ISSN: 2454-4248
215 – 225

Minimization of Total Waiting Time in Specially Structured Flow Shop Scheduling under Fuzzy Environment with Separated Set up Times

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Abstract: The present paper describes two stageflow shop Scheduling problem under uncertainty situation.An efficient heuristic technique is proposed to present the results.The processing time and the setup time of all the jobs on machines are uncertain and are presented by triangular membership function. The main objective of this paper is to attain a schedule which minimizethe total waiting time of jobs.Finally, computational results for this problem are provided to evaluate the performance of the proposed algorithm.

Keywords: *Flow Shop Scheduling, Waiting Time of Jobs, Average High Ranking, Setup Time*.

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1. Introduction

Shop scheduling has been premeditated extensively in a lot of varieties. Flow shop scheduling is a judgment making course of action that is used on an ordinary basis in many manufacturing and services industries. Flow shop scheduling sets a significant role in the majority of manufacturing and service systems as well as in most information processing environments. The fundamental shop scheduling model consists of machines and jobs each of which consists of a set of operations. Each process has an associated machine on which it has to be processed for a given length of time. The processing times of operations of a job cannot overlap. Each machine can practice at most one operation at a given occasion. Its aim is to optimize one or other objectives by means of the allocation of resources over given phase of time. The possessions may be machines in a workshop, crews at a building site and runways at an airport and many more. The jobs may be operations in a production process, stages in a building job, take – offs and landings in an airport etc. It is hard to discover an optimum solution in polynomial time. Thus it is vital to pick up the flow shop scheduling algorithms for sinking the running era of the machines which is useful in the area of production scheduling.

In the majority of the reviews concerned with the Scheduling problems, processing time of each job on each machine are assumed exact value. But in real world applications, there are a lot of situations in uncertain environment in which this assumption does not hold. Fuzzy sets are modeled to define this variation in the processing times in the flow shop problems. In literature, several techniques were proposed for managing uncertainty but to solve vague situations in real problems, the first systematic approach related to fuzzy sets theory was recommended by A LotfiZadeh [3] in 1965. McCahon and Lee [10] proposed an algorithm with fuzzy processing time in order to minimize the makespan. Ishibuchi and Lee [11] addressed the formulation of fuzzy flowshop scheduling problem with fuzzy processing time. Yager[7] has given a procedure for ordering fuzzy subsets of unit interval.

Flow shop scheduling problems with sequence dependent set up time (SDST) have been one of the most renowned problems in the area of scheduling. Sequence dependent setup times are usually found in the situation where the facility is a multipurpose machine. For instant in afabric industry: the fabric types are assigned to looms equipped with wrap chains,when the fabric type is changed on a machine, the wrap chain must be replaced and the time it takes depends on the previous and the current fabric types that means at that time setup time is required before processing the job on machine which play a vital role in the manufacturing industries.The instance of sequence-dependent setups can be found in various other industrial systems also, like chemical, printing, pharmaceutical and automobile industry etc.

The first systematic approach to scheduling problem was originatedby Johnson[1] in the mid-1950s. From that point forward, a huge number of papers on various scheduling problem have showed up in the writing.The majorityof these papers neglect the setup time or assumed that it is the part of job processing time.While this assumption adversely affects thesolution quality of many applications of scheduling that require an explicit treatment of setup times. The enthusiasm for scheduling problems that treat setup times as separate initiated by Yoshida[6]in themid-1960s. Palmer [2] initially anticipated a heuristic for the flow shop scheduling problem with the purpose of minimization of make span. Campbell et al. [4] proposedCampbellDudek, and Smith (CDS) heuristic which is a generalization of Johnson's two machine algorithm. Nawaz et al. [8] proposed Nawaz, Enscore, and Ham (NEH) heuristic algorithm which is most likely the most well recognized positive heuristic used in the general flow shop scheduling problem is based on the assumption that a job with high total processing time on all the machines should be given higher precedence than job with low total processing time. A lot of further heuristic techniques such as those of Gupta [5], Hundal and Rajgopal [9] formulate the exercise of a slope index allocated to every job. These heuristic algorithms arrange the list of jobs using that weight as a sort key to generate a feasible schedule. Yoshida et al. [6] explain two stage production scheduling by taking the set up time separated from processing time. Singh V. [12] put his efforts to study three machine flow shop scheduling problems with total rental cost.
There are so many objectives to be minimized for a flow shop scheduling problems such as job completion time, total waiting time of jobs, total elapsed time, total flow time etc. In this paper, we have considered the 2- machine n-jobs flow shop scheduling problem under fuzzy environment with objective as minimization of total waiting time of jobs. The present paper is an attempt to solve the problem made by Gupta D. & Goyal B.[14] under fuzzy environment. Hence the problem discoursed here has significant use in process industries.

