Application of DTM Method for Solving Electrical Engineering Problems of Simple Electric Circuits

Prof. Narhari Onkar Warade
Research Scholar Dept. of Mathematics,
J.J.T. University Churu, Rajasthan, India
e-mail: narhariwarade@gmail.com

Dr. Prabha Rastogi
Departmetn of Mathematic
J.J.T. University Churu, Rajasthan, India
e-mail: prabha.rastogi54@yahoo.com

Abstract - In this paper Chou's Method (DTM) for solving initial valve problems involving first order ordinary differential quotations we introduce the concept of DTM & applied it to obtain solution of three examples for demonstration. The results are compare with exact solution & DTM solutions.

Keywords - *Ordinary differential quotations chou's method, Initial valve problems LR-Circuits, RC- Circuit.*

*****

I. INTRODUCTION

The purpose of this paper is to employ the DTM method on example of LR- Circuits, RC- Cirenits, Which are very simple.

The purpose of this paper is to employ the DTM method on examples of ordinary differential equation of first order and compared with result obtain by exact solution by using complimentary function & particular integral. In recent years,

Bizar J. used for Riccati differential equation(1), Opanuga On numerical solution of systems of ordinary differential equitations by numeriacla analytical method (2), Chen used DTM to obtain the solutions of nonlinear system of differential quotations (3), DTM was first proposed by Zhou & Proved that DTM is an iterative procedure for obtaining analytic Taylor's series solution of differential equations DTM is useful to solve ordinary diff equations. & boundary value problems (4), Ayaz F has used DTM to find the series solution of system of differential equitation(5), Duen Y use DTM for Burger's equation to obtain the series solution(6), Bert W. has applied DTM on system of linear equation and analysis of its solutions(7), Chen C.L. has applied DTM technique for steady nonlinear beat conduction problems(8), Using DTM Hassan have find out series solution and that solution compared with DTM method for linear & non linear initial value problems & proved that DTM is reliable tool to find numerical solution(9), Khaled Batiha has been used DTM to obtain the Taylor's series as solution of linear, nonlinear system of ordinary differential equations(10), Montri Thangmoon has been used to find numerical solution of ordinary differential equations(11), Edeki, A semi method for solutions of a certain class of first order ordinary differential equations

II. BASIC DEFINITIONS & PROPERTIES OF DTM METHOD

v(t) can be expressed by Taylor's series, then v(t) can be represented as

$$v(t) = \sum_{k=0}^{\infty} \frac{(t-ti)^k}{k!} V(k)$$

v(t) is called inverse of V(k)

$$\therefore v(t) = \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] V(k) = D^{-1} V(k)$$

$$v(t) = \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] V(k) + R_{n+1}(t)$$

by Taylor's Series

$$V(k) = \frac{1}{k!}\left[\frac{d^k v(t)}{dt^k}\right] \quad \text{at} \quad t = t_0$$

III. FUNDAMENTAL THEOREMS ON DTM

Theorem 1 :- If $p(t) = n(t) \pm s(t)$ then $P(k) = N(k) \pm S(k)$

Theorem 2 :- If $p(t) = \propto(t)$ then $\propto n(t)$ then $P(k) = \propto N(k)$

Theorem 3 :- If $p(t) = \frac{dn(t)}{dt}$ then

$P(k) = (k+1) N(k+1)$

Theorem 4 :- If $p(t) = \frac{d^2 n(t)}{dt^2}$ then

$P(k) = (k+1)(k+2)(k+2) N(k+2)$

Theorem 5 :- If $p(t) = \frac{d^s n(t)}{dt^s}$ then

$P(k) = (k+1)(k+2)(k+3)\ldots(k+s) N(K+s)$

Theorem 6 :- If $p(t) = t^s$ them

$$P(K) = \sum_{l=0}^{k} S(l) P(k-l)$$

Theorem 7 :- If $p(t) = t^s$ them

$P(k) = \delta(k-s)$

$$\delta(k-s) = \begin{cases} 1 \text{ if } k = s \\ 0 \text{ if } k \neq s \end{cases}$$

