International Journal on Future Revolution in Computer Science & Communication Engineering
Volume: 4 Issue: 8

ISSN: 2454-4248
35 – 40

# Disconnected Topology Space

Dr. R. Balakumar, (M.Sc., M.Phil., Ph.D.)[1]
Assistant Professor, Department of Mathematics,
Prist University, Thanjavur

R. Karthika[2]
M.PhilResearch Scholar, Department of Mathematics,
Prist University, Thanjavur

***Abstract*:-** A more general definition of extremally μ-disconnected generalized topological space [3] is introduced and its properties are studied. We have further improved the definitions of generalized open sets [1] and upper(lower) semi-continuous functions defined for a generalized topological space in [5]. In this generalized framework we obtain the analogues of results in [1, 3, 5]. Examples of extremally μ-disconnected generalized topological spaces are given.

***Keywords:*** *Extremally μ-disconnected generalized topological spaces*

*****

## PRELIMINARIES

**Definition 1.1**

A topology on a set x is a collection J of subsets of x having the following properties,

i) $\phi$ and x are in J.

ii) The union of the elements of any sub-collection of J is in J.

iii) The intersection of the elements of any finite sub-collection of J is in J.

A set x for which topology J has been specified is called **Topological space**.

**Example 1.1.1**

Let x = {a, b, c, d)

i) $J\circ = \{\phi, X\}$ is a topology.

ii) $J_1 = \{\phi, X, \{a\}\}$ is a topology.

iii) $J_2 = \{\phi, X, \{b\}\}$ is a topology.

iv) $J_3 = \{\phi, X, \{a\}, \{b\}\}$ is not a topology.

**Definition 1.2**

A topology on X is said to be **discrete topology**, if the topology contains all the subsets of X.

**Example 1.2.1**

X = {1, 2, 3}

$J = \{\phi, X, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{3,2\}\}$ is a discrete topology.

**Definition 1.3**

If a topology which contains only $\phi$ and X is called **trivial (or) indiscrete topology**.

**Example 1.3.1**

X = {1, 2, 3}

$J = \{\phi, X\}$

**Definition 1.4**

ISSN: 2454-4248
35 – 40

Let X be a set. Let $J_f$ be the collection of all subsets U of X such that

X-U is either finite (or) is all of X.

Then $J_f$ is a topology on X. It is called the **finite complement topology**.

**Definition 1.5**

If X is a set. A basis for a topology on X is a collection Ɓ of subsets of X such that,

1) For each $x \in X$, there is atleast one basis element Ɓ containing X
2) If $x \in B_1 \cap B_2$ then there is a basis element $B_3$ such that,

$B_3 \subset B_1 \cap B_2$

The element of Ɓ are called **basis element**.

**Definition 1.6**

A subset U of X is said to be **open** in X. If for each $x \in U$ there is a basis element $B \in \mathrm{B}$ such that $x \in B$ and $B \subset U$.

**Definition 1.7**

Let X be a set with simple order relation. Assume X has more than one element.

Let Ɓ be the collection of all sets of the following types.

i) All open interval (a, b) in X.
ii) All intervals of the form [a ${}_{\circ}$ , b) where $a_{\circ}$ is the smallest element of X.
iii) All intervals of the form (a, b ${}_{\circ}$ ] where $b_{\circ}$ is the largest element of X.

The collection Ɓ is a basis for a topology on X which is called the **order topology**.

**CONNECTED TOPOLOGICAL SPACE**

**3.1 Definition**

Let (X,T) be a topological space, $\alpha \in [0,1]$ and let $T = \{0,1\} \cup (U : U \in T\}$ then any $U \in T$ is a **α-neighbourhood** for any point $X \in U$ (i.e) For any point of its support, but obviously U fails to be α-open in (X,T).

**3.2 Definition**

Let (X,T) be a topological space, $\alpha \in [0,1]$ and let $T = \{0,1\} \cup \{\alpha.u : u \in T\}$ then any $u \in T$ is a **α-neighbourhood** for any point $x \in u$ (i.e) For any point of its support, but obviously u fails to be α-open in (X,T).

**3.3 Definition**

A fuzzy topological space $(X, \tau)$ is called **α-compact**if every-shading of X by fuzzy open sets of X has a finite α-subshading.

**3.4 Definition**

Let $0 \le \alpha < 1$. A fuzzy topological space $(X, \tau)$ is said to be **α-Hausdorff** if for each $x, y \in X$ with $x \neq y$ there exist fuzzy open sets A and B in X, such that $A(x) > \alpha, B(x) > \alpha$ and $A \wedge B = 0_x$.

ISSN: 2454-4248
35 – 40

**3.5 Definition**

Let $(X,\tau)$ be a fuzzy topological space. A subset A of X is said to be **α-N-open set** in X if its complement $A^1$ is **α-closed** in X.

