On Unified Fractional Integral Operators Lommel-Wright Function and Generalized Astrophysical Thermonuclear Function

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Abstract - In this paper we introduce a unified class of fractional integral operators whose kernel involving generalized Lommel-Wright function [7] and generalized of astrophysical thermonuclear function [8]. First, we define the operator of our study and give the conditions of existence of these operators. Further, we get the images of multivariable H-function and $\overline{H}$ -function under them. Then, we obtain the three new integrals involving - generalized Lommel-Wright function, generalized form of astrophysical thermonuclear function, reduced Green function, generalized Mittag-Leffler function, Gaussian Model free energy.

Keywords: *Generalized Lommel- Wright function , generalized astrophysical thermonuclear function and Multivariable H-function,* $\overline{H}$ *-function.*

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Introduction

$J_{\eta,\delta}^{\omega,\kappa}(z)$ function occurring in this paper is generalized Lommel-Wright function introduced by [7]& [10]defined as follows

$$J_{\eta,\delta}^{\omega,m}(z)=\sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{z}{2}\right)^{2n+\eta+2\delta}}{\Gamma(\delta+n+1)^m\,\Gamma(\eta+n\omega+\delta+1)n!} \quad (1)$$

$$\left(z\in N/(-\infty,0),m\in N,\eta,\delta\in C,\mu\in R^+\right)$$

Also, we have the following relations of generalized Lommel -Wright function with trigonometric functions and the generalized Bessel function and Struve function:

$$J_{1/2,0}^{1,1}(z)=\sqrt{\frac{2}{\pi z}}\sin z \quad (2)$$

$$J_{-1/2,0}^{1,1}(z)=\sqrt{\frac{2}{\pi z}}\sin z \quad (3)$$

$$J_{\eta,\delta}^{\omega,m}(z)=J_{\eta,\delta}^{\omega,1}(z) \quad (4)$$

$$J_{\eta,\delta}^{\omega,m}(z)=J_{\eta,1/2}^{1,1}(z)=H_{\eta}(z) \quad (5)$$

$I_3(z,g,\nu,\rho,\mu,a,\alpha)$ function occurring in this paper is generalized form of astrophysical thermonuclear function introduced by Saxena [8,p.35,Eq.(4.1)] defined as

$$I_3(z,g,\nu,\rho,\mu,a,\alpha)=\int_0^{\infty} y^{\nu-1}\left[1+a(\alpha-1)y^{\rho}\right]^{-1/(\alpha-1)} e^{-z(y+g)^{-\mu}}dy \qquad (6)$$

The contour and H-function representation of generalized astrophysical thermonuclear function which is used in this paper is in the following form

$$I_3(z,g,\nu,\rho,\mu,a,\alpha)=\frac{\left[a(\alpha-1)\right]^{-\nu/\rho}}{\rho\Gamma\left(\frac{1}{\alpha-1}\right)}\sum_{i=0}^{\infty}\frac{\left[-g\left\{a(\alpha-1)\right\}^{1/\rho}\right]^{i}}{i!}\times$$
$$H_{2,3}^{2,2}\left[z\left[a(\alpha-1)\right]^{\mu/\rho}\left|\begin{matrix}\left(1-\frac{1}{(\alpha-1)}+\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right),(1-i,\mu)\\(0,1),\left(\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right),(1,\mu)\end{matrix}\right.\right]f(z)dz \qquad (7)$$

where $\rho\neq 0,\nu\in C,\alpha>1,\min\{z,\rho,\mu,a\}\geq 0$ (not all simultaneously zero)

FRACTIONAL INTEGRAL OPERATORS

In this paper we study the following two unified fractional integral operators involving the product of generalized Lommel-Wright function and generalized astrophysical thermonuclear function having general arguments.

