Performance Analysis of Unscented Kalman Filter Using Particle Swarm Optimization for Tracking Applications

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Abstract**:** The analysis of dynamic parameters of the system or process requires state estimation theory. State estimation theory is one of the best mathematical practices to analyze the variants of the system and this approach is used to generate the optimal estimate of the true state of the system. Based on the dynamics of the system state estimation process may be linear or nonlinear. When the system is linear Kalman Filter is used to estimate the states using minimum mean square error method. Mostly nonlinear systems were existed in the universe. For the state estimation of non-linear model, Unscented Kalman Filter (UKF) gives the better estimation performance. UKF is a transformation technique to measure the statistics of a random variable that changes in non-linear manner. Tuning of measurement noise and plant noise can be used for better estimation in tracking applications. The approach in this paper is to analyze the performance of Unscented Kalman Filter using Particle Swarm Optimization for sonar signal processing.

Key words*: State estimator, Unscented Kalman Filter, Particle Swarm Optimization*.

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I. INTRODUCTION

The main aim of this paper is to track the maneuvering object in sonar applications using bearing only measurements. In sonar application, the sonars are fixed to submarines and ships travelling in underwater. To get object position we have to pass sound signals as acoustic energy into the water. The energy of sound signal continuously illuminates the object. We have to measure object range and bearing taken in the presence of noise. To estimate the speed, course, range and bearing of the object using bearing and range measurements. The own-ship course and speed are considered without noise. In general object tracking is differentiated in two ways. One of them is a tracking a maneuvering object and another one is tracking a non-maneuvering. Non-maneuvering object tracking is a constant velocity method can be achieved using linear and non-linear estimators, which are implemented with the help of Kalman filter, Extended Kalman filter. In this paper, the performance of Unscented Kalman filter (UKF) is analyzed for maneuvering target tracking. The dynamic behaviour of the system can be achieved through the knowledge of measurements taken. UKF is a nonlinear transformation algorithm, which is mostly used for tracking a moving object.

To improve the performance of maneuvering object tracking tuning methods are used. Tuning methods are used to optimize the covariance matrices of plant noise and measurement noise. One of the best tuning methods is Particle Swarm Optimization (PSO).

II. UNSCENTED KALMAN FILTER

In general the Kalman filter is used to estimate the state of a linear model but in nature, most of the systems or processes are nonlinear, this will limit the usage of Kalman Filter. Unscented Kalman Filter can be used for the nonlinear systems. In this estimator, transfer of information in terms of mean and covariance vector matrices. For underwater applications, UKF is used. Generally, for nonlinear state estimator like EKF, first linearized the nonlinear function and then applied Kalman filter theory but in UKF, the estimation is based on the sigma point selection and this sigma points passing through the given nonlinear function. While selecting the sigma points we have to consider the mean, covariance and higher order moments should get equivalent to the Gaussian random variable .Now from this transformation points measured the mean and covariance can be used for better linearization than Taylor's series expansion. Sigma points selection procedure is not a random. For example consider a Gaussian random variable having Q dimensions that will generate 2Q+1 sigma points .In the selection of sigma points covariance matrix and matrix square root are used to match the covariance of the Gaussian random variable. Noise function is considered as a non-additive or non-Gaussian. The phenomenon that includes transferring the noise through nonlinear function is augmenting the state vector that consisting of noise generators. So from the augmented state vector the selected sigma points having noise values also.

The basic state model equation is given as follows:

$$S(Ꝁ+1) = F(S(Ꝁ), A(Ꝁ)) + w(Ꝁ); \qquad (2.1)$$

Where, w (Ꝁ) = plant noise.

The UKF selects (2q + 1) sigma points with their scalar weights. The scalar weights are calculated by using below equations.

$$W_0^{(m)} = \frac{g}{(q+g)} W_0^{(c)} = \frac{g}{(q+g)} + \beta \ ; \qquad (2.2)$$

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$$W_i^{(m)} = W_i^{(c)} = \frac{1}{2(q+g)} \quad ; \qquad (2.3)$$

Here i = 1, 2, ..., 2q .
Here $g = (a^2 - 1)q$ is a scaling variable it gives how much distance the points away from the mean. β Is a known value for state distribution in general β is equal to two.The basic UKF procedure consisting of following algorithmic steps.
Selection of (2q + 1) sigma points from the given initial conditions.

s(Ꝁ-1)= s(0) and C(Ꝁ-1)=C(0); (2.4)

S(Ꝁ-1)=[s(Ꝁ-1) s(Ꝁ-1)+$\sqrt{(q+g)C(Ꝁ-1)}$

s(Ꝁ-1)-$\sqrt{(q+g)C(Ꝁ-1)}$] ; (2.5)

Now the sigma points are transfer through the nonlinear process model and predict the state at 'n' from the 'n-1' measurements.

s(Ꝁ/Ꝁ-1)=$\sum_{i=0}^{2q} W_i^{(m)}\, s(i, Ꝁ/Ꝁ-1)$; (2.6)

Generally we consider the process noise is independent and additive .Next predicted covariance which is measure from the below equations.