Theorem 8:- If $p(t) = e^{\lambda t}$ then

 ISSN: 2454-4248
742 – 746

$P(k) = \frac{\lambda^k}{k!}$

Theorem 9:- If $p(t) = (1+t)^s$ then

$P(k) = \frac{S(s-1)..(s-k+1)}{k!}$

Theorem 10:- if $P(t) = \sin(wt + \propto)$ then
$P(k) = \frac{w^k}{ki} \sin(\frac{\pi k}{2} + \propto)$

Theorem 11:- if $p(t) = \cos(wt + \propto)$ then
$P(k) = \frac{w^k}{ki} \cos(\frac{\pi k}{2} + \propto)$

IV. EXPERIMENTATION AND VALIDATION OF RESULTS

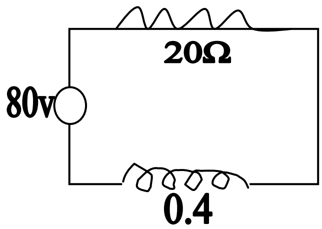
Example : 1

LR- Circuit

Find the current at any time t70 in a circuit having in series a constant electromotive force 80v, a resistar 20 and and induction 0.4 H given that initial current is zero find current in circuits.

→ Equation of LR – Circuit is

$L \frac{di}{dt} + RI = E(t)$



L = 0.4, R = 20, E- 80

$0.4 \frac{dI}{dt} + 20I = 80$

$4 \frac{dI}{dt} + 200I = 800$

$\frac{dI}{dt} + 50I = 200$

General solution is given by

I etc. $e^{5xt} = 200.\ e^{5xt}\ dt + C = 200. \frac{e^{50t}}{50} + C$

I (+) = e^{-50t} [4e^{50t}+c]

Given I (0) = 0 =

0 = 4 + C = C = -4

∴ $I(t) = 4(1 - e^{50t})$

$= 4 - 4(1 - 50t + \frac{(50t)^2}{2!} - \frac{(50t)^3}{3!} + \frac{(50t)^4}{4!})$

$= 4 - [4 - 200t + 5000t^2 - {}^2/_3 (50t)^3 + \frac{50t^4}{6}$

$R(t) = 200t - 5000t2 + {}^2/_3 (50t)^3 - \frac{(50t)^4}{6} + \ldots$

By Zhon's Method

I' (t) + 50 I (t) = 200

(K+1) I (K+1) + 50 I(k) = 200

Put K = 0, 1, 2, 3….

I (1) = 200

2 I (2) + 50 I (1) =

2I (2) + 10,000 =

2I (1) + 5000 =

I (2) = – 5000

3I (3) + 50 I (2) =0

3I (3) + 50 x (-5000) =0

$I(3) = \frac{250000}{3}$

4I (4) + 50 I (3) = 0

$4I(4) = \frac{-250000 \text{ x } 25}{3 \text{ x } 2}$

$= \frac{6250000}{6}$

I(5) =

I (6) =

$I(t) = I(0) + tI(1) + \frac{t^2}{2!} + t^3 I(3) + t^4 I(4) +$

$I(t) = 200t - 5000t^2 + 43 (50t)3 - \frac{-(50t)^5}{6} + \frac{(50t)^5}{30}$

$= 4[1 - e^{-50t}]$

Table-1

t	Exact Solution	DTM Solution	Error
0.00000	0.000000	0.000000	0.000000
0.1	3.97304842	3.97304842	0.000000
0.2	3.9998184	3.9998184	0.000000
0.3	3.999998776	3.999998776	0.000000
0.4	3.999999992	3.999999992	0.000000
0.5	4.00000	4.00000	0.000000
0.6	4.00000	4.00000	0.000000
0.7	4.00000	4.00000	0.000000
0.8	4.00000	4.00000	0.000000
0.9	4.00000	4.00000	0.000000
1.0	4.00000	4.00000	0.000000

