Analysis of Censored Sample Population with GA-SVM

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ABSTRACT: This paper is intended to propose a class of shrunken estimators for k^th power of scale parameter in censored samples from one-parameter exponential population when some apriori or guessed value of the parameter is available besides the sample information and analyses their properties. The proposed class of Shrunken estimator is compared with usual unbiased estimator and minimum mean square error (MMSE) estimator. Eventually, empirical study is carried out to exhibit the performance of some Shrunken estimators of the proposed class over the MSME estimator. It is found that certain of these estimators substantially improve the classical estimators even for the guessed values of the k^th power of scale parameter much away from the true value, specially for censored samples with small sizes.

Keywords*: SVM, GA-SVM, One-parameter exponential distribution, scale parameter, guessed value, Shrunken estimator, Bias, Mean squared error, Percent relative efficiency, MMSE estimator.*

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I. INTRODUCTION

The exponential distribution has considerable importance and widespread use in statistical procedures[1]. Currently among the most prominent applications are those in the field of life-testing. In life-testing research the simplest and the most widely exploited model is the one-parameter exponential distribution. Let $x_1<x_2<\ldots<x_m$ be the first m ordered observations of a random sample (x1, x2,..,xn) of size n drawn from the above said population, probability density function of which is given by:

$$f(x;\theta)=\frac{1}{\theta}\exp\left(\frac{-x}{\theta}\right);\quad x\geq 0,\ \theta>0 \qquad (1)$$

where θ is the mean life of the item

It also acts as scale parameter.

The word "moment" is used quite often in statistical context. The k^th raw moment of one parameter exponential distribution is

$$\theta^k \exp\left(\frac{-x}{\theta}\right) \qquad (2)$$

Therefore, our interest lies in estimating equation 2 so as to get the k^th moment which plays an important role in a variety of statistical problems.

An unbiased estimator of k^th power of the scale parameter is given by[2,3 and 4].

$$\theta^k=\frac{\sqrt{m}}{\sqrt{(m+k)}}S_m^{\ k} \qquad (3)$$

$$Variance\quad Var(\theta^k)=MSE(\theta^k)=\theta^{2k}\left[\frac{\sqrt{m}}{\sqrt{(m+k)}}-1\right] \qquad (4)$$

$$where\quad S_m=\left[\sum_{i=1}^{m}x_i+(n-m)x_m\right] \qquad (5)$$

Pandey and Singh(1978) showed in an application of their result that the minimum mean square error (MMSE) estimator given by

$$\theta^k{}_M=\frac{\sqrt{(m+k)}}{\sqrt{(m+2k)}}S_m^{\ k} \qquad (6)$$

Whose absolute relative bias (ARB)is given by

$$ARB(\theta^k{}_M)=\frac{\left|\sqrt{(m+k)}\right|}{\sqrt{m}} \qquad (7)$$

In many predicaments of practical pompposity, the guessed value of the parameter under study may be available either from past data or the experience generated in due course of time. Davis and Arnold (1970) have shown that, in terms of squared error risk, the usual unbiased estimator should not necessarily be considered. The have exhibited that one can improve upon the unique best mean squared error estimator. In this context, Thompson(1968) considered the problem of shrinking an unbiased estimator μ of the parameter € towards the natural origin μ_0 and suggested a srunken estimator

$$h\mu+(1-h)\mu_0 \qquad (8)$$

where 0<h<1 is a constant. The relevance of such type of shrunken estimators lies in the fact that , though perhaps they are biased, has smaller MSE than μ_0 of μ some interval around μ_0.

In the present investigation, it is desired to estimate equation 3 in the presence of a prior information μ_0 and the sample information contained in $S_m^{\ k}$. Consequently, this article is an endeavour in the direction of obtaining an efficient class of srunken estimators for k^th exponent of scale. The properties of the suggested class of estimators are further discussed theoretically and empirically[5,6 and 7].