This paper comprises of the following sections:
In section 2, we have defined the basic definitions on fuzzy number and numerous arithmetic operations used on that numbers. Section 3 formulated the proposed problem in a mathematical model which also includes the essential assumptions and notations. In Section 4, a mathematical theorem is established to get the optimal results. In section 5, a heuristic algorithm is developed on the basis of theorem formulated in the section 4. Section 6 providesa numerical example to illustrate the algorithm developed in the previous section.Comparison of the results are provided in section 7.Concluding remarks are given in section 8.

2. Preliminaries

The aim of this section is to present some fuzzy concepts whichare useful in further considerations.

2.1. Fuzzy Number :A fuzzy set ã defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{a}}$: R → [0, 1] has the following characteristics:
(i). ã is convex, that is ã $(\alpha x_1 + (1 - \alpha)x_2) \geq \min\{$ ã (x_1), ã $(x_2)\}$, for all $x_1, x_2 \in$R and $\alpha \in [0,1]$
(ii). ã is normal i.e. there exists an $x \in$R such that ã $(x) = 1$
(iii). ã is piecewise continuous.

2.2.Triangular Fuzzy Number:A fuzzy number $T_F = (\alpha,\beta,\gamma)$on R is said to be a triangular fuzzy number if its membership function T_F: R → [0, 1] has the following characteristics:

$$\mu_{T_F}(x)=\begin{cases}0;\ x\le\alpha,\\ \dfrac{x-\alpha}{\beta-\alpha};\ \alpha<x<\beta,\\ 1;\ x=\beta,\\ \dfrac{\gamma-x}{\gamma-\beta};\ \beta<x<\gamma,\\ 0;\ x\ge\gamma\end{cases}$$

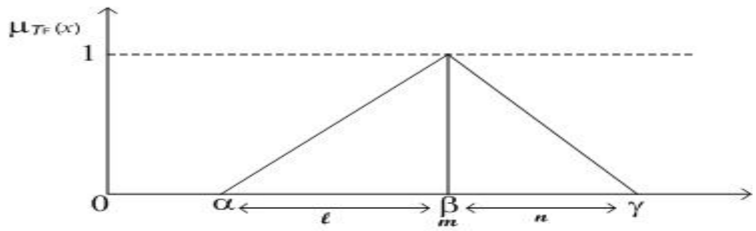


Fig.1: Triangular Fuzzy Number $T_F = (\alpha,\beta,\gamma) = (l, m, n)$

From Fig. 1 it is clear thatthe membership function $\mu_{\tilde{a}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{a}}: R \to [0,1]$ is continuous.
2. $\mu_{\tilde{a}}(x) = 0$ for every $x \varepsilon (-\infty, a] \cup (c, \infty]$
3. $\mu_{\tilde{a}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [b,c]
4. $\mu_{\tilde{a}}(x) = 1$ for x=b

2.3. Fuzzy Arithmetic Operations

Let $T_{F1} = (\alpha_1, \beta_1, \gamma_1)$and $T_{F2} = (\alpha_2, \beta_2, \gamma_2)$ be two triangular fuzzy numbers. Then the arithmetic operations on these fuzzy numbers can be defined as follows:

