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A Two Warehouse Inventory Model with Stock-Dependent Demand and variable deterioration rate

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Abstract— In this paper we discuss a two warehouses inventory model for non-instantaneous deteriorating items. Throughout last so many years, mostly researchers have consideration to the situation where the demand rate is dependent on the level of the on-hand inventory. For inventory systems, such as fashionable commodities, the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not. In real life situation, enterprises usually buy more goods than can be stored in their own warehouses (OW) for future production or sales. The surplus quantities are frequently stored in an extra storage space, represented by rented warehouses (RW).The rented warehouse is considered to charge high unit holding cost than the own warehouse. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown. We determine the optimal replenishment policy for non-instantaneous deteriorating items with partial backlogging and stock-dependent demand.

Keywords*-Inventory; Two warehouses; Variable deterioration; shortages.*

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I. INTRODUCTION

The problem of determine the optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand is considered in this paper. Ghare and Schrader [8] developed an EOQ inventory model for deteriorating items. Covert and Philip [6] discussed the inventory model with variable deterioration rate with two-parameter Weibull distribution. Many researchers such as Hollier and Mak [11] and Wee [15] developed the constant partial backlogging rates during the shortage period in their inventory models. Some others researchers such as Abad [1], Mishra et al. [3], Dye and Ouyang [7], Chang et al. [5] have modified some inventory policies by considering the ‘‘time-proportional partial backlogging rate’’. Singh and Malik [14] developed a demand dependent production inventory model with partial backlogging and two storage capacity. Maihami and Kamalabadi [2] discussed an inventory model for non-instantaneously deteriorating items and demand function is taken as a price-and-time-dependent. Philip [12] studied the model with a three parameter Weibull distribution rate and without shortages. Gupta and Vrat [10] have discussed the inventory models for stock-dependent consumption rate. Sarkar and Sarkar [4] discussed an improved inventory model for the time varying deterioration and stock-dependent demand with partial backlogging.

Gupta et al. [13] present a mathematical model for optimal ordering policy and stock-dependent demand inventory model with non-instantaneous deteriorating items. Yang et al. [9] discussed the model of supply chain coordination with stock-dependent demand rate and credit incentives. Vashisth et al. [17] discussed an inventory model for non-instantaneous deteriorating items. Recently, Chang et al. [16] derived a mathematical model with optimal pricing and ordering policies for non-instantaneously deteriorating items under the order-size-dependent delay in payments. Vashisth et al. [18] developed a trade credit inventory model with multi-variate demand for non-instantaneous decaying items. Kumar et al. [19] presents a mathematical model with linear holding cost and stock-dependent demand for non-instantaneous deteriorating items. Malik et al. [20] proposed analysis of an inventory model with non-instantaneous deteriorating Items.

In this paper we have developed a two warehouse inventory model to determining the optimal replenishment policy for non-instantaneous deteriorating items stock-dependent demand. In this particular model shortages are allowed. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are given.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions and notation have been used in this paper:

(1) The demand rate D (t) at time t is

$$D(t)=\begin{cases} a+bI(t), & I(t)>0, \\ a, & I(t)\le 0, \end{cases}$$

Where a, b are positive constants and I(t) is the inventory level at time t.

(2) Shortages are allowed to occur. $B(t)=\dfrac{1}{1+\lambda t}$ consider the backlogging rate.

(3) Replenishment rate is infinite and the lead time is zero.

(4) t_1 is the length of time in which the product has no deterioration (i.e., fresh product time). θ(t)=θt is the deterioration rate in RW and OW respectively.

(5) t_3 is the length of time in which the inventory is finished. T is the length of order cycle. Q is the order quantity per cycle. t_2, T and Q are decision variables.

(6) C_o, h_1, h_2, C_d, C_s *and* C_l denotes the ordering cost per order, inventory holding cost in RW per unit time, inventory

holding cost in OW per unit time, deteriorating cost for both RW and OW per unit, the shortage cost for backlogged items and the unit cost of lost sales, respectively. All of the cost parameters are positive constant.