II. SUGGESTED CLASS OF SHRUNKEN ESTIMATORS

Initially consider a class of estimators for equation (1) for MMSE, defined by

$$q^k_{(p,q)} = q^k\left[q + w\left(\frac{m^k}{S_m^{\ k}}\right)\right] \qquad (9)$$

where k is an integer, p and q are real numbers such that p and q are distinct from zero. W is constant to be chosen such that MSE is minimum [8,9 and 10].

By virtue of the result

$$E(S^{-b}) = \frac{m^b}{q^k\left[q + w\left(\frac{m^k}{S_m^{\ k}}\right)\right]} \qquad (10)$$

the MSE is derived as[11,12 and 13]

$$MSE\left[q^k\right] = q^{2k}\left[(q/\ -1)^2 + w^2 /^2 (p+1)\right] \qquad (11)$$

Now minimising (11) with respect to w and replacing with suitable parameter

$$w = \frac{\sqrt{(m-kp)}}{m^{kp}\sqrt{M-2kp}} \qquad (12)$$

The convex nature of the proposed statistic provides the criterion of selecting the scalar p. Therefore, the acceptable range of value of p. It seems from 12 that the MSE is minimum when q=π. Thus to obtain significant gain in efficiencyfor fixed π, one should choose q in the vicnity of q . It is interesting to note that if one should select smaller values of q then higher values of π will lead to a large gain in efficiency and vice-versa. This implies that for smaller values of q, the proposed class of estimators allow to choose guessed value much away from the true value, i.e., even if the experimenter is less experienced the risk of estimation using the proposed class of estimators is not higher. This is legitimate for all values of p and k[11,12 and 13].

III. EMPIRICAL STUDY

James and Stein (1961) promulgated that minimum MSE is a highly enviable characteristic and it is therefore used as a criterion to compare different estimators with each other[14,15 and 16]. The conditions under which the proposed class of estimators is more efficient than the unbiased estimator and the MMSEestimator are given below:

$$MSE = \frac{1-\sqrt{D-w(k,p)}}{q(1-w(k,p))} \qquad (13)$$

$$D = \frac{\sqrt{m}}{\sqrt{(m+k)}} - 1 \qquad (14)$$

Besides minimum MSE criterion[17,18 and 19], minimum bias should also be considered as ARB

$$ARB = \frac{1-\sqrt{D-w(k,p)}}{q(1-w(k,p))} + \frac{\sqrt{m}}{\sqrt{(m+k)}} - 1 \qquad (15)$$

It is interesting to note that for p=-1, both the interval of π in equation 11 and 12 exactly coincide with each other[20,21 and 22]. To elucidate the performance of the proposed of the proposed class of estimators against the MMSE estimator which is theoritically better than the unbiased estimator, percent relative efficiencies(PRE).

$$PRE = \frac{\left[1 - \frac{\sqrt{(m+k)}}{\sqrt{m}\sqrt{(m+2k)}}\right]}{q^{2k}\left[(q/\ -1)^2 + w^2 /^2 (p+1)\right]} \qquad (16)$$

The effective interval around the natural origin, i.e., the range of dominance of π in which the suggested class of estimators is preferable over MMSE estimator in terms of efficiency can be had by equation 16 and in terms of bias can be had by equation 11. The absolute relative bias can be had by equation 10 and that of by 10.

