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Profit Analysis of a Warm Standby Non-Identical Units System with Single Server Subject to Priority

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***Abstract*—**The present paper deals with the profit analysis of a warm standby non-identical (one is main unit another is duplicate unit) units system with single server. The model consists of two non-identical units –one is operative and the other kept as warm standby and one unit is sufficient to make the system in operative mode. The main unit may fail directly from normal mode and the warm standby unit can fail owing to remain unused for a longer period of time. There is a single server, who gives priority to repair of the main unit over the repair of the duplicate unit. The time is taken to repair activity by the server follows negative exponential distribution whereas the distributions of unit are taken as arbitrary with different probability density functions. The expressions of various efficiency measures are analyzed in steady state using semi-Markov process and regenerative point technique. Also, taking the arbitrary values for the parameters (i.e. λ, μ, ϕ and θ) to delineate the behavior of some important performance measures to check the efficacy of the system model under such situations shown in the graphs.

Keywords- *Profit Analysis,Redundant System,Warm-Standby,Regenerative point,Priority and semi-Markov process*

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I. INTRODUCTION

Redundancy is the provision of alternate means or parallel paths in a system for performing a given assignment. Application of redundancy in the system design is found in almost all types of system due to its numerous advantages to improve reliability and availability of a system. In literature, the stochastic behavior of warm standby system has been widely discussed by many researchers including [1] has discussed a two-unit standby redundant system with standby failure. [2] analyzed stochastic behavior of a two-unit priority standby redundant system with repair. [3] has explored reliability of a repairable system with standby failure. [4] analyzed cost-benefit of a one server two-unit cold standby system with repair and preventive maintenance. [5] analyzed reliability and availability of a system with standby and common cause failures. [6] has discussed profit of a system with two- units having guarantee periods and delayed operation of standby. [7] obtained a maintenance model for two-unit redundant system. [8] analyzed a two-unit warm standby system subject to degradation.

But, sometimes it is very difficult on the part of the users to keep a high cost identical unit in warm standby. And, in such a situation a duplicate unit may be kept as spare in order to its use in emergency and also to provide services to the customers for a considerable period. Each unit is capable of performing the same kinds of functions but their degree of reliability and desirability may differ from unit to unit. On the other hand, this unit can also be used to work in warm standby. [9] analyzed cost benefit of series systems with warm standby components and general repair time. [10] analyzed steady state of an operating system with repair at different levels of damages subject to inspection and weather conditions. [11] have studied stochastic modeling of a computer system with priority to PM over S/W replacement subject to maximum operation and repair times. [12] have examined reliability measures of a cold standby system with preventive maintenance and repair. [13] analyzed cost benefit of two similar warm standby systems subject to failure due to melting of glaciers and severe storms caused by global warming and failure rate as Gamma distribution.

Recently, [14] analyzed economically a warm standby system with single server. the purpose of the present study is to obtain reliability measures of a system of non identical units with warm standby approach. there are two units in the system- one is the main unit is initially operational and the second (duplicate) unit in warm standby. single repair facility is provided to the system immediately to rectify the faults. the units work as new after repair. each unit has operative and complete failure modes. the random variables related to failure and repair times are statistically independent. the expressions for various reliability characteristics including mean sojourn times, mean time to system failures (mtsf), availability, busy period analysis, expected number of visit by server and profit function are derived using semi-markov process and regenerative point technique. the results for particular cases have been obtained to depict the graphical behavior of some important reliability characteristic. this model is simulated with a water drain out system form the low level agriculture field in which rainy water accumulated in the rainy season. in this model the main unit is electric pump and warm stand unit is engine operated pump with different capacity generally used in such a situation.

II. NOTATIONS

M_0 / D_0: The main/duplicate unit is in working mode.

$Dws / DFwr$: Duplicate unit is in warm standby mode/failed waiting for repair.

$MFur / DFur$: Main/Duplicate unit under repair.

$\lambda / \lambda_1 / \lambda_2$: Constant failure rate of duplicate unit from warm standby mode/ constant failure rate of the main unit from operative mode /constant failure rate of duplicate unit from operative mode.

$g(t)/G(t)$: p.d.f./c.d.f. of repair time of the main unit.

$g_1(t)/G_1(t)$: p.d.f./c.d.f. of repair time of the duplicate unit.

