}(Q,m.r)}{\partial Q} = 0 \quad \text{we get}$$

$$Q_0 = \sqrt{\frac{2D\left[\alpha\left(A + \frac{S}{m}\right) + (1-\alpha)\left(E_0 + \frac{E_1}{m}\right) + \frac{t_0 d}{v} + \theta\eta(r) + \theta_0(1-b)\eta(r)\right]}{\alpha(h_b - E_b) + E_b + [E_v + \alpha(h_v - E_v)]\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)}}$$

similarly,

$$r = DL + \left(NORMSINV\left(1 - \frac{Q(\alpha h_b + (1-\alpha)E_b)}{Q(1-b)(\alpha h_b + (1-\alpha)E_b) + \theta D + \theta_0(1-b)D}\right)\right)\sigma_L$$

where $D = \left(a - gp + cn_g\right)\mathrm{T}_0 + \mathrm{Iq}_0\left(1 - e^{-\lambda \mathrm{T}_0}\right)/\lambda$

Now we need to determine the optimal value of Q for fixed m and r that satisfies the carbon constraint. The roots of the corresponding quadratic equation (3) of inequality is given by

$$Q_1 = \frac{\hat{C} - \pi D - \frac{\tau_Q dD}{v} - \alpha_b\left(r - DL + (1-b)\eta(r)\right) - \left[\begin{array}{l}\left(\alpha_b\left(r - DL + (1-b)\eta(r)\right) + \pi D + \frac{\tau_Q dD}{v} - \hat{C}\right)^2 \\ -2\left(\alpha_b + \alpha_v\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)\right)\left(\frac{fD}{m} + \frac{\tau_0 dD}{v}\right)\end{array}\right]^{1/2}}{\alpha_b + \alpha_v\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)}$$

and

$$Q_2 = \frac{\hat{C} - \pi D - \frac{\tau_Q dD}{v} - \alpha_b\big(r - DL + (1-b)\eta(r)\big) + \begin{bmatrix} \left(\alpha_b\big(r - DL + (1-b)\eta(r)\big) + \pi D + \frac{\tau_Q dD}{v} - \hat{C}\right)^2 \\ -2\left(\alpha_b + \alpha_v\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)\right)\left(\frac{fD}{m} + \frac{\tau_0 dD}{v}\right) \end{bmatrix}^{1/2}}{\alpha_b + \alpha_v\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right)}$$

where $D = (a - gp + cn_g)\mathrm{T}_0 + \mathrm{Iq}_0\left(1 - e^{-\lambda \mathrm{T}_0}\right)/\lambda$

where Q_1 and Q_2 are the lower and upper bound respectively, for the feasible range of Q.

The optimal $\hat{Q}$ can be obtained by satisfying the following condition

$$\hat{Q} = \begin{cases} Q_0, if\ Q_1 \le Q_0 \le Q_2 \\ Q_1, if\ Q_0 \le Q_1 \\ Q_2, if\ Q_0 \ge Q_2 \end{cases} \qquad (*)$$

Set $\sigma_L = 0$, since Q and r are interdependent and get an initial value of Q_0, Q_1, Q_2 and by using above condition, then we get an value of $\hat{Q}$. Using the value of $\hat{Q}$, we find the initial value of r. This r in turn used in Q_0, Q_1, Q_2 to determine the new value of $\hat{Q}$ by using above condition.

Stage 2: Demand model of discount shelf life at $T_0 < t \le T$

A perishable food have a limited shelf life. These items have an expiration date, such food will go bad if not eaten in a certain amount of time. Consumers are likely to purchase perishable goods when their expiration are near. For this reason, vendor implement a discount pricing policy when the products have reached closer to the expiry dates. Therefore , the demand function after discount imposed on the items is

$$D = a - glp + Iq(t) + F + \epsilon\,, T_0 < t \le T$$

The total demand on discount shelf life is

$$\int_{\mathrm{T}_0}^{\mathrm{T}} D(t)dt = (a - glp + F)(\mathrm{T} - \mathrm{T}_0) + \mathrm{Iq}_0\left(e^{-\lambda \mathrm{T}_0} - e^{-\lambda \mathrm{T}}\right)/\lambda + \epsilon(\mathrm{T} - \mathrm{T}_0)\,,$$

where ϵ belongs to $U[-L, L]$

Let D be the expected demand on discount shelf life is

$$D = (a - glp + F)(\mathrm{T} - \mathrm{T}_0) + \mathrm{Iq}_0\left(e^{-\lambda \mathrm{T}_0} - e^{-\lambda \mathrm{T}}\right)/\lambda$$