ISSN: 2454-4248
742 – 746

Ex. 2 A generator having emt 20 cosst volts is connected in series with 10 ohms resistor and inductor of 2 henries. Rt the switch is closed at a time t=0 determine the current at time t>0

$$L\frac{dI}{dt} + RI = E$$

$$2\frac{dI}{dt} + 10I = 20\cos 5t$$

$$\frac{dI}{dt} + 5I = 10\cos 5t$$

$$I \,.\, e^{st} = \int e^{5t}\, 10 \cos st\, dt + c$$

$$I.\ e^{st} = 10\left[\frac{e^{5t}}{(5^2+5^2)}\ (5\cos t + 5\sin 5t\ ) + C\right.$$

$$= 10\left[\frac{e^{5t}}{(50)}\ (5\cos t + 5\sin 5t\ )\right] + C$$

$$I(t) = \text{co}5t + \sin 5t + ce^{-st} = I(0) = 0$$

$$= c = -1$$

$$= \text{co}5t + \sin 5t - e^{-5t}$$

$$= \left[1 - \frac{(5t)^2}{2!} + \frac{(5t)^4}{4!} - \frac{(5t)^6}{6!} + \ldots\ldots\right.$$

$$+ \left[(5t) - \frac{(5t)^3}{3!} + \frac{(5t)^5}{5!} - \frac{(7t)^7}{7!} + \ldots\ldots\right.$$

$$- \left[1 - \frac{(5t)}{1!} + \frac{(5t)^2}{2!} - \frac{(5t)^3}{3!} + \frac{(5t)^4}{4!}\right]$$

$$= 10t - (5t)^2 + 2\frac{(5t)^5}{5!} + \ldots..$$

DTM

$$(K+1)\, I\,(K+1) + 5\, I\,(K) = 10 \left[\frac{s^k}{sk!}\ Cos\,({}^{\pi}/_{2}\ k)\right]$$

k = 0,1,2,3

I (1) + 5 (0) = 10

I (1) = 10

$$2I\,(2) + 5\,(1) = 10\left[\frac{s^1}{!!}\ \cos t/_2\right]$$

$$I\,(2) = \frac{-5}{2} \text{x}10 = \frac{-50}{2} = -25$$

$$3I\,(3) + 5\,I\,(2) = 10\left[\frac{5^2}{2!}(-1)\right]$$

$$3I\,(3) - 125 = \frac{5\ x\ 25}{1}$$

3I (3) = - 125 + 125 = 0

4I (4) + 5 I (3) = 0

4I (4) = -5 I (3) = 0

I (4) = 0

$$5I\,(5) + \frac{5I\,(4)}{0} = 10\left[\frac{5^4}{4!}\,.\,1\right]$$

$$I\,(0) + tI\,(1) + \frac{t^2}{2!} + t^3\, I\,(3) + t^4\, I\,(4) + \ldots.$$

$$0 + 10\,t - 25t^2 + \frac{(25)^2}{12}\ t^5 - \frac{3125}{72}\ t^6 + \ldots..$$

Table – 2

t	Exact Solution	DTM Solution	Error
0.00000	0.000000	0.000000	0.000000
0.1	0.75047744	0.75047744	0.000000
0.2	1.01389385	1.01389385	0.000000
0.314	0.7927508	0.7927508	0.000000
0.4	0.3578153	0.3578153	0.000000
0.5	-0.2847564	-0.2847564	0.000000
0.6	-0.898659	-0.898659	0.000000
0.7	-1.3174372	-1.3174372	0.000000
0.8	-1.42876175	-1.42876175	0.000000
0.9	-1.1994349	-1.1994349	0.000000
1.0	-0.68200	-0.68200	0.000000

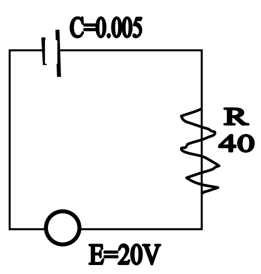
EX 3

RC – Circuit

A- Capacitor C = (.005) F in series with a resistor R = 40 ohms is changed from a battery E0=20 V assumes that initially the capacitor is completely uncharged, determine the change Q(t) voltage V(t) on the capacity and the current I(t) in the circuit.