- **Addition :**$T_{F1} + T_{F2} = (\alpha_1 + \alpha_2, \beta_1 + \beta_2, \gamma_1 + \gamma_2)$
- **Subtraction:**$T_{F1} - T_{F2} = (\alpha_1 - \alpha_2, \beta_1 - \beta_2, \gamma_1 - \gamma_2)$ if the following condition is satisfied $DP(T_{F1}) \ge DP(T_{F2})$, where $DP(T_{F1}) = (\gamma_1 - \alpha_1)/2$ and $DP(T_{F2}) = (\gamma_2 - \alpha_2)/2$. Here DP denotes difference point of a TFN. Otherwise, $T_{F1} - T_{F2} = (\alpha_1 - \gamma_2, \beta_1 - \beta_2, \gamma_1 - \alpha_2)$
- **Equality :** $T_{F1} = T_{F2}$ if $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2$
- **Multiplication :**Suppose A = (a_1, b_1, c_1) be any triangular fuzzy number and B = (a_2, b_2, c_2) be non-negative triangular fuzzy number, then we define:

$$A\times B=\begin{cases}(a_1a_2, b_1b_2, c_1c_2), a_1>0;\\ (a_1c2, b_1b_2, c_1c_2), a_1<0, c_1\ge 0;\\ (a_1c_2, b_1b_2, c_1a_2), c_1<0\end{cases}$$

- **Max**$[(T_{F1}), (T_{F2})] = T_{F1}$ if $\alpha_1 > \alpha_2;\ \beta_1 > \beta_2;\ \gamma_1 > \gamma_2$

3. Model Description

Thissection providessomenotations, assumptions and mathematical formulation of the proposed model.

3.1. Notations

The various notations used in the paper are given below:

Notations	Description
i	Index for jobs, i = 1,2,3,…..n.
f_{iA}	Fuzzy Processing time of ith job on machine A.
f_{iB}	Fuzzy Processing time of ith job on machine B.
h_i^A	AHR of fuzzy Processing time of ith job on machine A.
h_i^B	AHR of fuzzy Processing time of ith job on machine B.
σ_k	Optimal Sequence, k = 1,2,3……..n.
W_T	Total waiting time.

3.2 Assumptions

1. All the jobs and machines are available at time $t = 0$.
2. Set up times for operations are sequence dependent and has been excluded from the processing times.
3. The first machine is assumed to be ready whichever and whatever job is to be processed on it first.
4. Each machine is continuously available for assignment.
5. Machines never break down and are available throughout the scheduling period.

3.3. Mathematical Formulation

Let there are total 'n' jobs. Assume all jobs are processed on given machines in a specific order. Let f_{iA}andf_{iB}be the fuzzy processing time on machine A andB respectively. In the same way,$S_{\alpha_i}^{A}$and $S_{\alpha_i}^{B}$are the fuzzy setup time of ith jobon machine A and Brespectivelywhich are described by triangular fuzzy numbers.Objective is to achieve a schedule $\{\sigma_k, k = 1,2 \ldots. n.\}$resulting in the minimization of total waiting time of jobs. Mathematically, the problem is stated in the Table 1.