$$I_x^{\upsilon,\lambda}\left[f(t)\right]=$$
$$\frac{x^{-\upsilon-\lambda-1}}{\Gamma(\lambda)}\int_0^x t^{\upsilon}(x-t)^{\lambda}J_{\eta,\delta}^{\omega,\kappa}\left(z_0\left(\frac{t}{x}\right)^{\upsilon_0}\left(1-\frac{t}{x}\right)^{\lambda_0}\right)I_3\left(z^1\left(\frac{t}{x}\right)^{\upsilon^1}\left(1-\frac{t}{x}\right)^{\lambda^1},\mu,\nu,\rho,\gamma,a,\alpha\right)f(t)dt \qquad (8)$$

where $f(t)\in A$

Provided that

$$\left.\begin{matrix}\alpha>1;\eta,\delta,\mu,\nu,s\in C,\rho\neq 0;\mathrm{Re}(\upsilon+\varsigma+1)>0;\mathrm{Re}(\lambda+1)>0;\\ \min\left[(\omega,\kappa,\gamma,a,\rho,\mu,\upsilon_1,\lambda_1)\right]\geq 0,(\text{not all simultaneously zero})\end{matrix}\right\} \qquad (9)$$

ISSN: 2454-4248
78 – 86

$$J_x^{\upsilon,\lambda}\left[f(t)\right]=$$

$$\frac{x^{\upsilon}}{\Gamma(\lambda)}\int_x^{\infty} t^{-\lambda-\upsilon-1}(t-x)^{\lambda} J_{\eta,\delta}^{\omega,\kappa}\left(z_0\left(\frac{t}{x}\right)^{\upsilon_0}\left(1-\frac{t}{x}\right)^{\lambda_0}\right) I_3\left(z^1\left(\frac{t}{x}\right)^{\upsilon^1}\left(1-\frac{t}{x}\right)^{\lambda^1},\mu,\nu,\rho,\gamma,a,\alpha\right) f(t)\,dt$$

(10)

Provided that

$$\left.\begin{array}{c}\nu,s,\eta,\delta,\upsilon\in C;\alpha>1;\mathrm{Re}(w_2)>0 \text{ or } \mathrm{Re}(w_2)=0\,;\,\rho\neq 0;\mathrm{Re}(\nu-w_1)>0;\\ \mathrm{Re}(\lambda+1)>0,\min\left[(\omega,\kappa,\gamma,a,\rho,\mu,\nu_1,\lambda_1)\right]\geq 0,(\text{not all simultaneously zero})\end{array}\right\}$$

(11)

where $J_{\eta,\delta}^{\omega,\kappa}$ stands for generalized Lommel-Wright function defined by (1) and $I_3(z,g,\nu,\rho,\mu,a,\alpha)$ stands for generalization of astrophysical thermonuclear function defined by (7) and $f(t)\in A,$ where A stands for class of functions for which

$$f(t)=\begin{cases} O\left\{|t|^{\varsigma}\right\} & ,\max\left\{|t|\right\}\to 0\\ O\left\{|t|^{w_1}e^{-w_2|t|}\right\} & ,\min\left\{|t|\right\}\to\infty\end{cases}$$

IMAGES

In the present section, we obtain the images of some functions in our operators defined by (8) and (10), We have:

RESULT 1 :

$$I_x^{\upsilon,\lambda}\left[t^{\sigma} H_{r,s;r_1,s_1;\ldots;r_\ell,s_\ell}^{0,q;p_1,q_1;\ldots;p_\ell,q_\ell}\left[\begin{array}{c} z_1\, t^{\upsilon_1}(x-t)^{\lambda_1}\\ \vdots\\ z_i\, t^{\upsilon_\ell}(x-t)^{\lambda_\ell}\end{array}\right]\right]=$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n (z_0)^{2n+\eta+2\delta}}{2^{2n+\eta+2\delta}\,\Gamma(\delta+n+1)^m\,\Gamma(\eta+n\omega+\delta+1)\,n!}\sum_{i=0}^{\infty}\frac{\left[-g\left\{a(\alpha-1)\right\}^{1/\rho}\right]^i}{i!}$$

ISSN: 2454-4248
78 – 86

$$H^{0,q+2;p_1,q_1;\ldots;p_\ell,q_{i_\ell};2,2}_{r+2,s+1;r_1,s_1;\ldots;r_\ell,s_\ell;2,3}\left[\begin{array}{c} z_1\, x^{\upsilon_1+\lambda_1} \\ \vdots \\ z_\ell\, x^{\upsilon_\ell+\lambda_\ell} \\ z^1\left[a(\alpha-1)\right]^{\nu/\rho} \end{array}\left|\begin{array}{c} A^*:\left(c_j^{(\ell)},\gamma_j^{(\ell)}\right)_{1,r_1};\ldots;\left(c_j^{(\ell)},\gamma_j^{(\ell)}\right)_{1,r_\ell}; \\ B^*:\left(d_j^{(\ell)},\delta_j^{(\ell)}\right)_{1,s_1};\ldots;\left(d_j^{(\ell)},\delta_j^{(\ell)}\right)_{1,s_\ell}; \end{array}\right.\begin{array}{c} (1-i,\mu),\left(1-\frac{1}{(\alpha-1)}+\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right) \\ (0,1),\left(\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right),(1,\mu) \end{array}\right] \qquad (12)$$