$q_{ij}(t)/Q_{ij}(t)$: p.d.f./c.d.f. of first passage time from i^{th} to j^{th} regenerative state or to j^{th} failed state without staying in any other regenerative state in (0,t].

m_{ij} : The unconditional mean time taken by the system to transit from any regenerative state S_i when it (time) is counted from epoch of entrance in to the state S_j . Mathematically, it can be written as

$$m_{ij} = \int_0^\infty td[Q_{ij}(t)]dt = -q_{ij}'^{*}(0)$$

μ_i : Mean sojourn time in state S_i which is give by $\mu_i = E(T) = \int P(T_i > t)dt = \sum_j m_{ij}$ where T denotes the time to system failure.

$M_i(t)$: Probability that the system is initially up in the regenerative state S_i is up at time t without passing through any other regenerative state.

$W_i(t)$: Probability that the server is busy in the state S_i up to time t, without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states.

$\otimes / \oplus$: Symbol of Laplace Stieltjes Convolution/Laplace convolution.

$** / * / '$: Laplace Stieltjes transform/ Laplace transform/ derivative of the function.

III. SYSTEM DESCRIPTION

S_0 : It is regenerative operative state of the system; the main unit is in operative mode and duplicate unit is warm standby.

S_1 : It is also regenerative operative state, in this state main failed unit is under repair and the system is in operative mode due to duplicate unit is working.

S_2 : It is non-regenerative failed state, the main unit under repair and duplicate unit is waiting for repair because single server is busy for repairing the main unit.

S_3 : This state is also regenerative and operative state in which duplicate failed unit under repair.

S_4 : This is non-regenerative failed state; the main unit under repair and the duplicate unit are waiting for repair due to single server is busy for repairing the main unit.

IV TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

If it is assumed that $g(t) = \theta e^{-\theta t}$ and $g_1(t) = re^{-rt}$ then the transition probabilities are as follows

$$p_{01} = \frac{\lambda_1}{\lambda + \lambda_1}, \quad p_{03} = \frac{\lambda}{\lambda + \lambda_1}, \quad p_{10} = g^*(\lambda_2) = \frac{\theta}{\lambda_2 + \theta}$$

$$p_{12} = 1 - g^*(\lambda_2) = \frac{\lambda_2}{\lambda_2 + \theta}, \; p_{30} = g_1^*(\lambda_1) = \frac{r}{r + \lambda_1}$$

$$p_{34} = 1 - g^*(\lambda_2) = \frac{\lambda_1}{\lambda_1 + r}, \; p_{43} = p_{23} = 1 \qquad (1)$$

and transitions probabilities of via states are

$$p_{1,3;2} = p_{12} \cdot p_{23} = \frac{\lambda_2}{\theta + \lambda_2} \cdot 1 = \frac{\lambda_2}{\theta + \lambda_2} \quad \text{and}$$

$$p_{3,3;4} = p_{34} \cdot p_{43} = \frac{\lambda_1}{r + \lambda_1} \cdot 1 = \frac{\lambda_1}{r + \lambda_1} \qquad (2)$$

it is verified that

$$p_{01} + p_{03} = p_{10} + p_{12} = p_{30} + p_{34} = p_{23} = p_{43} = 1 \qquad (3)$$

Mean Sojourn Time

Let T denotes the time to system failure then the mean sojourn times (μ_i) in the state S_i are given by

$$\mu_i = E(t) = \int_0^\infty P(T > t)dt, \; \mu_0 = \frac{1}{\lambda + \lambda_1}, \; \mu_1 = \frac{1}{\lambda_2 + \theta}$$

$$\mu_2 = \frac{1}{\theta}, \; \mu_3 = \frac{1}{r + \lambda_1}, \; \mu_4 = \frac{1}{r}, \; \mu_1' = \frac{1}{\theta} \qquad (4)$$

V. MEAN TIME TO SYSTEM FAILURE

Let $\phi_i(t)$ be the c.d.f. of the passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$.

$$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t) + Q_{03}(t) \circledS \phi_3(t)$$
$$\phi_1(t) = Q_{10}(t) \circledS \phi_0(t) + Q_{12}(t)$$
$$\phi_3(t) = Q_{30}(t) \circledS \phi_0(t) + Q_{34}(t) \qquad (5)$$