The total expected cost per unit time for the supply chain can be expressed as

$$\begin{aligned} \boldsymbol{TC}\,(\boldsymbol{Q}, \boldsymbol{m}, \boldsymbol{r}) &= \alpha\left(\frac{AD}{Q} + \frac{SD}{mQ}\right) + \rho D + \alpha h_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right) \\ &+ \alpha h_v \frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + (1-\alpha)E_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right) \\ &+ (1-\alpha)\left(\frac{E_0 D}{Q} + \frac{E_1 D}{mQ}\right) + (1-\alpha)E_v \frac{Q}{2}\left(m\left(1 - \frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + \frac{t_0 dD}{vQ} + \frac{t_Q dD}{v} \\ &+ \frac{\theta D\eta(r)}{Q} + \frac{\theta_0(1-b)D\eta(r)}{Q} \end{aligned} \qquad (4)$$

where $D = (a - glp + F)(\mathrm{T} - \mathrm{T}_0) + \mathrm{Iq}_0\left(e^{-\lambda \mathrm{T}_0} - e^{-\lambda \mathrm{T}}\right)/\lambda$

The total expected carbon emission from unit time as

$$TE(Q,m,r) = \frac{fD}{mQ} + \pi D + \alpha_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right)$$

$$+\alpha_v \frac{Q}{2}\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + \frac{\tau_0 dD}{vQ} + \frac{\tau_Q dD}{v} \qquad (5)$$

As we considering strict carbon cap policy, The carbon emission constraint can be written as

$$\frac{fD}{mQ} + \pi D + \alpha_b\left(\frac{Q}{2} + r - DL + (1-b)\eta(r)\right) + \alpha_v \frac{Q}{2}\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right) + \frac{\tau_0 dD}{vQ} + \frac{\tau_Q dD}{v} \le \hat{C}$$
(6)

where $D = (a - glp + F)(\mathrm{T} - \mathrm{T}_0) + \mathrm{I}q_0\left(e^{-\lambda \mathrm{T}_0} - e^{-\lambda \mathrm{T}}\right)/\lambda$

Solution Methodology:

To find the optimal value of order quantity Q, reorder point r and number of shipments m.

The convexity of the TC$(Q, m.r)$ with respect to m for fixed (Q, r)

i.e, $\frac{\partial \mathrm{TC}(Q,m.r)}{\partial m} = 0$ then

$$m = \sqrt{\frac{2D(S\alpha + (1-\alpha)E_1)}{Q^2\left(1-\frac{D}{P}\right)[\alpha h_v + (1-\alpha)E_v]}}$$

where $D = (a - glp + F)(\mathrm{T} - \mathrm{T}_0) + \mathrm{I}q_0\left(e^{-\lambda \mathrm{T}_0} - e^{-\lambda \mathrm{T}}\right)/\lambda$

To derive the optimal value of Q and r for fixed m

$\frac{\partial \mathrm{TC}(Q,m.r)}{\partial Q} = 0$ we get

$$Q_0 = \sqrt{\frac{2D\left[\alpha\left(A + \frac{S}{m}\right) + (1-\alpha)\left(E_0 + \frac{E_1}{m}\right) + \frac{t_0 d}{v} + \theta\eta(r) + \theta_0(1-b)\eta(r)\right]}{\alpha(h_b - E_b) + E_b + [E_v + \alpha(h_v - E_v)]\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)}}$$

similarly,

$$r = DL + \left(NORMSINV\left(1 - \frac{Q(\alpha h_b + (1-\alpha)E_b)}{Q(1-b)(\alpha h_b + (1-\alpha)E_b) + \theta D + \theta_0(1-b)D}\right)\right)\sigma_L$$

where $D = (a - glp + F)(\mathrm{T} - \mathrm{T}_0) + \mathrm{I}q_0\left(e^{-\lambda \mathrm{T}_0} - e^{-\lambda \mathrm{T}}\right)/\lambda$