where

$$\left.\begin{array}{l} A^*=\left(-\upsilon-\sigma-\upsilon_0(2n+\eta+2\delta);\upsilon_1,\ldots,\upsilon_\ell,\upsilon^1\right),\left(-\lambda-\lambda_0(2n+\eta+2\delta);\lambda_1,\ldots,\lambda_\ell,\lambda^1\right),\left(a_j,\alpha_1,\ldots,\alpha_\ell,0\right)_{1,C} \\ B^*=\left(-1-\upsilon-\lambda-\sigma-(\upsilon_0+\lambda_0)(2n+\eta+2\delta);(\upsilon_1+\lambda_1),\ldots,(\upsilon_\ell+\lambda_\ell),\left(\upsilon^1+\lambda^1\right)\right),\left(b_j;\beta_1,\ldots,\beta_\ell,0\right)_{1,D} \end{array}\right\} \qquad (13)$$

Provided the conditions available from (8) and [12,252-253,Eqs.(C.4-C.6)] are satisfied.

RESULT 2:

$$J_x^{\nu,\lambda}\left[t^\sigma H^{0,q;p_1,q_1;\ldots;p_\ell,q_\ell}_{r,s;r_1,s_1;\ldots;r_\ell,s_\ell}\left[\begin{array}{c} z_1\, t^{-\upsilon_1}\left(1-\frac{x}{t}\right)^{\lambda_1} \\ \vdots \\ z_\ell\, t^{-\upsilon_\ell}\left(1-\frac{x}{t}\right)^{\lambda_\ell} \end{array}\right]\right]=\frac{\left[a(\alpha-1)\right]^{-\nu/\rho}}{\rho\,\Gamma\left(1/(\alpha-1)\right)}\sum_{i=0}^{\infty}\frac{\left[-g\left\{a(\alpha-1)\right\}^{1/\rho}\right]^i}{i!}$$

$$\sum_{n=0}^{\infty}\left(\frac{z_0}{2}\right)^{2n+\eta+2\delta}\frac{(-1)^n}{\Gamma(\delta+n+1)^m\,\Gamma(\eta+n\omega+\delta+1)\,n!}$$

$$H^{0,q+2;p_1,q_1;\ldots;p_\ell,q_\ell;2,2}_{r+2,s+1;r_1,s_1;\ldots;r_\ell,s_\ell;2,3}\left[\begin{array}{c} z_1\, x^{-\upsilon_1} \\ \vdots \\ z_\ell\, x^{-\upsilon_\ell} \\ z^1\left[a(\alpha-1)\right]^{\mu/\rho} \end{array}\left|\begin{array}{c} A^*:\left(c_j^{(\ell)},\gamma_j^{(\ell)}\right)_{1,r_1};\ldots;\left(c_j^{(\ell)},\gamma_j^{(\ell)}\right)_{1,r_\ell}; \\ B^*:\left(d_j^{(\ell)},\delta_j^{(\ell)}\right)_{1,s_1};\ldots;\left(d_j^{(\ell)},\delta_j^{(\ell)}\right)_{1,s_\ell}; \end{array}\right.\begin{array}{c} (1-i,\mu),\left(1-\frac{1}{(\alpha-1)}+\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right) \\ (0,1),\left(\frac{\nu-r}{\rho},\frac{\mu}{\rho}\right),(1,\mu) \end{array}\right] \qquad (14)$$