**3.6 Theorem**

Let (X,T) be a fuzzy topological spaces and let $\alpha \in I$ for each point

i) If $M \in N_\alpha(p)$ then $p \in M$

ii) If $M, N \in N_\alpha$ then $M \cap N \in N_\alpha(p)$

iii) If $M \in N_\alpha(p)$ and $M \subset N$ then $N \in N_\alpha(p)$

iv) If $M \in N_\alpha(p)$ then there is $N \in N_\alpha(p)$ such that $\sigma(N) \geq \alpha$ $N \subset M$ and $N \in N_\alpha(q)$ for every $q \in N.$

**Proof**

i) If $M \in N_\alpha(p)$

To prove $p \in M$

Thrn there exist $G \in T_\alpha$

Such that $p \in G \subset M$ and therefore $p \in M$

Hence $p \in M$

ii) If $M, N \in N_\alpha(p)$

To prove $M \cap N \in N_\alpha(p)$

There exist two α-open fuzzy sets that is $G_1$ and $G_2$ such that

$$p \in G_1 \subset M, p \in G_2 \subset N$$

Since $G_1 \cap G_2 \in T_\alpha$ and $p \in G_1 \cap G_2 \subset M \cap N$

We have $M \cap N \in N_\alpha(p)$

iii) If $M \in N_\alpha(p)$

To prove $N \in N_\alpha(p)$

There exists $G \in T_\alpha$ such that $p \in G \subset M \subset N$ and

Hence $N \in N_\alpha(p)$

iv) Suppose $M \in N_\alpha(p)$

To prove $(N) \geq \alpha$ and $N \in N_\alpha(q)$ for every $q \in N$

Then there exists $G \in T_\alpha$ such that $p \in G \subset M$

ISSN: 2454-4248
35 – 40

Let N=G

We have $(N) \geq \alpha$ and $N \in N_{\alpha}(q)$ for every $q \in N$

Hence the proof.

**DISCONNECTED TOPOLOGICAL SPACE**

A topological space that is not connected, i.e., which can be decomposed as the disjoint union of two nonempty open subsets. Equivalently, it can be characterized as a space with more than one connected component.

A subset $S$ of the Euclidean plane with more than one element can always be disconnected by cutting it through with a line (i.e., by taking out its intersection with a suitable straight line). In fact, it is certainly possible to find a line $r$ such that two points of $S$ lie on different sides of $r$. If the Cartesian equation of $r$ is

$$r : ax + by + c = 0 \tag{1}$$

for fixed real numbers $a$, $b$, $c$, then the set $S' = S \setminus r$ is disconnected, since it is the union of the two nonempty open subsets

$$U_{+} = S' \bigcap \{(x, y) \in \mathbb{R}^2 \mid ax + by + c > 0\} \tag{2}$$

and

$$U_{-} = S' \bigcap \{(x, y) \in \mathbb{R}^2 \mid ax + by + c < 0\}, \tag{3}$$

which are the sets of elements of $S$ lying on the two sides of $r$.

**Topology: Connected Spaces**

Let X be a topological space. Recall that if U is a clopen (i.e. open and closed) subset of X, then X is the topological disjoint union of U and X–U. Hence, if we assume X cannot be decomposed any further, there're no non-trivial clopen subsets of X.

**Definition**. The space X is **connected** if its only clopen subsets are X and the empty set. Equivalently, there're no non-empty disjoint open subsets U, V of X such that $X = U \cup V$. Otherwise, it's **disconnected**.

The following characterisation of connected sets is simple but surprisingly handy.

**Theorem 3.1**

X is connected if and only if any continuous map $f : X \rightarrow \{0, 1\}$, where {0, 1} has the discrete topology, is constant. Thus, it's disconnected if and only if there's a continuous surjective map $f : X \rightarrow \{0, 1\}$.

**Proof**.

If *f* is not constant, then $U := f^{-1}(0)$ and $V := f^{-1}(1)$ are non-empty disjoint open subsets such that $X = U \cup V$. Conversely, if $X = U \cup V$, where *U*, *V* are non-empty, disjoint and open in *X*, then we can set

$$f : X \rightarrow \{0, 1\}, \quad x \mapsto \begin{cases} 0, & \text{if } x \in U, \\ 1, & \text{if } x \in V. \end{cases}$$ ♦

With this theorem in hand, the remaining properties are surprisingly easy to prove.

**Definition 3.1**

Let X be a topological space. Then X is said to be disconnected if there exists two non-empty separated sets A and B such that $X = A \cup B$.

**Theorem 3.2**

Let X be a topological space then X is disconnected iff X has a non-empty proper subset which is both open and closed.

**Proof:**

Let A be a non-empty proper subset of X, which is both open and closed.

ISSN: 2454-4248
35 – 40

Then $(X-A)$ is also a non-empty proper subset of X which is both open and closed.