Now we need to determine the optimal value of Q for fixed m and r that satisfies the carbon constraint. The roots of the corresponding quadratic equation (6) of inequality is given by

$$Q_1 = \frac{\hat{C} - \pi D - \frac{\tau_Q dD}{v} - \alpha_b(r - DL + (1-b)\eta(r)) - \begin{bmatrix}\left(\alpha_b(r - DL + (1-b)\eta(r)) + \pi D + \frac{\tau_Q dD}{v} - \hat{C}\right)^2 \\ -2\left(\alpha_b + \alpha_v\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)\right)\left(\frac{fD}{m} + \frac{\tau_0 dD}{v}\right)\end{bmatrix}^{1/2}}{\alpha_b + \alpha_v\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)}$$

and

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$$Q_2 = \frac{\hat{C} - \pi D - \frac{\tau_Q dD}{v} - \alpha_b\left(r - DL + (1-b)\eta(r)\right) + \left[\begin{matrix}\left(\alpha_b\left(r-DL+(1-b)\eta(r)\right) + \pi D + \frac{\tau_Q dD}{v} - \hat{C}\right)^2 \\ -2\left(\alpha_b + \alpha_v\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)\right)\left(\frac{fD}{m} + \frac{\tau_0 dD}{v}\right)\end{matrix}\right]^{1/2}}{\alpha_b + \alpha_v\left(m\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right)}$$

where $D = (a - glp + F)(\mathrm{T} - \mathrm{T}_0) + \mathrm{I}q_0\left(e^{-\lambda \mathrm{T}_0} - e^{-\lambda \mathrm{T}}\right)/\lambda$

where Q_1 and Q_2 are the lower and upper bound respectively, for the feasible range of Q.

The optimal $\hat{Q}$ can be obtained by satisfying the following condition

$$\hat{Q} = \begin{cases} Q_0, if\ Q_1 \le Q_0 \le Q_2 \\ Q_1, if\ Q_0 \le Q_1 \\ Q_2, if\ Q_0 \ge Q_2 \end{cases} \qquad (*)$$

The following algorithm is developed to determine the optimal Q, r and m.

Algorithm:

i) Set $m = 1, \sigma_L = 0$
ii) Compute initial value of Q_0, Q_1, Q_2 respectively.
iii) Select appropriate value of $\hat{Q}$ satisfying the condition given.
iv) Compute r using $\hat{Q}$.
v) Find Q_0, Q_1, Q_2 using initial value of r respectively.
vi) Select the appropriate value of $\hat{Q}$ using the condition given.
vii) Repeat step (iv) and (v) until no change occur in the value of Q and r.
viii) Set $\hat{Q}_{(m)} = \hat{Q}$ and $r_m = r$. Thus $(\hat{Q}_{(m)}, r_m)$ is the optimal solution for fixed m and compute $TC\ (\hat{Q}_{(m)}, r_m, m)$
ix) Set $m = m + 1$, and repeat steps (i) to (viii) to get new $TC(\hat{Q}, r, m)$

