A Study of W_7 – Curvature Tensor in LP-Sasakian Manifold

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Abstract: In this paper the geometric properties of W_7- curvature tensor are studied in LP-Sasakian Manifold.
Keywords: *W_7- curvature tensor, LP-Sasakian Manifold, semi- symmetric , symmetric and W_7-flat.*

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I. INTRODUCTION

In this paper, we shall study the W_1-Curvature tensor in LP-sasakian manifold. An n-dimensional real differentiable manifold M_n is said to be Lorentzian para (LP)-sasakian manifold if it admits a (1,1) tensor field F, a C^∞ vector field T, a C^∞ 1-form A and a Lorentzian metric g which satisfy [Mishra (1)]:

(1.1) $A(T) = -1$

(1.2) $\bar{\bar{X}} = X + A(X)T$

(1.3) $g(\bar{X}, \bar{Y}) = g(X, Y) + A(X)A(Y)$,

(1.4) $g(X, Y) = A(X)$, $\quad D_X T = \bar{X}, A(Y)$,

(1.5) $(D_X F)(Y) = \{g(X, Y) + A(X)A(Y)\}T + \{X + A(X)A(Y)\}$,

Where $\bar{X} = F(X)$.

In an LP-Sasakian manifold M_n with structure (F, T, A, g), it can be seen that (Pokhariyal [2])

(1.6) $\bar{T} = 0$, $A(\bar{X}) = 0$,

(1.7) $rank\ (7) = n - 1$

If we put

(1.8) $F'(X, Y) = g(\bar{X}, Y)$,

then the tensor $F'(X, Y)$ is symmetric in X and Y.

In an n-dimensional LP-sasakian manifold with the structure (F, T, A, g), we have

(1.9) $R'(X, Y, Z, U) = g(X, U)g(Y, Z) - g(Y, U)g(X, Z)$,

Where $g(X, Z)$ is the metric tensor representing potential and

(1.10) $Ric(X, Y) = g(QX, Y) = (n - 1)g(X, Y)$, is the Ricci tensor representing the matter tensor .

(1.11) $S(X, Y) = Ric(X, Y)$, $\quad S(T, T) = R(T, T) = -(n - 1)$

Where R is the Riemannian (0,4) curvature tensor, $S = Ric(.,.)$ is the Ricci tensor.

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II. W_7 −CURVATURE TENSOR IN LP-SASAKIAN MANIFOLD

Mishra and Pokhariya [3] gave the definition of W_7-Curvature tensor as

(2.1) $W_7(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[g(Y,Z)QX - Ric(Y,Z)X]$.

or $W_7{}'(X,Y,Z,U) = R'(X,Y,Z,U) +$

$\frac{1}{n-1}[g(Y,Z)Ric(X,U) - Ric(Y,Z)g(X,U)]$ were Q is the linear endomorphism of a tangent space at each of its points to the Ricci tensor.

Definition 1: A LP-Sasakian manifold M_n is said to be flat if the Riemannian curvature tensor vanishes identically i.e $R(X,Y)Z$=0.

Definition 2: A LP-Sasakian manifold M_n is said to be W_7- flat if W_7-curvature tensor vanishes identically i.e. $W_7(X,Y)Z$=0.

Theorem 1. A W_7-flat LP-Sasakian manifold is a flat manifold.