where

$$\left.\begin{array}{l} C^{*}=\left(-\lambda-\lambda_{0}(2n+\eta+2\delta);\lambda_{1},\ldots,\lambda_{\ell},\lambda^{1}\right),\left(1+\sigma-\upsilon-\upsilon_{0}(2n+\eta+2\delta);\upsilon_{1},\ldots,\upsilon_{\ell},\upsilon^{1}\right),\left(a_{j};\alpha_{1},\ldots,\alpha_{\ell},0\right)_{1,r} \\ D^{*}=\left(-\upsilon-\lambda+\sigma-(\upsilon_{0}+\lambda_{0})(2n+\eta+2\delta);(\upsilon_{1}+\lambda_{1}),\ldots,(\upsilon_{\ell}+\lambda_{\ell}),\left(\upsilon^{1}+\lambda^{1}\right)\right),\left(b_{j};\beta_{1},\ldots,\beta_{\ell},0\right)_{1,s} \end{array}\right\} \tag{15}$$

Provided

$$\left.\begin{array}{l} \min\left[(\omega,\kappa,\gamma,a,\rho,\mu,\upsilon_{1},\lambda_{1})\right]\geq 0,(\text{not all simultaneously zero}) \\ \operatorname{Re}\left[\upsilon+\upsilon_{0}+\varsigma\right]>-1,\nu\in\square,\alpha>1,\operatorname{Re}\left[\lambda+\lambda_{0}\right]>-1 \end{array}\right\} \tag{16}$$

RESULT 3:

$$I_{x}^{\mu,\lambda}\left[t^{\sigma}\overline{H}\left(z_{1}t^{\mu_{1}}(t-x)^{\lambda_{1}}\left|\begin{array}{c}(a_{j},\alpha_{j};A_{j})_{1,N},(a_{j},\alpha_{j})_{N+1,P}\\(b_{j},\beta_{j})_{1,M},(b_{j},\beta_{j};B_{j})_{M+1,Q}\end{array}\right.\right)\right]=$$

$$\sum_{i=0}^{\infty}\sum_{h=1}^{M}\sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\frac{\left[a(\alpha-1)\right]^{-\nu/\rho}}{\rho\,\Gamma\left(1/(\alpha-1)\right)}\frac{(-1)^{n}\left[-g\left\{a(\alpha-1)\right\}^{1/\rho}\right]^{i}(z_{0})^{2n+\eta+2\delta}\,\bar{\theta}(s_{t,h})(z_{1})^{s_{t,h}}\,x^{\sigma+(\upsilon_{1}+\lambda_{1})s_{t,h}}}{2^{2n+\eta+2\delta}\,\Gamma(\delta+n+1)^{m}\,\Gamma(\eta+n\omega+\delta+1)\,i!\,n!}$$

$$H_{4,4}^{3,4}\left[z^{1}\left[a(\alpha-1)\right]^{\mu/\rho}\left|\begin{array}{c}(1-i,\mu),\left(1-\frac{1}{(\alpha-1)}+\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right),\left(-\upsilon-\sigma-\upsilon_{0}(2n+\eta+2\delta)+\upsilon_{1}s_{t,h};\upsilon^{1}\right),\left(-\lambda-\lambda_{0}(2n+\eta+2\delta)-\lambda_{1}s_{t,h};\lambda^{1}\right)\\(0,1),\left(\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right),(1,\mu),\left(-1-\upsilon-\sigma-(\upsilon_{0}+\lambda_{0})(2n+\eta+2\delta)-(\upsilon_{1}+\lambda_{1})s_{t,h};\left(\upsilon^{1}+\lambda^{1}\right)\right)\end{array}\right.\right] \tag{17}$$

Provided the conditions available from (8) and [12,252-253,Eqs.(C.4-C.6)] are satisfied.

RESULT 4:

ISSN: 2454-4248
78 – 86

$$J_x^{\upsilon,\lambda}\left[t^{\sigma}\overline{H}\left(z_1 t^{-\upsilon_1}(t-x)^{\lambda_1}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right)\right] =$$

$$\sum_{t=0}^{\infty}\sum_{h=1}^{M}\sum_{i=0}^{\infty}\sum_{n=0}^{\infty}\frac{[a(\alpha-1)]^{-\nu/\rho}}{\rho\,\Gamma(1/(\alpha-1))}\frac{(-1)^n\left[-g\{a(\alpha-1)\}^{1/\rho}\right]^i (z_0)^{2n+\eta+2\delta}\,\bar{\theta}(s_{t,h})(z_1)^{s_{t,h}}\,x^{\sigma+(\mu_1+\lambda_1)s_{t,h}}}{2^{2n+\eta+2\delta}\,\Gamma(\delta+n+1)^m\,\Gamma(\eta+n\omega+\delta+1)\,i!\,n!}$$