C(Ꝁ/Ꝁ-1)=$\sum_{i=0}^{2q} W_i^{(c)}\, [s(i,Ꝁ/Ꝁ-1) - s(Ꝁ/Ꝁ-1)][s(i,Ꝁ/Ꝁ-1) - s(Ꝁ/Ꝁ-1)]^T$ + Q(Ꝁ) ; (2.7)

In the next step sigma points are updated with the predicted covariance and mean .After updating sigma points are shown below:

S (Ꝁ/Ꝁ-1) =[s (Ꝁ/Ꝁ-1) s (Ꝁ/Ꝁ-1) + $\sqrt{(q+g)C(Ꝁ/Ꝁ-1)}$

s(Ꝁ/Ꝁ-1)- $\sqrt{(q+g)C(Ꝁ/Ꝁ-1)}$] ; (2.8)

After every updating sigma points are passing through the output or measurement equations.
Prediction of output (innovation) or measurement is done through the following equation:

z(Ꝁ/Ꝁ-1)=$\sum_{i=0}^{2q} W_i^{(m)}\, Z(i,Ꝁ/Ꝁ-1)$; (2.9)

Measurement noise is also taken as independent and additive. Measurement (innovation) covariance is given by

C_{zz}=$\sum_{i=0}^{2q} W_i^{(c)}\, [C(i,Ꝁ/Ꝁ-1) - z(Ꝁ/Ꝁ-1)]\, [Z(i,Ꝁ/Ꝁ-1) - z(Ꝁ/Ꝁ-1)]^T$ + R(Ꝁ) ; (2.10)

The measurement cross covariance is as follow

C_{sz}=$\sum_{i=0}^{2q} W_i^{(c)}\, [S(i,Ꝁ/Ꝁ-1) - s(Ꝁ/Ꝁ-1)]\, [Z(i,Ꝁ/Ꝁ-1) - z(Ꝁ/Ꝁ-1)]^T$ + R(Ꝁ) ; (2.11)

For the state updating Kalman gain is needed to calculated as

K(Ꝁ) =$C_{sz}C_{zz}{}^{-1}$; (2.12)

The state estimation from the above parameters is given as

S(Ꝁ/Ꝁ-1)= S(Ꝁ /Ꝁ-1)+ K(Ꝁ)((z(Ꝁ /Ꝁ)- z(Ꝁ/Ꝁ-1)) ; (2.13)

Where z(Ꝁ-1) is true output (measurement) and estimated error covariance is given as

C(Ꝁ/Ꝁ)= C(Ꝁ/Ꝁ-1)-K(Ꝁ)$C_{zz}K(Ꝁ)^T$; (2.14)

III. PARTICLE SWARM OPTIMIZATION(PSO)

PSO is a kind of algorithm that starts with the initialization of the flock of birds randomly over the solution space, in which every bird is called as a "particle".For each particle, the position and velocity vectors will be randomly initialized with the same size as the problem dimension. In each step, the fitness of each particle (*pbest*) is observed with the fitness value which is best among called as global best (*gbest*) of the particle. Then the velocity and position vectors are updated for each particle. The above procedure is repeated continuously until a best solution is obtained. The various steps of PSO algorithm are explained below

(a) The Particles Positions and Velocities Initialisation

In this step set of random particle velocityV_i^k and position d_i^k are initialized, with the following Equations

$$d_i^k = D_{min} + \big(rand(D_{max} - D_{min})\big); \qquad (3.1)$$

$$V_i^k = D_{min} + \big(rand(D_{max} - D_{min})\big); \qquad (3.2)$$

Where 'D_{max}' and 'D_{min}' are the maximum and minimum limits of the desired variable, '*rand*' is a random variable which is distributed uniformly between 0 and 1.

(b) Evaluating gbest and pbest

In this PSO searches for the best solution by updating the particles. The updating process depends on two best values called as particle best and global best in a consecutive iteration .Here pbest is the best solution so far for each particle and gbest is the best of all pbest's.