$$RI + \frac{Q}{C} = E$$

$$40I + \frac{Q}{(0.0005)} = 20$$



ISSN: 2454-4248
742 – 746

$40\ I + \frac{Q}{(0.005)} = 20$

$I + 5Q = 0.5$

$\frac{dQ}{dt} + 5Q = 0.5$

General Solution is

$Q = 0.1 + Ce^{-5t}$

at t $= 0, Q=0, C=0.1$

$Q(+) = 0.1 - 0.1\ e^{-5t}$

$= (0.1)\ (1 - e^{-5t})$

$v(+) = \frac{Q(+)}{c} = \frac{0.1(1-e^{-5t})}{0.005} = \frac{\frac{1}{10}}{\frac{2}{1000}}(1-e^{-5t})$

$= \frac{1000}{10+5}(1-e^{-5t})$

$= 20\ (1-e^{-5t})$

$I(t) = \frac{dQ}{dt} = 0.5\ e^{-5t}$

By DTM

$(K+1)\ Q\ (K+1) + 5\ Q(K) = 0.5$

When $K = 0,1,2,3\ldots\ldots$

$Q\ (1) + 5\ Q\ (0) = 0.5$

$Q\ (1) = 0.5$

$2Q\ (2) + 5\ Q\ (1) = 0$

$Q\ (2) = \frac{-5 \times 0.5}{2} = \frac{25/10}{\frac{2}{1}}$

$= -\frac{25}{20} = -1.25$

$3Q\ (3) + 5Q\ (2) = 0$

$3Q\ (3) = -5\ Q\ (2)$

$= -5 \times (-\frac{25}{20})$

$= \frac{25}{4}$

$Q\ (3) = \frac{25}{12}$

$4Q\ (4) + 5\ Q\ (3) = 0$

$4\ Q\ (4) = -5\ Q\ (3)$

$Q\ (4) = \frac{-5}{4}\ Q\ (3)$

$= \frac{-5}{4} \times \frac{25}{12} = \frac{-125}{48}$

$5Q\ (5) + 5\ Q\ (4) = 0$

$Q\ (5) = -Q\ (4)$

$= \frac{125}{48}$

$6Q\ (6) + 5Q\ (5) = 0$

$6Q\ (6) = -5\ Q\ (5)$

$= \frac{-5}{6}\left[\frac{125}{48}\right]$

$= 7 \times (t) + 5\ Q\ (6) = 0$

$Q\ (7) = \frac{-5}{6}\ [Q(6)]$

$= \frac{5}{6}\left[\frac{5}{6} \times \frac{125}{48}\right]$

GS

$Q\ (t) = Q\ (0) + t\ Q(1) + t^2\ Q(2) + t^2\ Q(3) + t^4\ Q(4) + t^5\ Q(5) + \ldots.$

$= 0 + (0.5)t + \left(\frac{-25}{20}\right) t2 + \left(\frac{25}{12}\right) t^3 + \left(\frac{-125}{48}\right) t^4 + \frac{125}{48}\ t^5\ldots.$

Table – 3

t	Exact	DTM	Error
0	0.00000	0.00000	0.00000
01	0.0993262053	0.0993262053	0.0993262053
02	0.09999546001	0.09999546001	0.09999546001
03	0.09999996941	0.09999996941	0.09999996941
04	0.09999999979	0.09999999979	0.09999999979
05	0.1	0.1	0.1
06	0.1	0.1	0.1
07	0.1	0.1	0.1
08	0.1	0.1	0.1
09	0.1	0.1	0.1
10	0.1	0.1	0.1

V. **CONCLUSION** : In this work we applied DTM for first order ordinary differential equation, it reduces the computational difficulties of other traditional methods (Laplace Transform).
DTM is best for solving initial value problems of first order on simple circuits electrical engineering problems

Fig. 1 (from table – 1)