Table 1: Jobs with uncertain processing time

Job i	Machine P		Machine Q	
	f_{iA}	$S_{\alpha_i}^{A}$	f_{iB}	$S_{\alpha_i}^{B}$
1.	$(\alpha_{11}, \beta_{11}, \gamma_{11})$	$(S_{\alpha_1}^{A}, S_{\beta_1}^{A}, S_{\gamma_1}^{A})$	$(\alpha_{12}, \beta_{12}, \gamma_{12})$	$(S_{\alpha_1}^{B}, S_{\beta_1}^{B}, S_{\gamma_1}^{B})$
2.	$(\alpha_{21}, \beta_{21}, \gamma_{21})$	$(S_{\alpha_2}^{A}, S_{\beta_2}^{A}, S_{\gamma_2}^{A})$	$(\alpha_{22}, \beta_{21}, \gamma_{21})$	$(S_{\alpha_2}^{B}, S_{\beta_2}^{B}, S_{\gamma_2}^{B})$
3.	$(\alpha_{31}, \beta_{31}, \gamma_{31})$	$(S_{\alpha_3}^{A}, S_{\beta_3}^{A}, S_{\gamma_3}^{A})$	$(\alpha_{31}, \beta_{31}, \gamma_{31})$	$(S_{\alpha_3}^{B}, S_{\beta_3}^{B}, S_{\gamma_3}^{B})$
.	.	.	.	.
n	$(\alpha_{n1}, \beta_{n1}, \gamma_{n1})$	$(S_{\alpha_n}^{A}, S_{\beta_n}^{A}, S_{\gamma_n}^{A})$	$(\alpha_{n1}, \beta_{n1}, \gamma_{n1})$	$(S_{\alpha_n}^{B}, S_{\beta_n}^{B}, S_{\gamma_n}^{B})$

 ISSN: 2454-4248
215 – 225

4. Theorem

Theorem statement: Let n-jobs (say) $j_1, j_2, j_3, \dots \dots \dots j_n$ be processed on machine X in a specific orderwith no passing allowed and satisfying the following structural condition:

$$\text{Max } h_{ji}^{P} \leq \text{Min } h_{ji}^{Q}$$

Where h_{ji}^{X} is the AHR value of the equivalent fuzzy processing time defined as $f'_{iP} = f_{iP} - S_{\alpha_i}^{Q}$ and $f'_{iQ} = f_{iQ} - S_{\alpha_i}^{P}$ of job j_i on machine X(X = P,Q); $(i = 1,2,3, \dots \dots, n)$, then for any job sequence $\sigma = \{j_1, j_2, j_3, \dots, \dots, j_n\}$,the total waiting time W_T (say) is given by:

$$W_T = n h_{\delta}^{P} + \sum_{r=1}^{n-1} z_{j_r} - \sum_{i=1}^{n} h_{ji}^{P}$$

where, $z_{j_r} = (n - r)x_{j_r}$; $j_r \in (1, 2, 3, \dots, n)$ and h_{δ}^{P} =processing time of the first job on machine P in sequence obtained by arranging the jobs of x_{j_r}.

Proof: Before the proof of theorem, first of all we will prove the following two lemmas **Lemma1:** For the n- job sequence $\sigma = \{ j_1, j_2, j_3, \dots, j_n\}$, $C_{jn}^{Q} = h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots . + h_{jn}^{Q}$ where C_{jn}^{Q} is the completion time of job j_n on machineQ.

Proof: We will prove the lemma by applying mathematical induction hypothesis on k:

Let the statement be S(k) defined as:

S(k): $C_{jn}^{Q} = h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots . + h_{jn}^{Q}$

Now$C_{j1}^{P} = h_{j1}^{P}$

$C_{j1}^{Q} = h_{j1}^{P} + h_{j1}^{Q}$

Hence for $k = 1$, the statement $S(1)$ is true.

Let the statement is true for n= m, i.e.

$C_{jm}^{Q} = h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots . + h_{jm}^{Q}$

Now, $C_{j(m+1)}^{Q} = \max\left(C_{j(m+1)}^{P}, \ C_{jm}^{Q}\right) + h_{j(m+1)}^{Q}$

$$= \max\left(h_{j1}^{P} + h_{j2}^{P} + h_{j3}^{P} \dots . + h_{j(m+1)}^{P}, h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots . + h_{jm}^{Q}\right) + \ h_{j(m+1)}^{Q}$$

asmin$h_{ji}^{Q} \ \geq \ maxh_{ji}^{P}$

Hence $C_{j(m+1)}^{Q} = h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots . + h_{jm}^{Q} + h_{j(m+1)}^{Q}$

Hence for $n = m + 1$, the statement $S(m + 1)$ holds true.