(c) Updating the Particles Position using Velocities

The particle velocity and position updates by using the following Equations:

$$V_i^{n+1} = w * V_i^n + \big(F_1 * rand1 * (pbest(i) - d_i^n)\big) + \big(F_2 * rand2 * (gbest - d_i^n)\big); \qquad (3.3)$$

$$d_i^{n+1} = d_i^n + V_i^{n+1} \;; \qquad (3.4)$$

V_i^{n+1}= ith particle velocity in (n + 1)th iteration

V_i^n = ith particle velocity in nth iteration

W = inertial weight

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$F_1$and F_2 are the self and swarm confidence factors. $rand1$ and $rand2$ are the uniformly distributed functions value lies between 0 to 1. $pbest(i)$is the best position of ith individual, $gbest$ is the best position among the individual.

(d) Termination Criteria

This is the last step in algorithm in this the above steps is continuously repeated until a best solution is obtained or maximum number of iterations.

IV. PROBLEM DESCRIPTION

The approach in this paper is to track the maneuvering target using bearing only measurements in sonar applications using Particle Swarm Optimization.

Initially distance between target and observer is considered as 5000 meters and 0 bearing value. Complete target motion is analyzed for 1800 samples. Initial values are assumed as Speed= 4 knots; course=135 degrees (up to 659th sample) & 225 degrees (from 660 to 1800 samples) [1].

The Target and Observer motion are shown in the following Fig1. The Line of Sight (LOS) is defined as the line joining between target and observer .The bearing is defined as angle between LOS and Y-Axis .The range of the target is defined as length of line of sight.

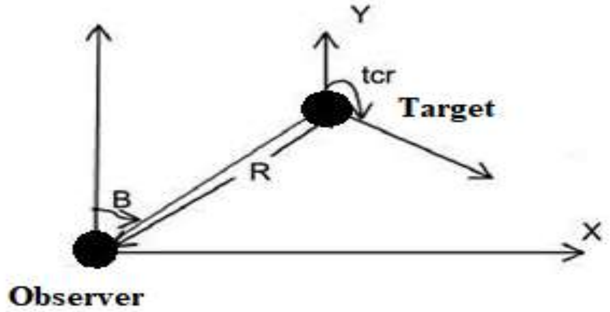

Fig 4.1: Target and observer positions

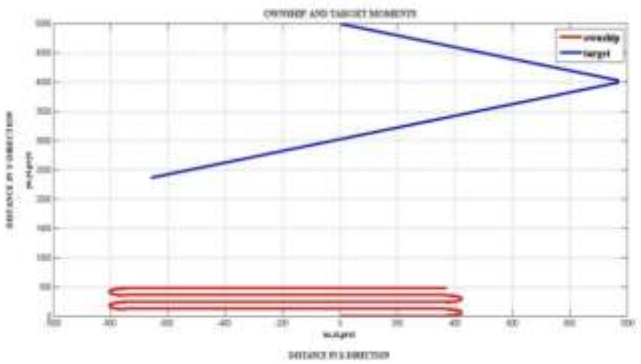

Fig 4.2: Target and observer moments

Here Observer motion is assumed to be 'S' maneuver on LOS for better tracking with a speed of 6 knots and Initial Course of observer is 90 degrees for the first 120 samples,270 degrees for next 420 samples,90 degrees for next 420 samples, 270 degrees for next 420 samples and 90 degrees for remaining 420 samples.

V. MODELLING OF A PROBLEM

Consider the object state vector is S(Ꝁ) ,
Where

$$S(Ꝁ)=\left[\overline{s(Ꝁ)z(Ꝁ)}R_s(Ꝁ)R_z(Ꝁ)\right]^T; \qquad (5.1)$$

$\overline{s(Ꝁ)}$and$\overline{z(Ꝁ)}$are object velocity components and, $R_s(Ꝁ) = R * \sin(B)$and $R_z(Ꝁ) = R * \cos(B)$ are range components in x and y directions.

The basic state dynamic equation of an object is representing in the following equation.

$$S(Ꝁ+1)= A(Ꝁ+1/Ꝁ)S(Ꝁ) + B(Ꝁ+1) + w(Ꝁ) ; \qquad (5.2)$$

A= transition matrix B= deterministic vector
'B' can be defined as follows

$$B(Ꝁ+1)=[0\ 0\ –[s_0(Ꝁ+1)-s_0(Ꝁ)]\ \ -[z_0(Ꝁ+1)-z_0(Ꝁ)] ; \qquad (5.3)$$

Where s_0 and z_0 are position components of observer. w(Ꝁ) is process noise .Here we have to take all angles with reference to y-axis.. The range measurement (Rm) and bearing measurement (Bm) are modelled as

$$B_m(Ꝁ+1) = tan^{-1}[R_s(Ꝁ+1)/R_z(Ꝁ+1)]+ εb(Ꝁ) ; \qquad (5.4)$$

$$R_m(Ꝁ+1) =\sqrt{R_s(Ꝁ+1)^{\mathbf{2}} + R_z(Ꝁ+1)^{\mathbf{2}}} + εr(n) ; \qquad (5.5)$$

Here εb(Ꝁ) = bearing error ε_B^2 =Bearing variance
εr(Ꝁ)= range error ε_R^2 =Range variance

The process and measurement noises are considered being not correlated to each other.