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Taking L.S.T. of relation (5) and solving for $\phi_0^*(s)$,

$$\text{MTSF}=\lim_{s\to 0} R^*(s)=\lim_{s\to 0}\frac{1-\phi_0^*}{s} \quad (6)$$

$$=\frac{\mu_0+\mu_1 p_{01}+p_{03}\mu_3}{1-p_{01}p_{10}-p_{03}p_{30}}$$

$$=\frac{(\lambda_2+\theta)(r+\lambda_1)+\lambda_1(r+\lambda_1)+\lambda(\lambda_2+\theta)}{(\lambda+\lambda_1)(r+\lambda_1)(\lambda_2+\theta)-r\lambda(\lambda_2+\theta)-\lambda_1\theta(r+\lambda_1)} \quad (7)$$

VI. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant t, given that the system entered the regenerative state S_i at t = 0.The recursive relations for $A_i(t)$ are as follows

$$A_0(t)=M_0(t)+q_{01}(t)©A_1(t)+q_{03}(t)©A_3(t)$$
$$A_1(t)=M_1(t)+q_{10}(t)©A_0(t)+q_{13;2}(t)©A_3(t)$$
$$A_3(t)=M_3(t)+q_{30}(t)©A_0(t)+q_{33;4}(t)©A_3(t) \quad (8)$$

$M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ at time t without visiting to any other regenerative state where

$$M_0(t)=e^{-(\lambda+\lambda_1)t}, \quad M_1(t)=e^{-\lambda_2 t}\overline{G(t)} \text{ and}$$
$$M_3(t)=e^{-\lambda_1 t}\overline{G_1}(t) \quad (9)$$

Taking Laplace transform of relation (8 and 9) and solving for $A^*{}_0(s)$, the steady state availability is given by

$$A_0=\lim_{s\to 0} sA_0^*(s)$$

$$=\frac{\mu_0(1-p_{33;4})+\mu_1 p_{01}(1-p_{33;4})+\mu_3(p_{01}p_{13;2}+p_{03})}{\mu_0(1-p_{33;4})+\mu_1' p_{01}(1-p_{33;4})+\mu_3'(p_{01}p_{13;2}+p_{03})}$$

$$=\frac{\theta[r(\lambda_2+\theta)+r\lambda_1+\lambda_1\lambda_2+\lambda(\lambda_2+\theta)]}{[r\theta(\lambda_2+\theta)+r\lambda_1(\lambda_2+\theta)+(\theta+\lambda_1)(\lambda_1\lambda_2+\lambda(\lambda_2+\theta))]} \quad (10)$$

VII. BUSY PERIOD OF THE SERVER DUE TO REPAIR OF THE FAILED UNIT

Let $B_i(t)$ be the probability that the server is busy due to repair of the failed unit at instant t, given that the system entered the regenerative state S_i at t = 0. The recursive relations for $B_i(t)$ are as follows

$$B_0(t)=q_{01}(t)©B_1(t)+q_{03}(t)©B_3(t)$$
$$B_1(t)=W_1(t)+q_{10}(t)©B_0(t)+q_{13;2}(t)©B_3(t)$$
$$B_3(t)=W_3(t)+q_{30}(t)©B_0(t)+q_{33;4}(t)©B_3(t) \quad (11)$$

where $W_i(t)$ is the probability that the server is busy in state S_i due to repairing of unit up to time t without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states so

$$W_1(t)=e^{-\lambda_2 t}\overline{G(t)} \text{ and } W_3(t)=e^{-\lambda_1 t}\overline{G_1(t)} \quad (12)$$

Taking Laplace Transform of relation (11 and 12) and solving for $B_0^*(s)$, the time for which server is busy is given as

$$B_0=\lim_{s\to 0} sB_0^*(s)$$

$$=\frac{W_1^*(0)p_{01}(1-p_{33;4})+W_3^*(0)(p_{01}p_{13;2}+p_{03})}{\mu_0(1-p_{33;4})+\mu_1' p_{01}(1-p_{33;4})+\mu_3'(p_{01}p_{13;2}+p_{03})}$$

$$=\frac{r\lambda_1(\lambda_2+\theta)+(\theta+\lambda_1)[\lambda_1\lambda_2+\lambda(\lambda_2+\theta)]}{[r\theta(\lambda_2+\theta)+r\lambda_1(\lambda_2+\theta)+(\theta+\lambda_1)(\lambda_1\lambda_2+\lambda(\lambda_2+\theta))]} \quad (13)$$

