# Perturbation Solution for Thermal Diffusion and Chemical reaction Effects on MHD Flow in Vertical Surface with Heat Generation

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**Abstract** - A study two-dimensional laminar incompressible flow continuously moving vertical surface porous plate has been analyzed to show the effect of an additional cross transport phenomenon, i.e. heat flux caused by concentration gradient in addition to the heat flux caused by temperature gradient. Further thermal Diffusion and chemical reaction effects on MHD flow in vertical surface with heat generation has been taken into consideration in the present study. Moreover, the Dufour effect has been considered in energy equation leaving the equation of thermal diffusion and mass diffusion coupled. The coupled non-linear equations are solved by applying perturbation technique. The effect of flow parameters are shown with the help of graphs.

Keywords: Dufour parameter, MHD, Heat generation

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## I. INTRODUCTION

Thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen and helium) and medium molecular weight. Combined hat and mass transfer problem with chemical reaction are of importance in many processes and have, therefore, received a considerable amount attention in recent years. In processes such as drying evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. On view of the above some of the authors studies Sparrow et. al [1] Transpiration induced buoyancy and thermal diffusion - diffusion thermo in a Helium - air free convection boundary layer, Kafoussias and Williams [2] Thermal diffusion and diffusion-thermo effects on mixed free forced convective and mass transfer boundary layer flow with temperature dependent viscosity, Anghel et.al [3] Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in porous medium, Ahmed [4] MHD convection with Soret and Dufour effect in a three dimensional flow past an infinite vertical plate, Srinivasachary et.al [5] Soret and Dufour effects on mixed convection along a vertical wavy surface in a porous medium with variable properties.

Many practical diffusive operations, the molecular diffusion of a species involved in the presence of chemical reaction within or at the boundary. Three are two types of chemical reactions, i.e. homogeneous and heterogeneous reactions. A homogeneous reaction occurs uniformly throughout a given phase. In such type of reaction the species generation is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary or a phase. It is so treated as a boundary condition similar to the constant heat flux condition in heat transfer. All industrial chemical processes are so designed that the cheaper raw material can be transformed to high value products by chemical reaction. For a specific chemistry, the reactor performance is a complex function of the underlying transport process. An analysis of the transport process and their interaction with chemical reactions are quite difficult and is directly connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes for core of chemical reaction engineering. Das et.al [6] illustrated the impact of chemical reaction on unsteady flow along a vertical plate by employing Laplace transformation technique. Bhattacharyya and Layek [7] scrutinized the impact of chemical reaction and transpiration on the flow over a flat plate. Later on, copious investigator [8] studied the chemical reaction effects on dissimilar flow geometries, Hussain et.al [9] discussed the chemical reaction effects on flow past an accelerated moving plate in a rotating system, Ch Kesavaiah et.al [10] Effects of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Bhavana et.al [11] has the Soret effect on free convective unsteady MHD flow over a vertical plate with heat source.

The aim of the present investigation is a study twodimensional laminar incompressible flow continuously moving vertical surface porous plate has been analyzed to show the effect of an additional cross transport phenomenon, i.e. heat flux caused by concentration gradient in addition to the heat flux caused by temperature gradient. Further thermal Diffusion and chemical reaction effects on MHD flow in vertical surface with heat generation has been taken into consideration in the present study.

### II. FORMULATION OF THE PROBLEM

Consider the steady, two - dimensional laminar, incompressible flow of a chemically reacting, viscous fluid on a continuously moving vertical surface in the presence of a uniform magnetic field and Dufour effect with heat generation, uniform heat and mass flux effects issuing a slot and moving with uniform velocity in a fluid at rest. Let the x- axis be taken along the direction of motion of the surface in the upward direction and y- axis is normal to the surface. The temperature and concentration levels near the surface are raised uniformly. The induced magnetic field, viscous dissipation is assumed to be neglected.



**Figure:** Flow configuration and coordinate system Now, under the usual Boussinesq's approximation, the flow field is governed by the following equations:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T' - T'_{\infty}) + g\beta^{*}(C' - C'_{\infty}) + v\frac{\partial^{2}u}{\partial y^{2}} - \frac{\sigma B_{0}^{2}}{\rho} - \frac{v}{K_{p}}u$$
<sup>(2)</sup>

Energy equation

$$\rho C_{p} \left( u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} \right) = k \frac{\partial^{2} T'}{\partial y^{2}} - Q_{0} \left( T' - T_{\infty}' \right) + \frac{D_{M} K_{T}}{C_{s} C_{p}} \frac{\partial^{2} C'}{\partial y^{2}}$$
<sup>(3)</sup>