The process noise covariance matrix is taken as

$$Q(Ꝁ) = \begin{bmatrix} ts^2 & 0 & ts^3/2 & 0 \\ 0 & ts^2 & 0 & ts^3/2 \\ ts^3/2 & 0 & ts^4/4 & 0 \\ 0 & ts^3/2 & 0 & ts^4/4 \end{bmatrix} * e(Ꝁ) \qquad (5.6)$$

$$\text{Where } e(Ꝁ) = E[w(Ꝁ)*w^T(Ꝁ)] \qquad (5.7)$$

VI. SIMULATION AND RESULTS

The performance of UKF for sonar application is simulated using MAT LAB. The Fig 6.1 indicates the own ship and target moments for Unscented Kalman Filter and Fig 6.2 indicates the own ship and target moments for Unscented Kalman Filter using Particle Swarm Optimization.

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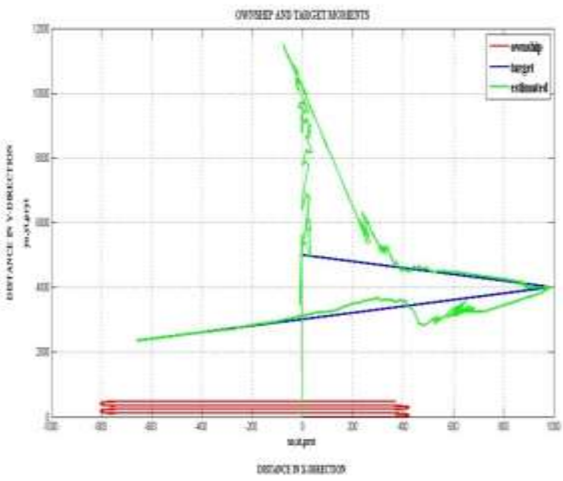

Fig 6.1: Own ship and Target moments using UKF

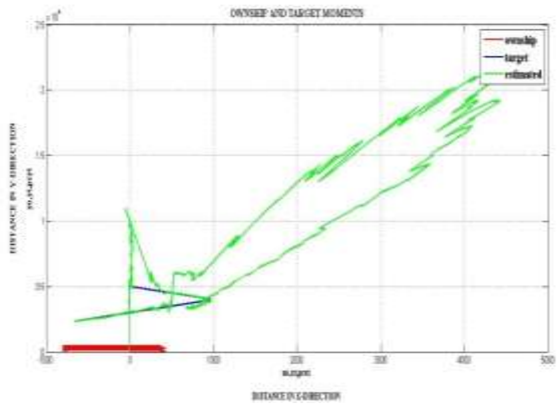

Fig 6.2: Own ship and Target moments using UKF-PSO

(a) *Performance analysis of UKF*

The estimated range in the presence of Gaussian noise converges with true range at 242 and 1263 Samples, the Estimated Course converges with true Course at 374 and 1412 Samples and the estimated Speed converges with true speed at 363 and 1354 Samples before and after target manoeuvres respectively. Effectively all the parameters converge at 374th sample before the manoeuvre and 1412th sample after target manoeuvring target respectively. The acquisition and reacquisition time for individual estimated parameters and their errors are shown in the tables given below.

	Range(m)	Course(deg)	Speed(knots)	Bearing(deg)
Initial acquisition time(sample)	242	374	363	363
Post maneuvering acquisition time(sample)	1263	1412	1354	1412

Table 6.1: Individual Parameter Acquisition and Reacquisition Time for UKF

Time (sec)	Bearing Error (degree)	RangeError(meter)	Course Error(degree)	SpeedError (degree)
242	0.07	203.02	19.29	2.74
363	0.03	36.41	7.35	0.75
374	0.00	49.01	4.19	0.40
1263	0.14	239.68	22.14	1.24
1354	0.14	147.87	10.44	0.80
1412	0.04	69.21	5.09	0.45

Table 6.2: Errors in Estimated Parameters using UKF

(b) *Performance analysis of UKF with PSO*

The estimated range in the presence of Gaussian noise converges with true range at 237 and 1081 Samples, the Estimated Course converges with true Course at 313 and 1159 Samples and the estimated Speed converges with true speed at 313 and 1162 Samples before and after target manoeuvres respectively. Effectively all the parameters converge at 313th sample before the manoeuvre and 1162th sample after target manoeuvring target respectively. The acquisition and reacquisition time for individual estimated parameters and their errors are shown in the tables given below.