5. Numerical Example

To illustrate the proposed model of both non-discount and discount shelf life of perishable product and solution procedure, a two-echelon supply chain is considered with the following data; $P = 2000$ units per year, $\theta = 50$ per unit, $\theta_0 = 65$ per unit, $d = 100$ km, $v = 50$ km/hr, $\rho = 200$ per unit, $t_0 = 10/hr$, $t_Q = 5/unit$ per hour, $\alpha_b = 0.80$ ton per unit, $\alpha_v = 0.80$ ton per unit,$f = 20$, $\pi = 2$ ton per unit, $p = 2$ per unit, $n_g = 1.9$ units per year, $a = 7.92$, $q_0 = 0.95$, $g = 4.86$, $c = 3$, $l = 0.85$, $I = 4.86$, $T = 72$, $T_0 = 60$, $F = 4$, $\lambda = 0.01$, $\tau_0 = 0.03$ ton per unit, $\tau_Q = 0.004$ ton per unit, $b = 0.6$, $\sigma = 1$unit per day , (Assume one year = 365 days) $L = 7$ days, $\hat{C} = 1550\ ton$, $E_0 = 245$ energy per order, $c_e = 2$, $\gamma_0 = 410$, $\gamma_H = 36$ per unit per year, $E_1 = 545$ energy per setup, $A = 900$ per order, $S = 1500$ per setup, $h_b = 60$ per unit per year, $h_v = 60$ per unit per year, $E_b = 12$ per unit per year, $E_v = 12$ per unit per year, $\alpha = 0.3$
It follows that,
At the time of Non-discount shelf life period: $0 \le t \le T_0$
$D = 460$units per year, $\hat{Q} = 200$, $m^* = 2$, $r^* = 12$, $TC = 104{,}974.75$, $TE = 1120$
At the time of Discount shelf life period: $T_0 < t \le T$
$D = 75$ units per year, $\hat{Q} = 54$, $m^* = 2$, $r^* = 4$, $TC = 18{,}393.7$, $TE = 236.29$
The total time of both Non-discount and Discount shelf life period have
$D = 535$ units per year, $\hat{Q} = 254$, $m^* = 4$, $r^* = 16$, $TC = 123368.45$, $TE = 1356.2$

6. Conclusion:

In this paper considered that perishable items and strict carbon cap policy to reduce carbon emission under stochastic demand. These perishable items are easily decay, So that we introduce energy usage for maintaining perishable items at certain temperature, note that it is assumed that required energy usage depends linearly on the number of orders and inventory size. This is motivated by the fact that energy consumption reduction initiatives with managing green house gas emission and the organisation carbon footprint. On this second stage vendor declare a discount for perishable items due to avoid of product spoilage. From the numerical result we can able to find that after considering perishable items it would reduce the total cost of the system. Further, one can consider a multi-echelon supply chain model can be a good direction for future study.

References:

[1]. N. Absi, S. Dauzère-Pérès, S. Kedad-Sidhoum, B. Penz, C. Rapine, Lot sizing with carbon emission constraints, Eur. J. Oper. Res. 227 (1) **2013** 55–61.

[2]. Arindam Ghosh , J.K. Jha , S.P. Sarmah , Optimal Lot-Sizing under Strict Carbon Cap Policy Considering Stochastic Demand, *Applied Mathematical Modelling* **2017**, doi: 10.1016/j.apm.2017.02.037

[3]. Benjaafar, Y. Li, M. Daskin, Carbon footprint and the management of supply chains : insights from simple models, IEEE Trans. Autom. Sci. Eng. 10 (1) **2013** 99–116.

[4]. M. Bonney, M.Y. Jaber, Environmentally responsible inventory models: Non-classical models for a non-classical era, Int. J. Prod. Econ. 133 **2011** 43–53.

[5]. Y. Bouchery, A. Ghaffari, Z. Jemai, Y. Dallery, Including sustainability criteria into inventory models, Eur. J. Oper. Res. 222 (2) **2012** 229–240

[6]. X. Chen, S. Benjaafar, A. Elomri, The carbon-constrained EOQ, Oper. Res. Lett. 41 (2) **2013** 172–179.

[7]. I. Dobos, Tradable permits and production-inventory strategies of the firm, Int. J. Prod. Econ. 108 **2007** 329–333.

[8]. A.M.A. El Saadany, M.Y. Jaber, M. Bonney, Environmental performance measures for supply chains, Manag. Res. Rev. 34 (11) **2011** 1202–1221.

[9]. Ferguson, M.E.; Ketzenberg, M.E. Information Sharing to Improve Retail Product Freshness of Perishables.Prod. Oper. Manag. **2006**, 15, 57–73.