$$H_{2,3}^{3,2}\left[z^1[a(\alpha-1)]^{\mu/\rho}\left|\begin{matrix}(-\upsilon-\sigma-\upsilon_0(2n+\eta+2\delta);\upsilon^1),(-\lambda-\lambda_0(2n+\eta+2\delta);\lambda^1),(1-i,\mu),\left(1-\frac{1}{(\alpha-1)}+\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right)\\(0,1),\left(\frac{\nu-i}{\rho},\frac{\mu}{\rho}\right),(1,\mu),\left(-1-\upsilon-\sigma-(\upsilon_0+\lambda_0)(2n+\eta+2\delta);(\upsilon^1+\lambda^1)\right)\end{matrix}\right.\right] \quad (18)$$

Provided the conditions available from (8) and [12,252-253,Eqs.(C.4-C.6)] are satisfied.

Proof: To prove Result 1, first of all we write the I-operator present in its left hand side in the integral form using (8).

$$I_x^{\upsilon,\lambda}\left[t^{\sigma}H_{r,s;r_1,s_1;\ldots;r_\ell,s_\ell}^{0,q;p_1,q_1;\ldots;p_\ell,q_\ell}\left\{\begin{matrix}z_1\,t^{\lambda_1}(x-t)^{\upsilon_1}\\ \vdots\\ z_\ell\,t^{\lambda_\ell}(x-t)^{\upsilon_\ell}\end{matrix}\right\}\right]=$$

$$=x^{-\upsilon-\lambda-1}\int_0^x t^{\upsilon+\sigma}(x-t)^{\lambda}\,J_{\eta,\delta}^{\omega,\kappa}\left(z_0\left(\frac{t}{x}\right)^{\upsilon_0}\left(1-\frac{t}{x}\right)^{\lambda_0}\right)I_3\left(z_1\left(\frac{t}{x}\right)^{\upsilon^1}\left(1-\frac{t}{x}\right)^{\lambda^1},\mu,\nu,\rho,\gamma,a,\alpha\right)$$

$$H_{r,s;r_1,s_1;\ldots;r_\ell,s_\ell}^{0,q;p_1,q_1;\ldots;p_\ell,q_\ell}\left[\begin{matrix}z_1\,t^{\upsilon_1}(x-t)^{\lambda_1}\\ \vdots\\ z_r\,t^{\upsilon_\ell}(x-t)^{\lambda_\ell}\end{matrix}\right]dt \quad (19)$$

Further , we define generalized Lommel-Wright function and generalized astrophysical thermonuclear function using (1) & (6) respectively, next the multivariable H- function express in terms of contour from with the help of [12].Now we interchanging the order of summation and t-integral and the left hand side assumes the following form (say Δ) :

ISSN: 2454-4248
78 – 86

$$\Delta = \frac{\left[a(\alpha-1)\right]^{-\nu/\rho}}{\rho\,\Gamma\left(1/(\alpha-1)\right)} \sum_{i=0}^{\infty} \frac{\left[-g\left\{a(\alpha-1)\right\}^{1/\rho}\right]^{i}}{i!} \times x^{\sigma} \frac{1}{(2\pi\omega)^{l+1}} \int_{L_1} \cdots \int_{L_{\ell+1}} \Phi(\xi_1)\ldots\Phi(\xi_\ell)\psi(\xi_1,\ldots,\xi_\ell)\Gamma(-\xi_{l+1})$$

$$\Gamma(i+\mu\xi)\Gamma\left(\frac{1}{(\alpha-1)} - \frac{\nu}{\rho} + \frac{\mu}{\rho}\xi_{\ell+1}\right)\Gamma\left(\frac{\nu}{\rho} - \frac{\mu}{\rho}\xi_{\ell+1}\right) z_1^{\xi_1},\ldots\ldots, z_\ell^{\xi_\ell},\left(z^1\right)^{\xi_{\ell+1}}\left[a(\alpha-1)\right]^{(\mu/\rho)\xi_{\ell+1}}$$