VIII. EXPECTED NUMBER OF VISITS BY THE SERVER DUE TO REPAIR OF THE UNIT

Let $N_0(t)$ be the expected number of visits by the server in (0, t], given that the system entered the regenerative state S_i at t=0 .The recursive relations for $N_i(t)$ are as follows

$$N_0(t)=Q_{01}(t)\,Ⓢ\,(N_1(t)+1)+Q_{03}(t)\,Ⓢ\,(N_3(t)+1)$$
$$N_1(t)=Q_{10}(t)\,Ⓢ\,N_0(t)+Q_{13;2}(t)\,Ⓢ\,N_3(t)$$
$$N_3(t)=Q_{30}(t)\,Ⓢ\,N_0(t)+Q_{33;4}(t)\,Ⓢ\,N_3(t) \quad (14)$$

Taking Laplace Stieltjes transform of the above relation and solving for $N_0^*(s)$, the expected number of visits by the server are given by

$$N_0=\lim_{s\to 0} sN^*_0(s)$$

$$=\frac{(p_{01}+p_{03})(1-p_{33;4})}{\mu_0(1-p_{33;4})+\mu_1' p_{01}(1-p_{33;4})+\mu_3'(p_{01}p_{13;2}+p_{03})}$$

$$=\frac{r\theta(\lambda+\lambda_1)(\lambda_2+\theta)}{[r\theta(\lambda_2+\theta)+r\lambda_1(\lambda_2+\theta)+(\theta+\lambda_1)(\lambda_1\lambda_2+\lambda(\lambda_2+\theta))]} \quad (15)$$

IX. PROFIT ANALYSIS

The profit occurred in the system model in steady state can be calculated as

$$P_0=T_0A_0-T_1B_0-T_2N_0 \quad (16)$$

where

T_0=(5000), Drain out capacity per unit up- time of the system.

T_1= (500), Cost per unit time for which server is busy.

T_2 = (1000), Cost per unit visits by the server

X. DISCUSSION

In this situation, the water is pumped out form the low level agriculture field by the two non-identical unit in which one is main unit (electric pump) and other is (engine pump) working with in limit in such a way that the field become free from rainy water as soon as possible. Keeping such situation in mind the effect of various parameters on performance measure of system model is envisioned. The behavior of mean time to system failure, availability and profit has been observed for

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arbitrary values of the parameters as shown in the figure 2, 3 & 4. It is observed that these reliability measures keep on decreasing with the increase of failure rate λ, λ_2. While the reliability measures increases with the increase of repair rate r and θ. Hence the performance of a system of non-identical units can be improved by increasing the repair rate of main unit and decreasing the failure rate of duplicate unit.

Figure-2: Shows the behavior of MTSF with respect to failure rate of the main unit interprets that the increase in failure rate causes MTSF decreases due to duplicate unit take place and system is in working mode, for fixed values of other parameters. It also depicts that as the repair rate of the main unit θ increases then MTSF is also decreasing in nature. The effect of other parameters can also be observed from the figure 2 on the MTSF of the system. Hence the mean time to system failure can control by increasing repair rate θ of the main unit which have more capacity to drain out the water from the field as compare with the duplicate unit.

Figure 3. The behavior of availability of the system corresponds to increasing failure rate λ_1 of the main unit having decreasing pattern. Curve L_5 and L_3 of the figure 3 clearly shows that when the repair rate θ of main unit and repair rate r of duplicate unit increased than the availability of the system at high level as compared to other parameters but slightly in decreasing in order. The curve L_5 indicated when the repair rate of the main unit θ having highest value i.e.4 at the maximum level as compare with other and the curve L_3 corresponding to the repair rate r having its value 3 at the runner up position in the figure. Hence, to enhance the availability of the system we confine our attention about the repair rate of main unit and duplicate unit.

Figure 4. In this figure of profit (i.e. capacity of pump per unit time to drain out the stored rainy water from the agriculture field) can be analyzed easily. Only the pumping system can drain out the stored rainy water from the agriculture filed at its maximum capacity whenever the system is available for use with main unit in working condition. So, by increasing the repair rate θ of the main unit and repair rate 'r' of the duplicate unit can increase the availability of the system for the sake of drain out the stored water into agriculture field.

XI. CONCLUSION

It is very difficult on the part of the users to keep a high cost identical unit in warm standby. And, in such a situation a duplicate unit may be kept as spare in order to its use in emergency and also to provide services to the customers for a considerable period. Each unit is capable of performing the same kinds of functions but their degree of reliability and desirability may differ from unit to unit. The main finding of this model to drain out the rainy stored water in low level agriculture field is profitable to increasing the repair rate θ of main unit and repair rate r of duplicate unit respectively in the above said situation.