	Range(m)	Course(deg)	Speed(knots)	Bearing(deg)
Initial acquisition time(sample)	237	313	313	313
Post maneuvering acquisition time(sample)	1081	1162	1159	1159

Table 6.3: Individual Parameter Acquisition and Reacquisition Time for UKF with PSO

Time(sec)	Bearing Err (degree)	RangeErr (meter)	Course Err(degree)	Speed Err(degree)
237	0.13	294.5	6.03	0.54
313	0.02	46.4	6.55	0.58
313	0.02	46.4	6.55	0.58
1081	0.1	205.48	5.16	0.35
1162	0.01	31.49	10.85	1.09
1159	0.01	31.49	10.85	1.09

Table 6.4: Errors in Estimated Parameters using UKF with PSO

From the above results, UKF-PSO gives better performance when compared to UKF. In UKF-PSO, the initial acquisition time is 61 seconds earlier and post manevuering acquisition time is 206 seconds earlier than UKF. The comparison of

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mean square error values of bearing, range, course and speed in UKF and UKF-PSO are shown in the following figures.

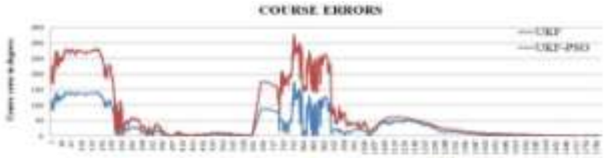

Fig 6.3: Course error of UKF and UKF-PSO

Fig 6.4: Range error of UKF and UKF-PSO

Fig 6.5: Speed error of UKF and UKF-PSO

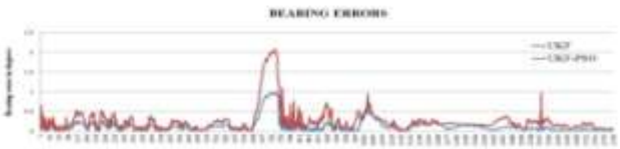

Fig 6.6: Bearing error of UKF and UKF-PSO

VII. CONCLUSIONS

The performance of Unscented Kalman Filter using Particle Swarm Optimization is analysed. From the results it is ensured that UKF-PSO gives better performance when compared to the UKF. The performance can be further improved by using Particle Filter (PF) and Interactive Multiple Model Filter (IMM) using proper Optimization technique.

REFERENCES

[1] Rao.SK, Rajeswari.K, Lingamurty KS, "Unscented Kalman Filter with application to bearing only Target Tracking. IETE Journal of Research 2009-Apr 55(2)-63-7.

[2] M. Ravi Shankar, V.Sunanda Babu S. Koteswara Rao, Sourav Majumdar "Unscented Kalman Filter Based Tracking of Maneuvering Active Targets using Sonar Measurements" ©20 11 IEEE.

[3] X.Rong Li, Vesselin P.Jilkov, "Survey of Maneuvering Target Tracking Part I: Dynamic Models", IEEE Transactions on Aerospace and Electronic systems, vol. 39, No.4, pp.1333-1364, October 2003.

[4] N.J. Gordon, D.J. Salmond, and A.F.M.Smith. A novel approach to nonlinear/non-Gaussian Bayesian state estimation. In IEE Proceedings on Radar and Signal Processing, volume 140, pages 107{113, 1993.

[5] B.Leela Kumari and K.Padma Raju,"Application of Unscented Kalman Filter for Sonar Signal Processing", International Journal of Advanced Research in Computer Engineering & Technology Volume 1, Issue 4, June 2012.

[6] Xia, G., Ludwig, S. A. (2016). Target tracking using particle filter with PSO optimization. In Evolutionary Computation (CEC), 2016 IEEE Congress on (pp. 4151-4158).

[7] Anthony F. Genovese, "The Interacting Multiple Model Algorithm for Accurate State Estimation of Maneuvering Targets",johns Hopkinsapl technical digest, volume 22, number 4 (2001).

[8] B.Leela Kumari and K.Padma Raju,"Performance Evaluation of Unscented Kalman Filter for Gaussian and Non-Gaussian Tracking Application", International Journal of Electrical and Computer Engineering (IJECE) Vol. 3, No. 1, February 2013, pp. 93~101 ISSN: 2088-8.