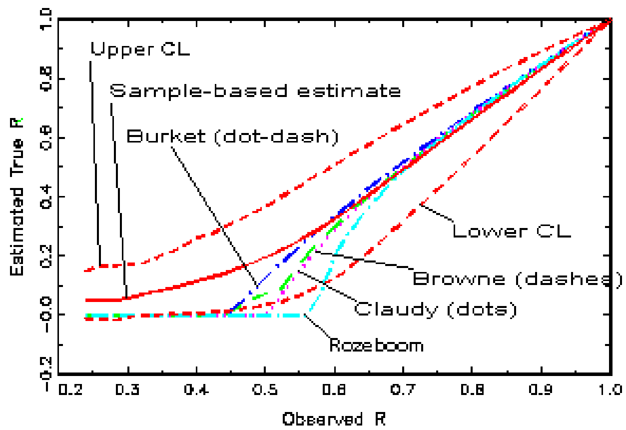

Fig1. Results of ARB Estimate

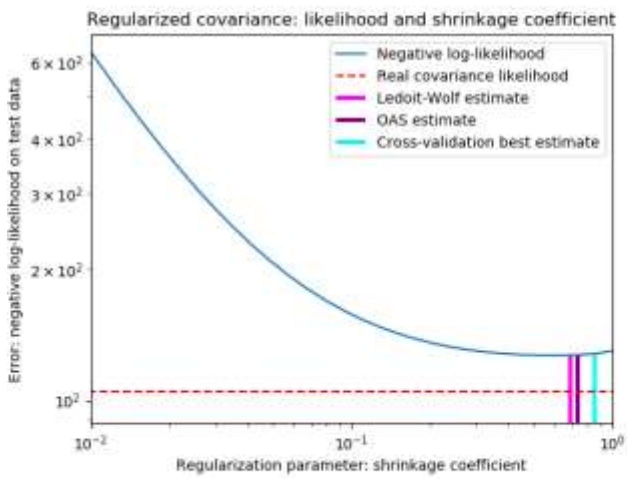

Fig2. Relative MSE Estimator

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300 – 303

The effect of change in size of censored sample m is also a matter of great interest. For fixed k,p,q and π the gain in relative efficiency and ARBs both decreases with increment in m, i.e., the proposed class of srunken estimators is beneficial especially for small censored smaples where the MSE and ARB both are small. Besides, it appears that to get better estimators in the calss, the value of w should be as small as possible in the interval (0,1). That is to say, a value of w near 0 implies strong belief in parameter p where as near 1 implies strong belief in S^k. Thus, to choose p one should not consider the smaller values of w in isolation, but also wider length of effective interval of π.

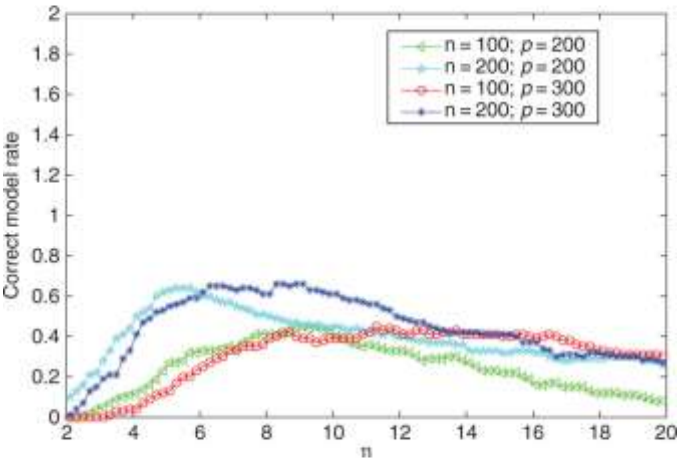

Fig.3 Results of PRE Estimator.

Consider an example from Jani, when 10 items were put on a life-test; the test is terminated when the sixth item failed. The life-times (in hours) were recorded as 273, 307 , 344, 376, 415 and 455. Assuming that the samples has come from an exponential population defined by 10. Our interest is to estimate the mean life. A prior point estimates of the mean life.

	Unbiased	MMSE	PROPOSED
Estimate	670	580	690
ARB	0	.1432	0.017
Relative MSE	0.166	.1465	0.127
PRE	140	117	100