(7) I_{R1} denotes the inventory level in RW at time [0, t_1] in which the product has no deterioration. I_{R2} denotes the inventory level in RW at time [t_1, t_2] in which the product has deterioration. I_{O1} is the inventory level in OW at time [0, t_1] in which the product has no deterioration. I_{O2} denote the inventory level in OW at time [t_1, t_2] in which only deterioration. I_{O3} denote the inventory level in OW at time [t_2, t_3] in which the product has deterioration. I_N denote the inventory level in shortage at time t in which the product has shortage.

(8) TIC (t_1, t_2, t_3, T) is the total relevant inventory cost per unit time of inventory system.

III. MATHEMATICAL MODEL

The inventory level at RW and OW are governed by the following differential equations:

$$\frac{dI_{R1}(t)}{dt} = -[a + b\, I_{R1}(t)] \qquad 0\le t\le t_1 \qquad \dots (1)$$

$$\frac{dI_{R2}(t)}{dt} + \theta(t) I_{R2}(t) = -[a + b\, I_{R2}(t)] \qquad t_1\le t\le t_2 \qquad \dots (2)$$

$$\frac{dI_{O1}(t)}{dt} = 0 \qquad 0\le t\le t_1 \qquad \dots (3)$$

$$\frac{dI_{O2}(t)}{dt} + \theta(t)\; I_{O2}(t) = 0 \qquad t_1\le t\le t_2 \qquad \dots (4)$$

$$\frac{dI_{O3}(t)}{dt} + \theta(t)\; I_{O3}(t) = -[a + b\, I_{O3}(t)], \qquad t_2\le t\le t_3 \qquad \dots (5)$$

$$\frac{dI_N(t)}{dt} = -\frac{a}{1+\lambda(T-t)}, \qquad t_3\le t\le T \qquad \dots (6)$$

with the boundary conditions $I_{R1}(0) = W_R$, $I_{R2}(t_2) = 0,\; I_{O1}(t_1) = W_O = I_{O2}(t_1),\; I_{O3}(t_3) = 0,\; I_N(t_3) = 0$ respectively.

The solutions of the above equations are:

$$I_{R1}(t) = W_R\; e^{-bt} + \frac{a}{b}\left(e^{-bt} - 1\right) \qquad \dots (7)$$

$$\begin{aligned} I_{R2}(t) = {} & a\left\{(t_2 - t) + \frac{b}{2}\left(t_2^2 - t^2\right) + \frac{\theta}{6}\left(t_2^3 - t^3\right)\right\} \\ & - ab\left\{\left(tt_2 - t^2\right) + \frac{b}{2}\left(tt_2^2 - t^3\right) + \frac{\theta}{6}\left(tt_2^3 - t^4\right)\right\} \\ & - \frac{a\theta}{2}\left\{\left(t^2 t_2 - t^3\right) + \frac{b}{2}\left(t^2 t_2^2 - t^4\right) + \frac{\theta}{6}\left(t^2 t_2^3 - t^5\right)\right\} \end{aligned} \qquad \dots (8)$$

$$I_{O1}(t) = W_O \qquad \dots (9)$$

$$I_{O2}(t) = W_O\; e^{\frac{\theta}{2}\left(t_1^2 - t^2\right)} \qquad \dots (10)$$

$$\begin{aligned} I_{O3}(t) = {} & a\left\{(t_3 - t) + \frac{b}{2}\left(t_3^2 - t^2\right) + \frac{\theta}{6}\left(t_3^3 - t^3\right)\right\} \\ & - ab\left\{\left(tt_3 - t^2\right) + \frac{b}{2}\left(tt_3^2 - t^3\right) + \frac{\theta}{6}\left(tt_3^3 - t^4\right)\right\} \\ & - \frac{a\theta}{2}\left\{\left(t^2 t_3 - t^3\right) + \frac{b}{2}\left(t^2 t_3^2 - t^4\right) + \frac{\theta}{6}\left(t^2 t_3^3 - t^5\right)\right\} \end{aligned} \qquad \dots (11)$$

$$I_N(t) = \frac{a}{\lambda}\log\left(\frac{1+\lambda(T-t)}{1+\lambda(T-t_3)}\right) \qquad \dots (12)$$