$$x^{-\upsilon-\lambda-(\upsilon_0+\lambda_0)(2n+\eta+2\delta)-\left(\upsilon^1+\lambda^1\right)\xi_{\ell+1}-1}$$

$$\left\{\int_0^x (t)^{\sigma+\upsilon+\upsilon_0(2n+\eta+2\delta)+\sum_{k=1}^{\ell}\upsilon_k\xi_k+\upsilon^1\xi_{\ell+1}} (x-t)^{\lambda+\lambda_0(2n+\eta+2\delta)+\sum_{k=1}^{\ell}\lambda_k\xi_k+\lambda^1\xi_{\ell+1}}\, dt\right\} d\xi_1\ldots d\xi_{\ell+1} \qquad (20)$$

Now solving the above integral and re-interpreting the result obtained in terms of H-function of ℓ +1variable, we therefore conclude result after little simplification.

The proof of Result 2 can be obtained by proceeding on similar lines to those given above.

To prove Result 3 , we define generalized Lommel-Wright function and generalized of astrophysical thermonuclear function with the help of (1) & (6) respectively, next the $\bar{H}$ -- function express in series form with the help of [2]. Further we replace the order of summation and t-integral and finally, on the evaluation of the t-integral and re-interpretation of the result obtained in terms of H-function of ℓ +1 variable, we arrive at the present result .

APPLICATIONS:

In thispart we get some important applications in our image of study

(I) If we take $i=1, \upsilon_0 = \lambda^1 = \lambda_1 = 0$ and reduce $I_3(z,g,\nu,\rho,\mu,a,\alpha)$ to generalized Krätzel function to generalized Krätzel $D^{\nu,\alpha}_{\rho,\mu}$ [1,p.840,eq.(36)] and generalized Lommel Wright function $J^{\omega,k}_{\eta,\delta}$ to generalized Bessel function $J^{\omega}_{\eta,\delta}$ and multivariable H function to reduced Green function $K^{\theta}_{\ell,\chi}$ [4,p.11,Eq.(10)]. We obtain the following integral valid under the conditions derivable from the conditions stated with (8) & [12, p.252-253,Eqs.(C.4-C.6)].

$$\int_0^x \frac{1}{x}\left(\frac{t}{x}\right)^{\upsilon+\sigma+\upsilon_1}\left(1-\frac{t}{x}\right)^{\lambda} J^{\omega}_{\eta,\delta}\left\{z_0\left(1-\frac{t}{x}\right)^{\lambda_0}, s, b\right\} D^{\nu,\alpha}_{\rho,\mu}\left(z_1\left(\frac{t}{x}\right)^{\upsilon^1}\right) K^{\theta}_{\ell,\chi}\left[\left(\frac{t}{x}\right)^{\upsilon_1}\right] dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\left[a(\alpha-1)\right]^{-\nu/\rho}}{\ell\,\rho\,\Gamma\left(1/(\alpha-1)\right)\Gamma(\delta+n+1)\Gamma(\eta+n\omega+\delta+1)n!}\left(\frac{z_0}{2}\right)^{2n+\eta+2\delta} x^{\sigma}$$

$$H^{0,1:2,2;2,1}_{1,1:3,3;1,2}\left[\frac{1}{z^1\left[(\alpha-1)\right]^{1/\rho}}\left|\begin{matrix}(-\sigma-\upsilon;\upsilon_1,\upsilon^1): & (1,1/\ell),(1,\chi/\ell),(1,\varepsilon); & \left(1-\frac{1}{(\alpha-1)}+\frac{\nu}{\rho},\frac{1}{\rho}\right)\\ (-1-\sigma-\upsilon-\lambda_0(2n+\eta+2\delta);\upsilon_1,\upsilon^1): & (1,1/\ell),(1,1),(1,\varepsilon); & (0,1),\left(\frac{\nu}{\rho},\frac{1}{\rho}\right)\end{matrix}\right.\right] \quad (21)$$

(II) If we take $\lambda_0=\upsilon^1=\upsilon_1=.....=\upsilon_\ell=0$ and reduce generalized form of astrophysical thermonuclear function to Krätzel function to $D^{\nu}_{\rho}(z)$ [3, p.604,Eq.(5)] and Lommel-Wright function $J^{\omega,\kappa}_{\eta,\delta}$ to Struve function $H_\eta(z)$ with the help of (5) and multivariable H function to generalized Mittag Leffler function[9,p.5.Eq.(2.1)], We obtain the following integral valid under the conditions derivable from the conditions stated with (8) & [12,p.252-253,Eqs.(C.4-C.6)].