Table1. Estimates, ARBs and PREs of Competitive

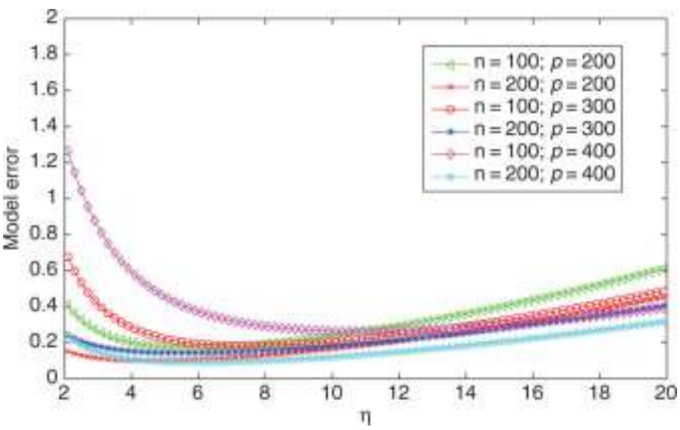

Fig. 4 Results of ARB with GA-SVM

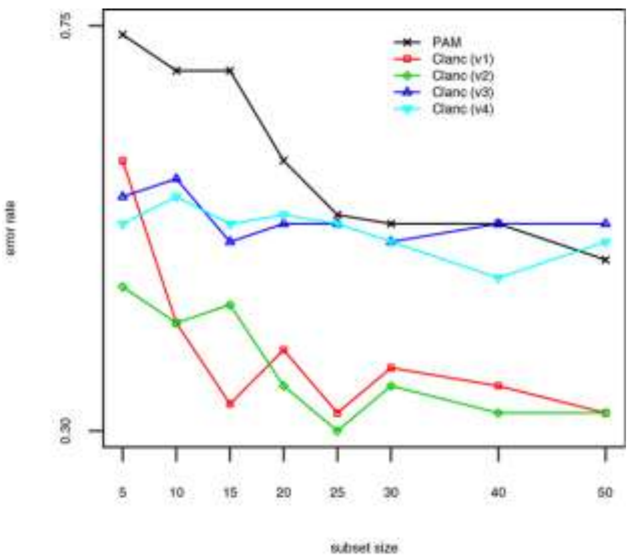

Fig. 5 Results of PRE with GA-SVM

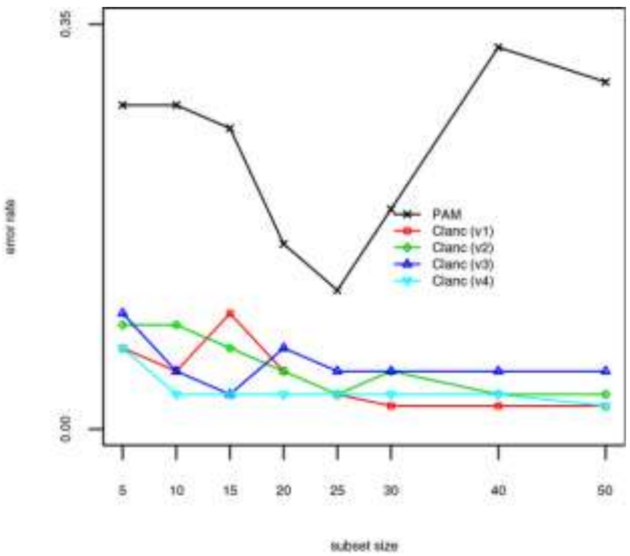

Fig 6. Results of MSE Estimator with GA-SVM

From Table1 it implies that there is substantial gain in efficiency by using the proposed estimator over unbiased estimator as well as MMSE estimator. It is also interesting to note that the proposed is less biased than the MMSE estimator. This escorts us to quote that the proposed estimator performs tremendously better than the conventional estimators.

IV. CONCLUSION AND RECOMMENDATIONS

It has been seen that the suggested class of shrunken estimators has considerable gain in efficiency and are less biased than the MMSEestimator for a number of choices of scalars comprehend in it, particularly for heavily sensored samples, i.e, for small m. Even for ;large censored sample sizes, so far as the proper selection of scalars is concerned, a number of estimators from the suggested class of shrunken estimators are not only more efficient but also less biased than the MMSE estimator. Accordingly, even if the experimentor has less confidence in the guessed value of estimated parameter, the efficiency of the suggested class of shrunken estimators can be increased considrerably by choosing the scalars p and q appropriately.

While dealing with the suggested class of srunken estimators, it is recommended that one should not consider the substantial gain in efficiency in isolation, but also the wider range of dominance of π will lead to increase the possibility of getting better estimators from the proposed class. The suggested class of shrunken estimators are therefore recommended for its use in practice.

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