Due to considering continuity of $I_R(t)$ at $t=t_1$, using (7) and (8) that

$$I_{R1}(t_1) = I_{R2}(t_1)$$

$$\begin{aligned} \Rightarrow\; W_R = {} & \frac{a}{b}\left\{1 - e^{bt_1}\right\} + a\left\{(t_2 - t_1) + \frac{b}{2}\left(t_2^2 - t_1^{\,2}\right) + \frac{\theta}{6}\left(t_2^3 - t_1^{\,3}\right)\right\} \\ & - \frac{a\theta}{2}\left\{\left(t_1^{\,2} t_2 - t_1^{\,3}\right) + \frac{b}{2}\left(t_1^{\,2} t_2^2 - t_1^{\,4}\right) + \frac{\theta}{6}\left(t_1^{\,2} t_2^3 - t^5\right)\right\} \end{aligned} \qquad \dots (13)$$

According to given conditions at $t=t_2$, $I_{O2}(t_2) = I_{O3}(t_2)$, we have

$$\begin{aligned} W_O = {} & a\left\{(t_3 - t_2) + \frac{b}{2}\left(t_3^2 - t_2^{\,2}\right) + \frac{\theta}{6}\left(t_3^3 - t_2^{\,3}\right)\right\} \\ & - ab\left\{\left(t_2 t_3 - t_2^{\,2}\right) + \frac{b}{2}\left(t_2 t_3^2 - t_2^{\,3}\right) + \frac{\theta}{6}\left(t_2 t_3^3 - t_2^{\,4}\right)\right\} \\ & - \frac{a\theta}{2}\left\{\left(t_1^{\,2} t_3 - t_2 t_1^{\,2}\right) + \frac{b}{2}\left(t_1^{\,2} t_3^2 - t_1^{\,2} t_2^{\,2}\right) + \frac{\theta}{6}\left(t_1^{\,2} t_3^3 - t_1^{\,2} t_2^{\,3}\right)\right\} \end{aligned} \qquad \dots (14)$$

Putting t=T in Equation (12), we obtain the maximum amount of demand backlogged per cycle as follows:

$$S = -I_N(T) = \frac{a}{\lambda}\log\left(1 + \lambda(T - t_3)\right) \qquad \dots (15)$$

The total order quantity, Q; as

$$Q = W_R + W_O + S$$

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$$
\begin{aligned}
&= \frac{a}{b}\left\{1-e^{bt_1}\right\} + a\left\{(t_2-t_1)+\frac{b}{2}\left(t_2^2-t_1^{\,2}\right)+\frac{\theta}{6}\left(t_2^3-t_1^{\,3}\right)\right\} \\
&\quad -\frac{a\theta}{2}\left\{\left(t_1^{\,2}t_2-t_1^{\,3}\right)+\frac{b}{2}\left(t_1^{\,2}t_2^2-t_1^{\,4}\right)+\frac{\theta}{6}\left(t_1^{\,2}t_2^3-t^5\right)\right\} \\
&\quad + a\left\{(t_3-t_2)+\frac{b}{2}\left(t_3^2-t_2^{\,2}\right)+\frac{\theta}{6}\left(t_3^3-t_2^{\,3}\right)\right\} \\
&\quad - ab\left\{\left(t_2t_3-t_2^{\,2}\right)+\frac{b}{2}\left(t_2t_3^2-t_2^{\,3}\right)+\frac{\theta}{6}\left(t_2t_3^3-t_2^{\,4}\right)\right\} \\
&\quad -\frac{a\theta}{2}\left\{\left(t_1^{\,2}t_3-t_2t_1^{\,2}\right)+\frac{b}{2}\left(t_1^{\,2}t_3^2-t_1^{\,2}t_2^{\,2}\right)+\frac{\theta}{6}\left(t_1^{\,2}t_3^3-t_1^{\,2}t_2^{\,3}\right)\right\} \\
&\qquad +\frac{a}{\lambda}\log\left(1+\lambda(T-t_3)\right)
\end{aligned}
$$

…. (16)

Next, the total relevant inventory cost per cycle consists of the following elements:

1. Ordering cost per cycle is C_O ….(17)

2. Inventory holding cost per cycle in RW is

$$
IHC_{RW} = h_1\left\{\int_0^{t_1} I_{R1}(t)dt + \int_{t_1}^{t_2} I_{R2}(t)dt\right\}
$$

$$
= h_1\left[
\begin{aligned}
&\left\{\frac{a}{b^2}\left(1-bt_1-e^{-bt_1}\right)-\frac{W_R}{b}\left(e^{-bt_1}-1\right)\right\} \\
&+\left(\begin{aligned}
&a\left\{\frac{t_2^{\,2}}{2}+\frac{b}{3}t_2^{\,3}+\frac{\theta}{8}t_2^{\,4}\right\}-ab\left\{\frac{t_2^{\,3}}{6}+\frac{b}{8}t_2^{\,4}+\frac{\theta}{20}t_2^{\,5}\right\} \\
&-\frac{a\theta}{2}\left\{\frac{t_2^{\,4}}{12}+\frac{b}{15}t_2^{\,5}+\frac{\theta}{36}t_2^{\,6}\right\}
\end{aligned}\right) \\
&-\left(\begin{aligned}
&a\left\{\left(t_1t_2-\frac{t_1^{\,2}}{2}\right)+\frac{b}{2}\left(t_1t_2^2-\frac{t_1^{\,3}}{3}\right)+\frac{\theta}{6}\left(t_1t_2^3-\frac{t_1^{\,4}}{4}\right)\right\} \\
&-ab\left\{\left(\frac{t_1^{\,2}}{2}t_2-\frac{t_1^{\,3}}{3}\right)+\frac{b}{2}\left(\frac{t_1^{\,2}}{2}t_2^2-\frac{t_1^{\,4}}{4}\right)+\frac{\theta}{6}\left(\frac{t_1^{\,2}}{2}t_2^3-\frac{t_1^{\,5}}{5}\right)\right\} \\
&-\frac{a\theta}{2}\left\{\left(\frac{t_1^{\,3}}{3}t_2-\frac{t_1^{\,4}}{4}\right)+\frac{b}{2}\left(\frac{t_1^{\,3}}{3}t_2^2-\frac{t_1^{\,5}}{5}\right)+\frac{\theta}{6}\left(\frac{t_1^{\,3}}{3}t_2^3-\frac{t_1^{\,6}}{6}\right)\right\}
\end{aligned}\right)
\end{aligned}
\right]
$$

…. (18)

3. Inventory holding cost per cycle in OW is

$$
IHC_{OW} = h_2\left\{\int_0^{t_1} I_{O1}(t)dt + \int_{t_1}^{t_2} I_{O2}(t)dt + \int_{t_2}^{t_3} I_{O3}(t)dt\right\}
$$

$$
= h_2\left[
\begin{aligned}
&W_O t_1 \\
&+W_O\, e^{\frac{\theta}{2}t_1^2}\left\{(t_2-t_1)-\frac{\theta}{6}\left(t_2^{\,3}-t_1^{\,3}\right)\right\} \\
&+\left(\begin{aligned}
&a\left\{\frac{t_3^{\,2}}{2}+\frac{b}{3}t_3^{\,3}+\frac{\theta}{8}t_3^{\,4}\right\}-ab\left\{\frac{t_3^{\,3}}{6}+\frac{b}{8}t_3^{\,4}+\frac{\theta}{20}t_3^{\,5}\right\} \\
&-\frac{a\theta}{2}\left\{\frac{t_3^{\,4}}{12}+\frac{b}{15}t_3^{\,5}+\frac{\theta}{36}t_3^{\,6}\right\}
\end{aligned}\right) \\
&-\left(\begin{aligned}
&a\left\{\left(t_2t_3-\frac{t_2^{\,2}}{2}\right)+\frac{b}{2}\left(t_2t_3^2-\frac{t_2^{\,3}}{3}\right)+\frac{\theta}{6}\left(t_2t_3^3-\frac{t_2^{\,4}}{4}\right)\right\} \\
&-ab\left\{\left(\frac{t_2^{\,2}}{2}t_3-\frac{t_2^{\,3}}{3}\right)+\frac{b}{2}\left(\frac{t_2^{\,2}}{2}t_3^2-\frac{t_2^{\,4}}{4}\right)+\frac{\theta}{6}\left(\frac{t_2^{\,2}}{2}t_3^3-\frac{t_2^{\,5}}{5}\right)\right\} \\
&-\frac{a\theta}{2}\left\{\left(\frac{t_2^{\,3}}{3}t_3-\frac{t_2^{\,4}}{4}\right)+\frac{b}{2}\left(\frac{t_2^{\,3}}{3}t_3^2-\frac{t_2^{\,5}}{5}\right)+\frac{\theta}{6}\left(\frac{t_2^{\,3}}{3}t_3^3-\frac{t_2^{\,6}}{6}\right)\right\}
\end{aligned}\right)
\end{aligned}
\right]
$$