Proof:

If LP-space is W_7-flat then $W_7 = 0$ in

$W_7{}'(X,Y,Z,U) = R'(X,Y,Z,U) + \frac{1}{n-1}[g(Y,Z)QX - Ric(Y,Z)X]$

or $W_7{}'(X,Y,Z,U) = R'(X,Y,Z,U) + \frac{1}{n-1}[g(Y,Z)Ric(X,U) - Ric(Y,Z)g(X,U)]$

if LP-space is W_7-flat then we have,

$0 = R'(X,Y,Z,U) + \frac{1}{n-1}[g(Y,Z)Ric(X,U) - Ric(Y,Z)g(X,U)]$

Where $Ric(X,Y) = g(QX,Y) = (n-1)g(X,Y,)$

we have

$R'(X,Y,Z,U) = \frac{1}{n-1}[Ric(Y,Z)g(X,U) - g(Y,Z)Ric(X,U)]$

$= \frac{1}{(n-1)}\{(n-1)g(Y,Z)g(X,U) - (n-1)g(Z,Y)g(X,U)\}$

i.e. $R'(X,Y,Z,U) = g(Y,Z)g(X,U) - g(Z,Y)g(X,U)$

but in LP-Sasakian manifold we have

$R'(X,Y,Z,U) = g(Y,Z)g(X,U) - g(X,Z)g(Y,U)$

$\Rightarrow R'(X,Y,Z,U) = 0$ or $Ric(X,Y) = 0$

Hence the theorem.

Corollary 1: A W_7-flat LP-Sasakian manifold is neither Einstein or n-Einstein manifold.

III. A W_7-SEMISYMMETRIC LP-SASAKIAN MANIFOLD

U.C. De and N. Guha [4] gave the definition of semisymmetric as $R(X,Y)R(Z,U)V = 0$

Definition 3. A LP-Sasakian manifold is said to be W_7 –semisymmetric if $R(X,Y)W_7(Z,U)V = 0$

Theorem 2: *A* W_7-semisymmetric LP-Sasakian manifold is said to be W_7-flat manifold.

Proof:

If LP-space is a W_7-semisymmetric then $R(X,Y)W_7(Z,U)V = 0$

$$\Rightarrow g(R(X,Y)W_7\,(\mathrm{Z},\mathrm{U})\mathrm{V},\mathrm{T}) = R'(X,Y,W_7,(\mathrm{Z},\mathrm{U})\mathrm{V},\mathrm{T})$$

$$= g(X,T)g(W_7(\mathrm{Z},\mathrm{U})\mathrm{V},\mathrm{Y}) - g(Y,T)g(W_7(\mathrm{Z},\mathrm{U})\mathrm{V},\mathrm{X})$$

$= A(X){W_7}'(Y,Z,U)V - A(Y){W_7}'(X,Z,U)V = 0$ but since $A(X)$ and $A(Y)$ are non-zero $\Rightarrow {W_7}'(Y,Z,U)V = 0$ and ${W_7}'(X,Y,U)V = 0$ from $R(X,Y)W_7(Z,U)V = 0$

Hence the theorem.

Corollary 2: A W_7-semisymmetric LP-Sasakian manifold is neither a Einstein or η-Einstein manifold.

IV. A W_7-SYMMETRIC LP-SASAKIAN MANIFOLD

A LP-Sasakian manifold is said to W_8-symmetric if

$$\nabla_U W_7(X,Y)Z = {W_7}'(U,X,Y)Z = 0$$

Theorem 4: A W_7-symmetric and W_7-flat LP-Sasakian manifold is a flat manifold.

Proof:

From the previous theorem, we found out that a W_7-semisymmetric is a W_7-flat manifold and if LP-space is a W_7-symmetric this implies $R\big(X,Y,W_7(Z,U,V)\big) - W_7(R(X,Y,Z),U,V) - W_7(Z,R(X,Y,U),V) - W_7\big(Z,U,R(X,Y,V)\big) = 0$ which on expanding the expressions we have

(4.4.1)

$$R'(X,Y,W_7(Z,U,V),T) = g(X,T)g\big(Y,W_7(Z,U,V)\big) - g(Y,T)g\big(X,W_7(Z,U,V)\big)$$
$$= A(X){W_7}'(Y,Z,U,V) - A(Y){W_7}'(X,Z,U,V)$$

(4.4.2)