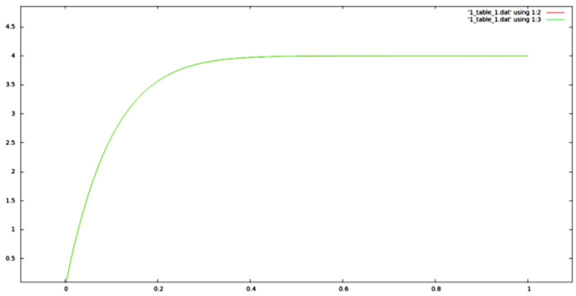


Fig. 2 (from table – 2)

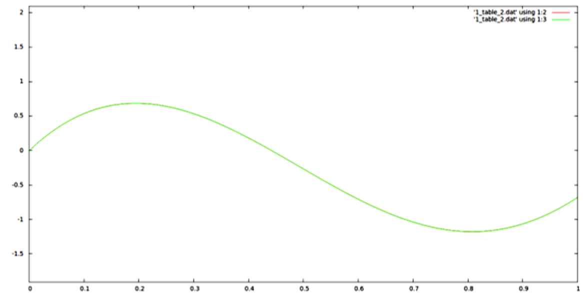
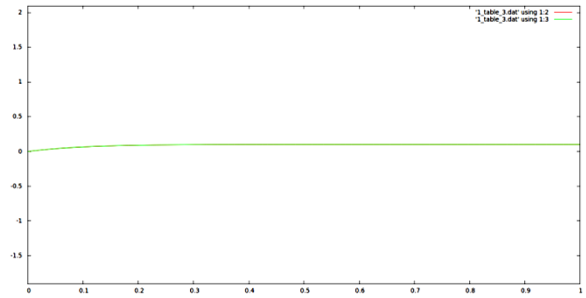


Fig. 3 (from table – 3)



VI. REFERENCES :

[1] Biazar, J., and Eslami, M., 2010, "Differential transform method for quadratic Riccati differential equations," International Journal of Nonlinear Science, 9(4),pp. 444-447.

[2] Opanuga, A. A. Edeki, S.O., Okagbue, H. I. Akinloabi, G. O., Osheku, A. S. and Ajayi, B., 2014, "On numerical solutions of system of ordinary differential equations by numerical-analytical method," Applied Mathematical Sciences. 8, pp. 8199-8207. doi.org/10-12988/ams.2014-410807.

[3] Chen C. K. and S.S. Chen, Application of the differential transform method to a non-linear conservative system, applied Mathematics and computation, 154, 431-441 (2004)

[4] Zhou X., Differential Transformation and its Applications for Electrical Circuits, Huazhong University Press, Wuhan, China, 1986 (in Chinese)

[5] Anyaz F., Solutions of the system of differential equations by differential transform method. Applied Mathematics and Computation, 147, 547-567 (2004).

[6] Duan Y., R. Liu and Y. Jaing, Lattice Boltzmann model for the modified Burger's equation, Appl. Math. Computer, 202, 489-497 (2008)

[7] Bert. W. H. Zeng, Analysis of axial vibration of compound bars by differential transform method. Journal of Sound and Vibration, 275, 641-647 (2004)

[8] Chen C.L. and Y.C. Liu. Differential transformation technique for steady nonlinear heat conduction problems, Appl. Math. Computer, 95, 155164 (1998)

[9] Hassanl H. Abdel-Halim Comparison differential transformation technique with Admain decomposition method for linear and nonlinear initial value problems, Choas Solution Fractals, 36(I) , 53-65 (2008)

[10] Khaled Batiha and Belal Batiha, A New Algorithm for Solving Lnear Ordinary Differential Equations, World Applied Sciences Journal, 15(12), 1774-1779, (2011), ISSN 1818-4952, IDOSI Publications.

[11] Montri Thongmoon, Sasitorn Pusjuso The numerical solutions of differential transform method and the Laplace transforms method for a system of differential equations, Nonlinear Analysis: Hybrid System, 4 425-431 (2010)