…. (19)

4. Deterioration cost per cycle in RW is

$$
DC_{RW} = C_d\left\{\int_{t_1}^{t_2}\theta(t)\, I_{R_2}(t)dt\right\}
$$

$$
= C_d\theta\left[
\begin{aligned}
&a\left\{\frac{t_2^{\,3}}{3}+\frac{b}{2}\left(\frac{t_2^{\,4}}{4}\right)+\frac{\theta}{6}\left(\frac{3t_2^{\,5}}{10}\right)\right\} \\
&-ab\left\{\frac{t_2^{\,4}}{12}+\frac{b}{2}\left(\frac{2t_2^{\,5}}{15}\right)+\frac{\theta}{6}\left(\frac{t_2^{\,6}}{6}\right)\right\} \\
&-\frac{a\theta}{2}\left\{\frac{t_2^{\,5}}{20}+\frac{b}{2}\left(\frac{t_2^{\,6}}{12}\right)+\frac{\theta}{6}\left(\frac{3t_2^{\,7}}{28}\right)\right\} \\
&-a\left\{\left(\frac{t_1^{\,2}}{2}t_2-\frac{t_1^{\,3}}{3}\right)+\frac{b}{2}\left(\frac{t_1^{\,2}}{2}t_2^2-\frac{t_1^{\,4}}{4}\right)+\frac{\theta}{6}\left(\frac{t_1^{\,2}}{2}t_2^3-\frac{t_1^{\,5}}{5}\right)\right\} \\
&+ab\left\{\left(\frac{t_1^{\,3}}{3}t_2-\frac{t_1^{\,4}}{4}\right)+\frac{b}{2}\left(\frac{t_1^{\,3}}{3}t_2^2-\frac{t_1^{\,5}}{5}\right)+\frac{\theta}{6}\left(\frac{t_1^{\,3}}{3}t_2^3-\frac{t_1^{\,6}}{6}\right)\right\} \\
&+\frac{a\theta}{2}\left\{\left(\frac{t_1^{\,4}}{4}t_2-\frac{t_1^{\,5}}{5}\right)+\frac{b}{2}\left(\frac{t_1^{\,4}}{4}t_2^2-\frac{t_1^{\,6}}{6}\right)+\frac{\theta}{6}\left(\frac{t_1^{\,4}}{4}t_2^3-\frac{t_1^{\,7}}{7}\right)\right\}
\end{aligned}
\right]
$$

… (20)

5. Deterioration cost per cycle in OW is

$$
DC_{OW} = C_d\left\{\int_{t_1}^{t_2}\theta(t)I_{O2}(t)dt + \int_{t_2}^{t_3}\theta(t)I_{O3}(t)dt\right\}
$$