${W_7}'(R(X,Y,Z),U,V,T) = R'(R(X,Y,Z),U,V,T) + \frac{1}{n-1}[Ric(R(X,Y,Z),T)g(U,V) - Ric(U,V)g(R(X,Y,Z),T)]$ then using

$Ric(X,Y) = S(X,Y) = (n-1)g(X,Y)$, we get

${W_7}'(R(X,Y,Z),U,V,T) = R'(R(X,Y,Z),U,V,T) + \frac{1}{n-1}[(n-1)R'(X,Y,Z,T)g(U,V) - (n-1)g(U,V)R'(X,Y,Z,T)] = R'(R(X,Y,Z),U,V,T) + g(U,V)R'(X,Y,Z,T) = R'(R(X,Y,Z),U,V,T) = g(R(X,Y,Z),T)g(U,V) - g(U,T)R'(X,Y,Z,V)$

(4.4.3)

${W_7}'(Z,R(X,Y,U),V,T) = R'(Z,R(X,Y,U),V,T) + \frac{1}{\eta-1}[g\big(V,R(X,Y,U)\big)Ric(Z,T) - Ric(R(X,Y,U),V)g(Z,T)]$ then using $g(X,T) = (n-1)g(X,Y)$ and $g\big(Z,R(X,Y,U)\big) = R'(X,Y,U,Z)$ we have $R'(Z,R(X,Y,U),V,T) + \frac{1}{n-1}[(n-1)R'(X,Y,U,V)A(Z) - (n-1)R'(X,Y,U,V)A(Z)] =$

$A(Z)R'(X,Y,U,V) - R'(X,Y,U,T)g(Z,V) + A(Z)R'(X,Y,U,Z) - A(Z)R'(X,Y,U,V) = A(Z)R'(X,Y,U,Z) - g(Z,V)R'(X,Y,U,T)$

(4.4.4)

$W_7{}'(Z,U,R(X,Y,V),T) = R'(Z,U,R(X,Y,V),T) + \frac{1}{n-1}\Big[g\big(U,R(X,Y,V)\big)Ric(Z,T) - A(Z)\Big(R'(X,Y,V,U)\Big)\Big]$, then using $S(X,Y) = (n-1)g(X,Y)$, then we have $W_7{}'(Z,U,R(X,Y,V),T) = R'(Z,U,R(X,Y,V),T) + \frac{1}{n-1}[(n-1)A(Z)R'(X,Y,V,U) - (n-1)R'(X,Y,V,U)A(Z)] = A(Z)R'(X,Y,V,U) - A(U)R'(X,Y,V,Z) + g(Z,U)R'(X,Y,V,T) - A(Z)R'(X,Y,V,U) = g(Z,U)R'(X,Y,V,T) - A(U)R'(X,Y,V,Z)$

Next we put together 4.4.1, 4.4.2, 4.4.3, and 4.4.3 we have $A(X)W_7{}'(Y,Z,U,V) - A(Y)W_7{}'(X,Z,U,V) + A(Z)R'(X,Y,V,U) - A(U)R'(X,Y,Z,V) + A(Z)R'(X,Y,U,V) - g(U,V)R'(X,Y,Z,T) - g(Z,V)R'(X,Y,U,T) - A(U)R'(X,Y,V,Z) = 0$

Terms which are coefficients of $A(Z)$ cancelled's out since they are skew-symmetric with respect to the last variables, same applies to those of $A(U)$, $W_7{}' = 0$ because of symmetric property. Thus we remain with

$\Rightarrow g(U,V)R'(X,Y,Z,T) - g(Z,V)R'(X,Y,U,T) = 0$

Since $\nabla_V W_7(Y,Z,U) = W_7{}'(Y,Z,U,V) = 0 \Rightarrow g(Z,U)R'(X,Y,V,T) = 0$ since $g(Z,U) and\ g(Z,V) \neq 0 \Rightarrow R'(X,Y,V,T) = 0$ thus follows the theorem.

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