Since $S(n)$ is true for$n = 1, n = m, n = m + 1$, and m being arbitrary.

Hence $S(n)$: $C_{jn}^{Q} = h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots . + h_{jn}^{Q}$ is true.

Lemma 2: For the n job sequence $\sigma = \{ j_1, j_2, j_3, \dots, \dots j_m \dots j_n\}$, we have $W_{j_1} = 0$&

$$W_{j_m} = h_{j1}^{P} + \sum_{r=1}^{m-1} Z_{j_r} - h_{jm}^{P} \ ; m = 2,3, \dots . n$$

where W_{j_m} is the waiting time of jobj_m for sequence $\{j_1, j_2, j_3, \dots, \dots j_n\}$

 ISSN: 2454-4248
215 – 225

$\&\mathrm{Z}_{\mathrm{j_r}} = h_{jr}^{Q} - h_{jr}^{P}$; $\mathrm{j_r}\ \epsilon\ (1, 2, 3, \dots, \mathrm{n})$

Proof: $\mathrm{W}_{\mathrm{j_1}} = 0$

$$\mathrm{W}_{\mathrm{j_m}} = \max\left(C_{jm}^{P}, C_{j(m-1)}^{Q}\right) - C_{jm}^{P}$$

$$= \max(h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots + h_{j(m-1)}^{Q}, h_{j1}^{P} + h_{j2}^{P} \dots + h_{jm}^{P}) - (h_{j1}^{P} - h_{j2}^{P} \dots - h_{jm}^{P})$$

$$= h_{j1}^{P} + h_{j1}^{Q} + h_{j2}^{Q} \dots + h_{j(m-1)}^{Q} - h_{j1}^{P} - h_{j1}^{P} - h_{j2}^{P} \dots - h_{jm}^{P}$$

$$= h_{j1}^{P} + (h_{j1}^{Q} - h_{j1}^{P}) + (h_{j2}^{Q} - h_{j2}^{P}) + \cdots \dots + (h_{j(m-1)}^{Q} - h_{j(m-1)}^{P}) - h_{jm}^{P}$$

$$= h_{j1}^{P} + \sum_{\mathrm{r}=1}^{\mathrm{m}-1} (h_{jr}^{Q} - h_{jr}^{P}) - h_{jm}^{P}$$

$$= h_{j1}^{P} + \sum_{\mathrm{r}=1}^{\mathrm{m}-1} (\mathrm{Z}_{\mathrm{j_r}}) - h_{jm}^{P}$$

Now we are able to attempt the proof of the main theorem as follows:

Proof : From Lemma 2, we have

$$\mathrm{W}_{\mathrm{j_1}} = 0,$$

$$\mathrm{W}_{\mathrm{j_m}} = h_{j1}^{P} + \sum_{\mathrm{r}=1}^{\mathrm{m}-1} \mathrm{Z}_{\mathrm{j_r}} - h_{jm}^{P} \ ; m = 2,3, \dots . n$$

For $\mathrm{m} = 2$, we have

$$\mathrm{W}_{\mathrm{j_2}} = h_{j1}^{P} + \sum_{\mathrm{r}=1}^{1} \mathrm{Z}_{\mathrm{j_r}} - h_{j2}^{P}$$

For $\mathrm{m} = 3$, we have

$$\mathrm{W}_{\mathrm{j_3}} = h_{j1}^{P} + \sum_{\mathrm{r}=1}^{2} \mathrm{Z}_{\mathrm{j_r}} - h_{j3}^{P}$$

Continuing in this way

…………………….

…………………….