$$\int_x^\infty \frac{1}{t}\left(\frac{x}{t}\right)^{-\sigma+\upsilon}\left(1-\frac{x}{t}\right)^{\lambda} H_\eta\left\{z_0\left(\frac{x}{t}\right)^{\upsilon_0}\right\} D^{\nu}_{\rho}\left(z^1\left(1-\frac{x}{t}\right)^{\lambda^1}\right) E^{\gamma_j}_{\delta_j,\chi}\left[-z_1\left(1-\frac{x}{t}\right)^{\lambda_1},.......,-z_\ell\left(1-\frac{x}{t}\right)^{\lambda_\ell}\right]dt$$

$$=\frac{1}{\rho}\Gamma\left(-\sigma+\upsilon+\upsilon_0(2n+\eta+1)\right)$$

$$H^{0,1:1,1;\ldots;1,1;1,1}_{1,2:1,1;\ldots;1,1;1,1}\left[\begin{matrix}z_1\\ \vdots\\ z_r\\ z^1\end{matrix}\left|\begin{matrix}C^*: & (1-\gamma_1,1);.......;(1-\gamma_r,1); & -\\ D^*: & (0,1);.......;(0,1); & (0,1),\left(\frac{\nu}{\rho},\frac{1}{\rho}\right)\end{matrix}\right.\right] \quad (22)$$

Where

$$\left.\begin{aligned}C^*&=\left(-\lambda;\lambda_1,\ldots,\lambda_r,\lambda^1\right)\\ D^*&=\left(-\upsilon-\lambda+\sigma-\upsilon_0(2n+\eta+1);\lambda_1,\ldots,\lambda_r,\lambda^1\right),\left\{1-\chi;\delta_1,....,\delta_r,0\right\}\end{aligned}\right\} \quad (23)$$

Provided that the conditions easily obtainable from the existing conditions of (10) & [12, p.252-253,eqs(C.4-C.6)].

(iii)In the result 3, if we take $\upsilon_0=\lambda^1=\lambda_1=0$ and $I_3(z,g,\nu,\rho,\mu,a,\alpha)$ to generalized Krätzel function $D^{\nu}_{\rho}(z)$ [3, p.604,Eq.(5)] and generalized Lommel-Wright function $J^{\omega,\kappa}_{\eta,\delta}$ to Struve function $H_\eta(z)$ and $\overline{H}$ function in terms of Gaussian Model free energy [5,pg.(4126 &4127), Eqs.(23 &28)]. We get the following integral exists under the conditions obtainable from (10) & [12, p.252-253,Eqs.(C.4-C.6)].

$$\int_0^x \frac{1}{x}\left(\frac{t}{x}\right)^{\upsilon+\sigma+\upsilon^1\nu-\frac{\upsilon_1}{2}}\left(1-\frac{t}{x}\right)^{\lambda} H_\eta\left\{z_0\left(1-\frac{t}{x}\right)^{\lambda_0},s,b\right\}K_{-\nu}\left(2\left(\frac{t}{x}\right)^{\upsilon^1/2}\right)\beta F\left[d,\left(\left(\frac{t}{x}\right)^{-\upsilon_1/2}-1\right)\right]dt$$

$$=\frac{1}{8\rho(\pi)^{d/2}}\sum_{t=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{\Gamma(\eta+n+3/2)\Gamma(n+3/2)n!}\frac{\{\Gamma(1+t)\}^2\{\Gamma(3/2+t)\}^d}{\{\Gamma(2+t)\}^{1+d}\,t!}\left(\frac{z_0}{2}\right)^{2n+\eta+1}\Gamma(1+\lambda+\lambda_0(2n+\eta+1))$$

$$H^{3,1}_{1,3}\left[z^1\left|\begin{matrix}(-\upsilon-\sigma,\upsilon^1)\\(0,1),\left(\frac{\nu}{\rho},\frac{1}{\rho}\right),(-1-\upsilon-\sigma-\lambda_0(2n+\eta+1),\upsilon^1)\end{matrix}\right.\right]$$

(24)

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