Diffusion equation

$$u\frac{\partial C'}{\partial x} + v\frac{\partial C'}{\partial y} = D\frac{\partial^2 C'}{\partial y^2} - Kr'(C' - C'_{\infty})$$
(4)

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The initial and boundary conditions

$$u = u_{w}, v = -v_{0} \text{ const},$$
  

$$\frac{\partial T}{\partial y} = -\frac{q}{k}, \quad \frac{\partial C}{\partial y} = -\frac{j''}{k} \quad at \quad y = 0 \quad (5)$$
  

$$u \to 0, T \to T'_{\infty}, C \to C'_{\infty} \text{ as } y \to \infty$$

where u, are velocity components in x and y directions respectively. g is the acceleration due to gravity,  $\beta$  is volumetric coefficient of thermal expansion,  $\beta^*$  is the volumetric coefficient of expansion with concentration, is the temperature of the fluid, C' is the species concentration,  $T'_{w}$  is the wall temperature,  $C'_{w}$  is the concentration at the plate,  $T'_{\infty}$  is the free steam temperature far away from the plate,  $C'_{\infty}$  is the free steam concentration in fluid far away from the plate, v is the kinematic viscosity, D is the species diffusion coefficient, Kr is the chemical reaction parameter. The term is assumed to be the amount of heat generated or absorbed per unit volume.  $Q_0$  is a constant, which may take on either positive or negative values. When the wall temperature  $T'_{w}$  exceeds the free steam temperature  $T'_{\infty}$ , the source term represents the heat source  $Q_0 > 0$  when and heat sink when  $Q_0 < 0$ . The first term and second term on the right hand side of the momentum equation (2) denote the thermal and concentration buoyancy effects respectively.

In order to write the governing equations and the boundary conditions the following non-dimensional quantities are introduced.

$$Y = \frac{yv_{o}}{v}, \ U = \frac{u}{u_{w}}, \ \Pr = \frac{\mu C_{p}}{k}, \ Q = \frac{Q_{0}v}{\rho C_{p}v_{0}^{2}}$$

$$k = \frac{K_{p}v_{0}^{2}}{v^{2}}, \ M = \frac{\sigma B_{0}^{2}v}{\rho}, \ Gr = \frac{vg\beta\left(\frac{qv}{kv_{0}}\right)}{u_{w}v_{0}^{2}}$$

$$Gc = \frac{vg\beta^{*}\left(\frac{j''v}{kv_{0}}\right)}{u_{w}v_{0}^{2}}, \ T = \frac{T'-T_{\infty}'}{\left(\frac{qv}{kv_{0}}\right)}, \ C = \frac{C'-C_{\infty}'}{\left(\frac{j''v}{kv_{0}}\right)}$$

$$Ql = \frac{Ql'j''v}{qv_{0}^{2}\rho C_{p}}, \ R = \frac{4qvI}{kv_{0}}, \ Kr = \frac{Kr'v}{v_{0}^{2}}Sc = \frac{v}{D}$$

In view of (6) the equations (2) - (4) are reduced to the following non-dimensional form

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} - \left(M + \frac{1}{k}\right)U = -GrT - GrC \quad (7)$$

$$\frac{d^2T}{dY^2} + \Pr\frac{dT}{dY} - Q\Pr T = -Du\Pr\frac{d^2C}{dY^2}$$
(8)

$$\frac{d^2C}{dY^2} + Sc\frac{dC}{dY} - KrScC = 0$$
(9)

The corresponding initial and boundary conditions in nondimensional form are

$$U = 1, \frac{\partial T}{\partial Y} = -1, \frac{\partial C}{\partial Y} = -1 \quad at \quad Y = 0$$
  
$$U \to 0, T \to 0, C \to 0 \quad as \quad Y \to \infty$$
 (10)

The radiative heat flux  $q_r$  is given by equation (5) in the spirit of Cogly et.al [6]

$$\frac{\partial q_r}{\partial y} = 4 \left( T - T_{\infty} \right) I \tag{11}$$

where 
$$I = \int_{0}^{\infty} K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$$
,  $K_{\lambda w}$  – is the absorption

coefficient at the wall and  $e_{b\lambda}$  – is Planck's function, I is absorption coefficient

Where Gr is the thermal Grashof number, Gc is the solutal Grashof number, Pr is the fluid Prandtl number, Sc is the Schmidt number and Kr is the chemical reaction parameter, Q is the heat generation/absorption parameter and  $Q_l$  is the radiation absorption parameter.