For $\mathrm{m} = \mathrm{n}$,

$$\mathrm{W}_{\mathrm{j_n}} = h_{j1}^{P} + \sum_{\mathrm{r}=1}^{\mathrm{n}-1} \mathrm{Z}_{\mathrm{j_r}} - h_{jn}^{P}$$

Hence total waiting time

$$\mathrm{W}_{\mathrm{T}} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{W}_{\mathrm{j_i}}$$

$$\mathrm{W}_{\mathrm{T}} = \mathrm{n} h_{j1}^{P} + \sum_{\mathrm{r}=1}^{\mathrm{n}-1} \mathrm{z}_{\mathrm{j_r}} - \sum_{\mathrm{i}=1}^{\mathrm{n}} h_{ji}^{P}$$

Where, $\mathrm{z}_{\mathrm{j_r}} = (\mathrm{n} - \mathrm{r})\mathrm{x}_{\mathrm{j_r}}$; $\mathrm{j_r}\ \epsilon\ (1, 2, 3, \dots, \mathrm{n})$

5. Heuristic Algorithm

To obtain optimal schedule we proceed as follows:

Step 1: Calculate expected fuzzy processing time f'_{iA} and f'_{iB} on machines A &B respectively as follows:

(i) $f'_{iA} = f_{iA} - S_{\alpha_i}^{B}$

(ii) $f'_{iB} = f_{iB} - S_{\alpha_i}^{A}$

Step 2: Evaluate <AHR> of the expected fuzzy processing time for all the jobs using Yager's (1981)formula.

Step 3: Check the Structural condition i.e. Max $h_i^A \leq$ Min h_i^B

Step 4: Calculate $z_{ir} = (n-r)x_i$ where $x_i = h_i^A - h_i^B$ for r = 1,2,3,…….n-1 and place all computed values in the following format.

Job	Machine A	Machine B		$z_{ir} = (n-r)x_i$				
I	h_i^A	h_i^B	$x_i = h_i^B - h_i^A$	r = 1	r = 2	r = 3	…….	r = n − 1
j_1	h_1^A	h_1^B	x_1	z_{11}	z_{12}	z_{13}	…….	$z_{1(n-1)}$
j_2	h_2^A	h_2^B	x_2	z_{21}	z_{22}	z_{23}	…….	$z_{2(n-1)}$
j_3	h_3^A	h_3^B	x_3	z_{31}	z_{32}	z_{33}	…….	$z_{3(n-1)}$
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
j_n	h_n^A	h_n^B	x_n	z_{n1}	z_{n2}	z_{n3}	…….	$z_{n(n-1)}$

Step 5: Obtain a Sequence $\sigma_1 = \{ j_1, j_2, j_3, \ldots \ldots \ldots j_n \}$ (say) by arranging the jobs in increasing order of x_i.

Step 6: Find minimum of h_i^A on machine A. Let it is h_α^A. Now check the condition:

$\min\{ h_i^A \} = h_\alpha^A = h_\delta^A$ where h_δ^A is the expected processing time of the first job on machine A in above obtain sequence σ_1. if this condition satisfies then the sequence obtained in above step is optimal sequence. Otherwise we will move to step 7.

Step 7: Obtain all the possible sequences σ_i's for i = 2,3…n by placing ith job in the sequence σ_1 to the first position and rest of the sequence remaining same.

Step 8: Calculate the total waiting time W_T for all the sequence $\sigma_1, \sigma_2, \sigma_3, \ldots \ldots, \sigma_n$ using the following formula:

$$W_T = n h_\delta^A + \sum_{r=1}^{n-1} z_{ar} - \sum_{i=1}^{n} h_i^A$$

h_δ^A = Expected processing time of the first job on machine A in sequence σ_i

$z_{ar} = (n-r)x_{ar}$; $a = j_1, j_2, j_3, \ldots \ldots \ldots j_n$

Step 9: Choose a sequence with minimum total waiting time which is the required optimal sequence.