$$= C_d\,\theta\left\{\begin{array}{l} W_O\, e^{\frac{\theta}{2}t_1^2}\left(\dfrac{t_2^{\,2}-t_1^2}{2}-\dfrac{\theta}{8}\left(t_2^{\,4}-t_1^4\right)\right) \\ + \left(\begin{array}{l} a\left\{\dfrac{t_3^{\,3}}{6}+\dfrac{b}{2}\left(\dfrac{t_3^{\,4}}{4}\right)+\dfrac{\theta}{6}\left(\dfrac{3t_3^{\,5}}{10}\right)\right\}-ab\left\{\dfrac{t_3^{\,4}}{12}+\dfrac{b}{2}\left(\dfrac{2t_3^{\,5}}{15}\right)+\dfrac{\theta}{6}\left(\dfrac{t_3^{\,6}}{6}\right)\right\} \\ -\dfrac{a\theta}{2}\left\{\dfrac{t_3^{\,5}}{20}+\dfrac{b}{2}\left(\dfrac{t_3^{\,6}}{12}\right)+\dfrac{\theta}{6}\left(\dfrac{3t_3^{\,7}}{28}\right)\right\} \end{array}\right) \\ - \left(\begin{array}{l} a\left\{\left(\dfrac{t_2^{\,2}}{2}t_3-\dfrac{t_2^{\,3}}{3}\right)+\dfrac{b}{2}\left(\dfrac{t_2^{\,2}}{2}t_3^2-\dfrac{t_2^{\,4}}{4}\right)+\dfrac{\theta}{6}\left(\dfrac{t_2^{\,2}}{2}t_3^3-\dfrac{t_2^{\,5}}{5}\right)\right\} \\ -ab\left\{\left(\dfrac{t_2^{\,3}}{3}t_3-\dfrac{t_2^{\,4}}{4}\right)+\dfrac{b}{2}\left(\dfrac{t_2^{\,3}}{3}t_3^2-\dfrac{t_2^{\,5}}{5}\right)+\dfrac{\theta}{6}\left(\dfrac{t_2^{\,3}}{3}t_3^3-\dfrac{t_2^{\,6}}{6}\right)\right\} \\ -\dfrac{a\theta}{2}\left\{\left(\dfrac{t_2^{\,4}}{4}t_3-\dfrac{t_2^{\,5}}{5}\right)+\dfrac{b}{2}\left(\dfrac{t_2^{\,4}}{4}t_3^2-\dfrac{t_2^{\,6}}{6}\right)+\dfrac{\theta}{6}\left(\dfrac{t_2^{\,4}}{4}t_3^3-\dfrac{t_2^{\,7}}{7}\right)\right\} \end{array}\right) \end{array}\right\}$$

…(21)

6. Shortage cost per cycle due to backlog is

$$SC = C_s \int_{t_3}^{T}\left[-I_N(t)\right]dt = \frac{aC_s}{\lambda}\left[\left(T-t_3\right)-\frac{1}{\lambda}\log\left(1+\lambda\left(T-t_3\right)\right)\right]$$

…(22)

7. Opportunity cost per cycle due to lost sales is

$$LS = C_l \int_{t_3}^{T} a\left[1-B\left(T-t\right)\right]dt$$

$$= aC_l\left[\left(T-t_3\right)-\frac{1}{\lambda}\log\left(1+\lambda\left(T-t_3\right)\right)\right] \qquad \text{….. (23)}$$

Therefore, the total relevant inventory cost per unit time is

$$TIC = \frac{1}{T}\left[OC + HC_{RW} + HC_{OW} + CD_{RW} + CD_{OW} + SC + LS\right]$$

….. (24)

The total relevant inventory cost (TIC) per unit time is minimum if

$$\frac{\partial TC}{\partial t_1}=0,\quad \frac{\partial TC}{\partial t_2}=0,\quad \frac{\partial TC}{\partial T}=0$$

$$\frac{\partial^2 TC}{\partial t_1^2},\ \begin{vmatrix} \dfrac{\partial^2 TC}{\partial t_1^2} & \dfrac{\partial^2 TC}{\partial t_1 t_2} \\ \dfrac{\partial^2 TC}{\partial t_1 t_2} & \dfrac{\partial^2 TC}{\partial t_2^2} \end{vmatrix},$$

and

$$\begin{vmatrix} \dfrac{\partial^2 TC}{\partial t_1^2} & \dfrac{\partial^2 TC}{\partial t_1 t_2} & \dfrac{\partial^2 TC}{\partial T \partial t_1} \\ \dfrac{\partial^2 TC}{\partial t_1 t_2} & \dfrac{\partial^2 TC}{\partial t_2^2} & \dfrac{\partial^2 TC}{\partial t_2 \partial T} \\ \dfrac{\partial^2 TC}{\partial T \partial t_1} & \dfrac{\partial^2 TC}{\partial t_2 \partial T} & \dfrac{\partial^2 TC}{\partial T^2} \end{vmatrix}$$

are all positive.

CONCLUSION

This paper proposed a two warehouses inventory model for non-instantaneous deteriorating items with partial backlogging. Shortages are allowed. Here we assume stock-dependent demand which is very realistic in nature. Furthermore, the proposed inventory model can be used in inventory control of certain non-instantaneous deteriorating items such as food items, electronic components such as computers, laptops, mobiles, machines, circuit, toys and fashionable commodities etc. Further incorporate the developed inventory model into more realistic assumptions, such as probabilistic demand quadratic, ramp type, production dependent and a finite rate of replenishment.

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