#### III. METHOD OF SOLUTION

The study of ordinary differential equations (7), (8) and (9) along with their initial and boundary conditions (10) have been solved by using the method of ordinary linear differential equations with constant coefficients. We get the following analytical solutions for the velocity, temperature and concentration

$$U = (F_1 + F_3)e^{m_2 y} + F_2 e^{m_4 y} + F_4 e^{m_6 y}$$
$$T = E_1 e^{m_2 y} + E_2 e^{m_4 y}$$
$$C = -\frac{1}{m_2} e^{m_2 y}$$

Skin friction

$$\tau = \left(\frac{\partial U}{\partial y}\right)_{y=0} = m_2 \left(F_1 + F_3\right) + F_2 m_4 + m_6 F_4$$

Nusselt number

$$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0} = m_2 E_1 + m_4 E_2$$

Sherwood number

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0} = -1$$

Appendix

$$\begin{split} \beta &= \left(M + \frac{1}{K}\right), m_2 = -\left(\frac{Sc + \sqrt{Sc^2 + 4KrSc}}{2}\right) \\ m_4 &= -\left(\frac{Sc + \sqrt{Sc^2 + 4KrSc}}{2}\right), m_6 = -\left(\frac{1 + \sqrt{1 + 4\beta}}{2}\right) \\ E_1 &= \left(\frac{Du \operatorname{Pr} m_2}{m_2^2 + \operatorname{Pr} m_2 - Q \operatorname{Pr}}\right), E_2 = -\left(\frac{1 + E_l m_2}{m_4}\right) \\ F_1 &= -\left(\frac{GrE_1}{m_2^2 + m_2 - \beta}\right), F_2 = -\left(\frac{GrE_2}{m_4^2 + m_4 - \beta}\right) \\ F_3 &= \frac{1}{m_2}\left(\frac{Gr}{m_2^2 + m_2 - \beta}\right), F_4 = \left(1 - F_1 - F_2 - F_3\right) \end{split}$$

#### IV. RESULTS AND DISCUSSION

Figure (1) reveals that the velocity variation with parameters Grashof number (Gr) from this figures it is found that the fluid velocity increases with increases in Gr. It is because that increase in the values Gr has the tendency to increase the thermal buoyancy effect. This gives rise to an increase in the induced flow transport. Figure (2) depicted that the velocity profiles for different values of heat source parameter (Q), it is clear that an increases in heat source parameter the results are decreases. The velocity variation for different values of permeability of the porous medium (K) observed in figure (3), it is making known the velocity increases with increasing values of permeability of the porous medium. Figure (4) displays the velocity profiles for different values of Dufour effects (Du), it was found that the velocity increases with increasing values of Dufour parameter. From figure (5) observed the velocity of the fluid decreases with the increase of the magnetic parameter values. It is because the application of the transverse magnetic field will result in a resistive type (Lorentz similar to the drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decrease with an increase in the magnetic parameter. We also had seen that velocity profiles decrease with the increase of magnetic effect indicating that the magnetic field trends to retard the motion of the fluid. The magnetic field may control the flow characteristics. The effect of the chemical reaction parameter (Kr) has shown figure (6). It should be mentioned that the case studied relates to a destructive chemical reaction. In fact, as the chemical reaction parameter increases, a considerable reduction in the velocity occurs, and the presence of the peak indicates that the maximum velocity takes place in the fluid body close to the surface, but not at the surface itself. It is evident that an increase in this parameter significantly alters the concentration boundary layer thickness but does not change the momentum one. Figure (7) shows that velocity profiles for different values of Prandtl number. It is observed that increase in the value of Prandtl number results in decrease in the velocity profile. The velocity profiles observed in figure (8) for various values of Schmidt number (Sc), it is clear that increases in Schmidt number the velocity decreases. From figures (9) and (10) it is observed that Dufour parameter (Du) and heat source parameter (Q) as increases, the temperature of the flow field increases at the all points in flow region, but the reverse effect observed in heat source parameter. From figure (11), it is observed that the temperature for conducting air (Pr = 0.71) is higher than that of water (Pr = 7.0) it is because of the fact that thermal conductivity of the fluid decreases with increasing values of Pr resulting decreases in thermal boundary layer thickness. The temperature of the flow field is mainly affected by the flow parameter, namely Prandtl number. The effect of concentration profiles for different values of chemical reaction parameter (Kr), Schmidt number (Sc)illustrated in figure (12) and (13), it is found that the concentration decreases as chemical reaction parameter or Schmidt number. Skin friction for different values of heat source parameter (Q) versus thermal Grashof number (Gr) observed in figure (14), it is accurately says that an increasing values of heat source parameter the results were decreases.

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y Figure (8): Velocity Profiles for different Values of Sc

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