 ISSN: 2454-4248
215 – 225

6. Numerical Illustration

To evaluate the performance of the proposed algorithm, a numerical illustration is given below:

Let 5 jobs say j_1, j_2, j_3, j_4, j_5 are processed on two machines namely A &B. The processing time and the set up time of each job on each machine are described by triangular fuzzy numbers.Let the processing time matrix be seen as given below:

Table 3: Uncertain processing time and setup time of jobs

Job	Machine A		Machine B	
i	$\mathbf{f_{iA}}$	$S_{\alpha_i}{}^A$	$\mathbf{f_{iB}}$	$S_{\alpha_i}{}^B$
j_1	(11,15,20)	(4,7,10)	(22,25,27)	(2,6,10)
j_2.	(10,14,22)	(1,3,5)	(15,19,24)	(4,5,9)
j_3	(8,12,16)	(3,5,8)	(17,23,28)	(1,2,4)
j_4	(13,17,20)	(3,7,10)	(19,26,31)	(2,5,11)
j_5	(17,21,29)	(1,2,3)	(11,18,25)	(3,7,10)

Our objective is to obtain an optimal schedule that minimizes the total waiting time of the jobs on given machines.

Solution: To evaluate the performance of the proposed algorithm a numerical illustration is given below:

Step 1: The Calculated values of expected fuzzy processing timesare shown in Table 4:

Table 4: Expected fuzzy processing time

Job	Machine A	Machine B
i	$\mathbf{f'_{iA}}$	$\mathbf{f'_{iB}}$
j_1	(9,9,10)	(12,18,23)
j_2.	(6,9,13)	(14,16,19)
j_3	(7,10,12)	(14,18,20)
j_4	(2,12,18)	(16,19,21)
j_5	(14,14,19)	(10,16,22)

Step 2: The AHR values of the given fuzzy processing times are calculated using Yager's formulae and are given as below:

Table 5: Crisp Values of expected fuzzy processing time

Job	Machine A	Machine B
i	h_i^A	h_i^B
j_1	28/3	65/3
j_2.	34/3	53/3
j_3	35/3	60/3
j_4	48/3	62/3
j_5	47/3	60/3

Step 3: Here the structural condition for the specially structured Problem is satisfied i.e. Max $h_i^A \leq$ Min h_i^B

Step 4: All theComputed values ofz_{ir} for $r = 1,2,3,4$ andx_i for i = 1,2,…...5 are described in the Table 6.

 ISSN: 2454-4248
215 – 225

Table 6: Description of calculated z_{ir}

Job	Machine A	Machine B	x_i	$z_{ir} = (5 - r)x_i$			
i	h_i^A	h_i^B		r = 1 $z_{i1} = 4x_1$	r = 2 $z_{i2} = 3x_2$	r = 3 $z_{i3} = 2x_3$	r = 4 $z_{i4} = x_4$
j_1	28/3	65/3	37/3	148/3	111/3	74/3	37/3
j_2	34/3	53/3	19/3	76/3	57/3	38/3	19/3
j_3	35/3	60/3	25/3	100/3	75/3	50/3	25/3
j_4	48/3	62/3	14/3	56/3	42/3	28/3	14/3
j_5	47/3	60/3	13/3	52/3	39/3	26/3	13/3

Step 5: As per step 5, provided in the algorithm we got sequence σ_1 ={5,4,2,3,1}.

Step 6: Here Min $\{h_i^A\} \neq h_\delta^A$. So we will proceed to next step**.**

Step 7: All the possible sequencesσ_i's for i = 2,3…5 by rearranging the positions of jobs in Sequence σ_1 **:** $\{j_5,j_4,j_2,j_3, j_1\}$ are given as σ_2: $\{j_4,j_5,j_2,j_3,j_1\}$, σ_3: $\{j_2,j_5,j_4,j_3,j_1\}$, σ_4: $\{j_3,j_5,j_4,j_2,j_1\}$, σ_5**:** $\{j_1,j_5,j_4,j_2,j_3\}$

Step 8: The total waiting time for the sequences σ_i's for i = 1, 2,3…5 are given in Table 7

Table7: Optimal Sequence Table

Sequence	Total Waiting Time (W_T)
σ_1	214/3
σ_2	206/3
$\boldsymbol{\sigma_3}$	**146/3**
σ_4	169/3
σ_5	182/3