[10]. Goyal, S.K.; Giri, B.C. Recent trends for modeling deteriorating inventory. Eur. J. Oper. Res. **2001**, 134, 1–16.

[11]. G. Hadley, T. Whitin, Analysis of Inventory Systems, Prentice-Hall, Englewood Cliffs, New Jersey,**1963**

[12]. Hertog, M.L.A.T.M.; Uysal, I.; McCarthy, U.; Verlinden, B.M.; Nicolaï, B.M. Shelf life modelling for first-expired-first-out warehouse management. Philos. Trans. R. Soc. A Math. Phys. Eng. Sci. **2014**,372, 1–15.

[13]. G. Hua, T.C.E. Cheng, S. Wang, Managing carbon footprints in inventory management, Int. J. Prod.Econ. 132 **2011** 178–185.

[14]. M.Y. Jaber, C.H. Glock, A.M. A. El Saadany, Supply chain coordination with emissions reduction incentives, Int. J. Prod. Res. 51 (1) **2013** 69–82.

[15]. Karkkainen, M. Increasing efficiency in the supply chain for short shelf life goods using RFID tagging. Int. J.Retail Distrib. Manag. **2003**, 31, 529–536.

[16]. La Scalia, G.; Settanni, L.; Corona, O.; Nasca, A.; Micale, R. An innovative shelf Life model based on smart logistic unit for an efficient management of the perishable food supply chain. J. Food Process Eng. **2015**.

[17]. S. Li, M. Gu, The effect of emission permit trading with banking on firm's production–inventory strategies, Int. J. Prod. Econ. 137 (2) **2012** 304–308.

[18]. S. Li, Optimal control of the production–inventory system with deteriorating items and tradable emission permits, Int. J. Syst. Sci. 45 (11) **2014** 2930–2401.

[19]. Mandal, B.N.; Phaujdar, S. An inventory model for deteriorating items and stock-dependent consumption rate. J. Oper. Res. Soc. **1989**, 40, 483–488.

[20]. Petruzzi, N.; Dada, M. Pricing and the newsvendor problem: A review with extensions. Oper. Res. **1999**, 47, 183–194.

[21]. H. Rosič, W. Jammernegg, The economic and environmental performance of dual sourcing: a newsvendor approach, Int. J. Prod. Econ. 143(1) **2013** 109–119.

[22]. Sciortino, R.; Micale, R.; Enea, M.; La Scalia, G. A WebGIS-based System for Real Time Shelf Life Prediction.Comput. Electron. Agric. **2016**, 127, 451–459.

[23]. Shuai Yang, Yujie Xiao and Yong-Hong Kuo "The supply chain design for perishable food with stochastic demand".*Sustainability* **2017,** 1-12

[24]. A. Toptal, H. Özlü, D. Konur, Joint decisions on inventory replenishment and emission reduction investment under different emission regulations, Int. J. Prod. Res. 52 (1) **2014** 243–269.

[25]. Tsan - Ming Choi, Handbook of EOQ inventory problems: Stochastic and deterministic models and Applications. International series in operation research and management science. Springer doi: 10.1007/978-1-4614-7639-9

[26]. M.I.M. Wahab, S.M.H. Mamun, P. Ongkunaruk, EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact, Int. J. Prod. Econ. 134 (1) **2011** 151–158.

[27]. Wang, X.; Li, D. A dynamic product quality evaluation based pricing model for perishable food supply chains. Omega **2012**, 40, 906–917.

[28]. Wee, H.M. Joint pricing and replenishment policy for deteriorating inventory with declining market. Int. J. Prod. Econ. **1995**, 40, 163–171.

[29]. World Health Organization. Climate change and health. (2014). Available from <http://www.who.int/mediacentre/factsheets/fs266/en/>, 2015 (17. 07. 2015).

[30]. Xiao, Y.; Yang, S. The retail chain design for perishable food: The case of price strategy and shelf space allocation. Sustainability **2017**, 9, 12.

[31]. Zanoni, S.; Zavanella, L. Chilled or frozen? Decision strategies for sustainable food supply chains. Int. J.Prod. Econ. **2012**, 140, 731–736.