So X is the union of two non-empty separated sets, showing that X is disconnected.

**Conversely,**

Let X be a disconnected space, then there exists two non-empty separated sets A and B such that $X = A \cup B$

Since A and B are separated, therefore $A \cap \overline{B} = \phi$ and C.

So $\overline{A} \cap B = X$ , $A \cap \overline{B} = X$ and $A \cap B = \phi$ ……(i)

Now,

$A \cup B = X$, $A \cap B = \phi \Rightarrow A = X - B$......(ii)

And $A \cup \overline{B} = X, A \cap \overline{B} = \phi \Rightarrow A = X - \overline{B}$ ......(iii)

Also $\overline{A} \cup B = X, \overline{A} \cup B = \phi \Rightarrow B = X - \overline{A}$ ……(iv)

Since $A \neq \phi, B \neq \phi$

It follows from (ii) that A is a non-empty proper subset of X and (iii) shows that A is open. (ii) and (iv) both shows that A is closed.

Thus X has a non-empty proper subset which is both open and closed.

**Example 3.3**

Every discrete space which contains more than one point is disconnected.

**Proof:**

Let X be discrete space and let $x \in X$ . Then $\{x\}$ is a non-empty proper subset of X.

Which is both open and closed in X. Hence X is disconnected.

**Definition 3.4**

A subset A of a topological space X is said to be closed. If its complement $X - A$ is open.

**Theorem 3.5**

Let X be a topological space then X is disconnected. Iff $X = A \cup B$ ,where A and B are non-empty disjoint open sets.

**Proof:**

Let X is disconnected then there exist a non-empty proper subset A of X.

Which is both open and closed.

Then $X - A$ is also a non-empty subset of X which is both open and closed.

This shows that X is the union of two non-empty disjoint open sets.

**Conversely,**

Let X be the union of two non-empty disjoint open sets A and B, then $X - B = A$.

Since B is open this implies A is closed and since $B \neq \phi$ .

$\Rightarrow$ A is non-empty proper subset of X, that is both open and closed.

Hence X is disconnected.

**Note:**

The rational Q are disconnected.

**Definition 3.6**

ISSN: 2454-4248
35 – 40

A subset U of X is said to be open in X. If for each $x \in U$ there is a basis element $B \in \mathrm{B}$ and $B \in U$.

**Theorem 3.7**

Let x be a topological space then X is disconnected. Iff $X = A \cup B$,where A and B are non-empty disjoint closed sets.

**Proof:**

Let X is disconnected. Then there exists a non-empty proper subset A of X.

Which is both open and closed.

And $X - A$ is also a non-empty subset of X.

Which is both open and closed.

This shows that X is union of two non-empty disjoint closed set.

**Conversely,**

Let X be the union of two non-empty disjoint closed sets A and B.

Then $X - B = A$.

Since B is closed this implies A is open.

Since $B \neq \phi$. This implies that A is a non-empty proper subset of X.

That is both open and closed.

Hence X is disconnected.

**SUGGESTIONS**

The connected and disconnected space is an intermediate product of the project, but it has many interesting properties. It might be worthwhile to continue the research, for example, we can choose another space other than the one used in strongly connected spaces.

The other point to note is that the conditions for a connected space to be strongly connected that presented in this report is still too strict, thus finding more suitable conditions will be a challenge.

**CONCLUSION**

In this report, we first review the concept of connectedness. It is a well-developed concept, and its definition, properties and examples are all ready to use. Thus, a quick revision enables us doing further study. By slightly modifying its definition, we gain a connected and disconnected space, which has almost the same features as a connect space. The connectedness may vary greatly, from the usual connectedness to one-point connectedness; therefore, being a general connected space, it does not have anything very interesting.

However, a specific connected space the strongly connected space is an ideal notion of stronger connectedness. Since the space is carefully chosen, the strongly connectedness is only a little stronger than the usual connectedness. We have shown that the strongly connectedness not only has the same property as the connected space, but also has many new properties. Moreover, it is also mentioned in the report that how to make a connected space strongly connected.

**REFERENCE:**

[1] Bing, R. H. (1953), “A connected countable Hausdorff space”, Proc, A.M.S.
[2] Duda, E. &Whyburn, G. (1978), “Dynamic Topology”, Undergraduate texts in
[3] Mathematics, Springer_Berlag.
[4] Hocking, J. G. & Young, G. S. (1988), “ Topology”, Addison-Wesley Publishing
[5] Company, Inc.
[6] Hu, S.T. (1966), “Introduction to general topology”, Holden-Day inc.
[7] Kuratowski, K. (1948), “Topologie”, Vol 1, Warsaw.
[8] Munkers, J. R. (1975), “Topology, a first course”, Rrentice-Hall inc. New Jersey.