Step 9: Here Min(W_T) =146/3 = 48.66 units of time which is calculated for Sequence σ_3

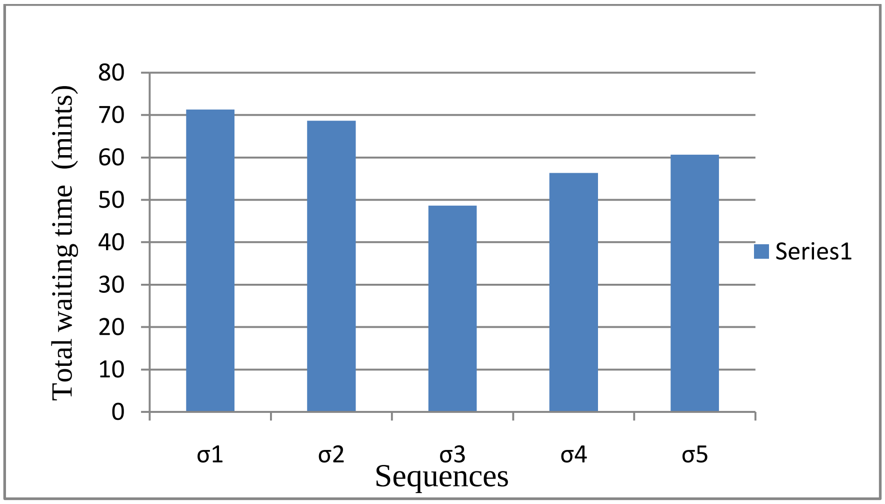


Fig. 2: Depiction of total waiting time for different sequences

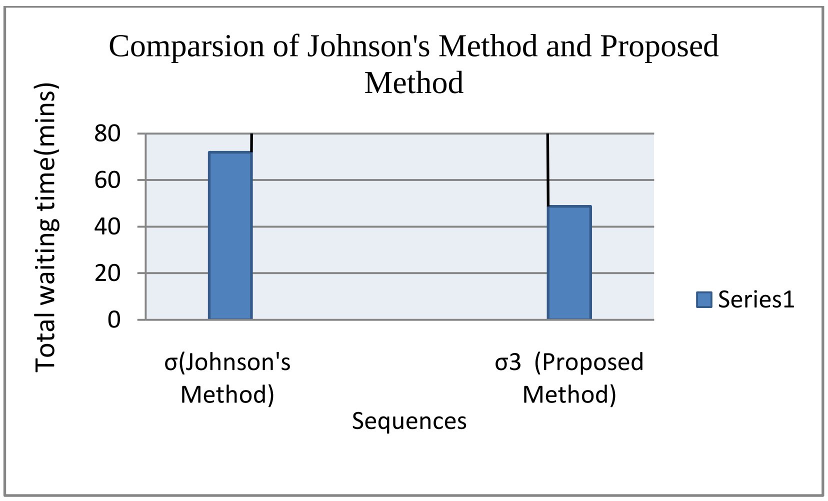
Hence σ_3 is the required optimal sequence achieving our objective function for the proposedproblem.

7. Result Analysis

We have compared our results with the results obtained by applying Johnson's technique and conclude that proposed technique provides better results than the existing algorithm given by Johnson. Comparative analysis of the above define problem is shown in Table 8.

Table 8: Comparison of the results

Technique	Optimal Sequence	Total waiting time(units)
Johnson's Heuristic	σ: $j_1 - j_2 - j_3 - j_5 - j_4$	216/3 = 72
Proposed Heuristic	$\sigma3$: $j_2 - j_5 - j_4 - j_3 - j_1$	146/3 = 48.66



8. Conclusion

In the present paper, we have developed a new algorithm to minimize the total waiting time of jobs using a heuristic technique which provides the more legitimate outcomes when compared with the existing algorithm given by Johnson. The present work can additionally be extended by taking trapezoidal fuzzy numbers, considering weighted jobs and by presenting the idea of breakdown of machines and so on.

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