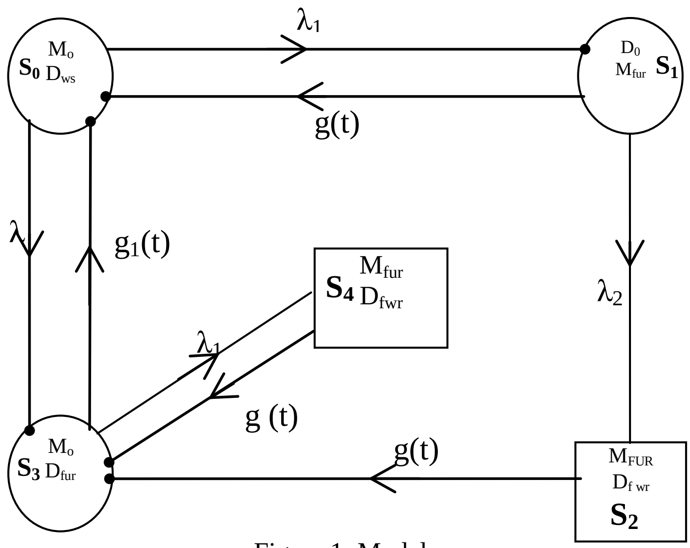

Figure 1: Model

State-Transition Diagram

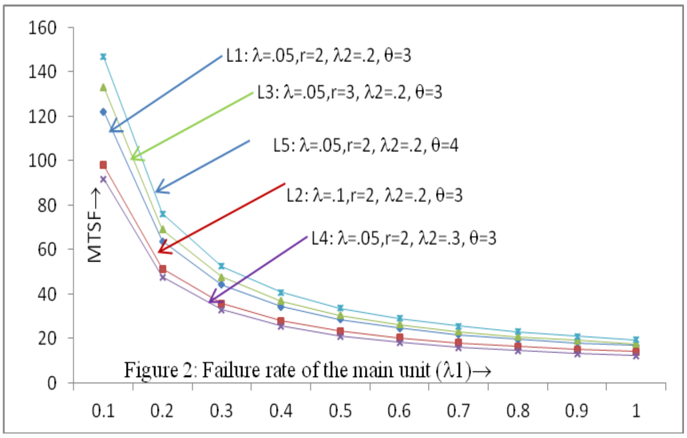

Figure 2: Failure rate of the main unit (λ1)→

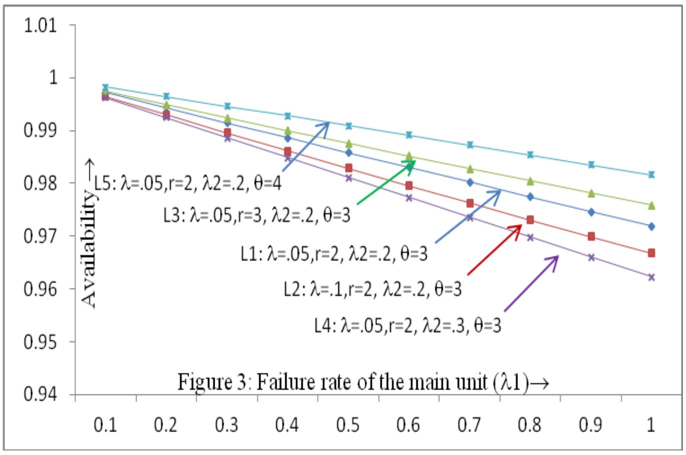

Figure 3: Failure rate of the main unit (λ1)→

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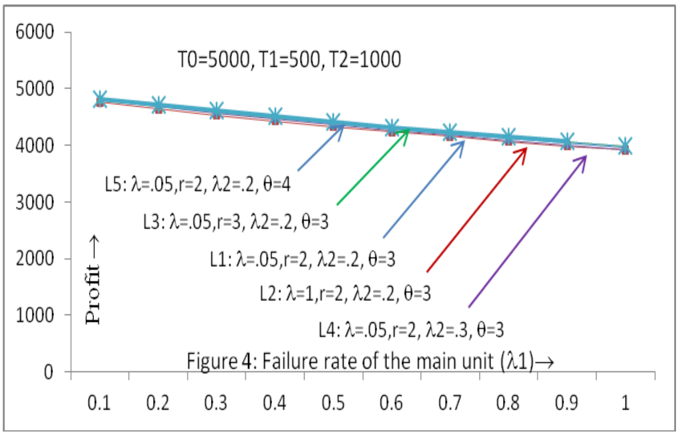

Figure 4: Failure rate of the main unit (λ1)→

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