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Anti Homomorphism in Fuzzy Subgroups

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Abstract:- On the basis of fuzzy sets introduced by L.A. Zadeh, we first gave the definition of $\alpha-$ fuzzy subgroups and $\alpha-$ fuzzy normal subgroups. This paper proves a necessary and sufficient condition of $\alpha-$ fuzzy subgroup (normal subgroup) to be fuzzy subgroup (normal subgroup). We introduced the notion of $(\alpha,\beta)-$ anti fuzzy subgroups and their related properties. Finally, behaviour of these $(\alpha,\beta)-$ anti fuzzy subgroups (normal subgroups) under group homomorphism have been discussed. We define a notion of homomorphism and anti homomorphism of multi L-fuzzy subgroup and investigate some of its properties. This paper contains the definition and result of anti L-fuzzy normal subgroup is being given using homomorphism and anti homomorphism.

Keywords: *$\alpha-$ fuzzy subgroups, $(\alpha,\beta)-$ anti fuzzy subgroups (normal subgroups), homomorphism, anti homomorphism multi L-fuzzy subgroup, anti L-fuzzy normal subgroup.*

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INTRODUCTION

"Fuzzy set theory is a marvellous tool for modelling the kind of uncertainty associated with vagueness, with Imprecision and with a lack of information regarding to a particular element of the problem at hand". Thus the idea of fuzziness is one of "enrichment" not of "replacement".

Zadeh's ideas have found application in computer science, artificial intelligence, decision analysis, information science, system science, control engineering, expert systems, pattern recognition, management science, operation research and robotics.

Rosenfield introduced the concept of fuzzy subgroup. Motivated by this many mathematicians started to review various concept of theorems of abstract algebra in the boarder frame work of fuzzy settings. Biswas introduced the concept of anti-fuzzy subgroups of groups. Palaniappan. N and Muthuraj defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy groups.

Chakrabatty and Khare introduced the notion of fuzzy homomorphism between two groups and studied its effect to the fuzzy subgroups. In Ajmal defined the notion of containment of an ordinary kernel of a group homomorphism in fuzzy subgroups and gave the idea of quotient group in a natural way. Consequently, the fundamental theorem of homomorphism was established in fuzzy subgroups.

In this paper, we introduce the notion of $\alpha-$ fuzzy subgroups and $\alpha-$ fuzzy normal subgroups. $(\alpha,\beta)-$ anti fuzzy subgroups (normal subgroups) under group homomorphism, homomorphism of multi L-fuzzy subgroup and anti L-fuzzy normal subgroup and some properties are discussed.

The idea of fuzzy set theory have been introduced in topology, abstract algebra, geometry graph and analysis.

PRELIMINARIES

Fuzzy set theory was introduced by Zadeh in 1965.

Definition 1.1

The concept of a **fuzzy set** is an extension of the concept of a crisp set.

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Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1}, a fuzzy set on a domain U is defined by its membership function from U to [0,1].

Let U be a non-empty set, to be called the universal set (or) the universe of discourse (or) simply a domain.

Then by a fuzzy set on U is meant a function, $A : U \rightarrow [0,1]$. 'A' is called the **membership function**, A(x) is called the **membership grade**

of x.

We also write,

$$A = \{x, A(x) : x \in U\}$$

Example

Let U = {1, 2, 3, 4, 5}.

A = The set of all numbers in U very close to 1.

Where $A(x) = \frac{1}{x}$

$\Rightarrow$ A = {(1, 1), (2, 0.5), (3, 0.3), (4, 0.25), (5, 0.2)}

Definition 1.2

Let σ and τ be two fuzzy sets of a set V. Then σ is said to be **fuzzy subset** of τ, written as $\sigma \subseteq \tau$, if $\sigma(x) \leq \tau(x)$ for every $u \in V$.

Definition 1.3

A is said to be **included (or) contained** in B, iff $A(x) \leq B(x)$ for all x in U. In symbol we write

$A \subseteq B$,

$(i.e)$ A is a subset of B.

Definition 1.4

A is said to be **equal** to B (or) same as B, iff $A \subseteq B \,\&\, B \subseteq A$.

$(i.e)$, $A(x) = B(x)$ all in U.

we write $A = B$.

Definition 1.5

The **union** of A and B is denoted by $A \cup B$ and it is defined on U as,

$$A \cup B(x) = \max\{A(x), B(x)\}$$
$$= A(x) \vee B(x) \text{for every x in U.}$$

Where '$\vee$' refers to the **maximum** of two fuzzy sets.

Definition 1.6

The **intersection** of A and B denoted by $A \cap B$ and it is defined as a fuzzy set on U for which

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$$A \cap B(x) = \min\{A(x), B(x)\}$$

$= A\ (x)\ \wedge B(x)$for every x in U.

Where ‘ $\wedge$ ’ refers to the **minimum** of two fuzzy sets.

Definition 1.7

The **complement of A** is denoted by A' and it is defined as a fuzzy set on U for which,

$A'(x) = 1 - A(x)$ for every X in U.

Definition 1.8

Let A be a fuzzy set on U. Then the **Height of a fuzzy set A** is the largest membership grade obtained by any element of the set.

$$h(A) = \sup_{x \in X} A(x)$$

Example

(i) Let A = {(a, 0.2), (b, 0.3), (c, 0.8), (d, 0.9)}

Then $h(A) = 0.9$

(ii) If A is a non-empty crisp set then $h(A) = 1.$

Definition 1.9

Let A be a fuzzy set on U. Then A is said to be **normal fuzzy set**

if, $A(x) = 1$ for atleast one x in U.

Example

All non-empty crisp sets are normal.

Definition 1.10

Let A be a fuzzy set on U. Then the set $\{x \in U / A(x) = 1\}$ is called the

core of the fuzzy set A and it is denoted by core(A).

Example

Let A = {(x, 0.2), (y, 0.3), (z, 0.8), (w,1)}

$\therefore core(A) = \{w\}$

Definition 1.11

Let A be a fuzzy set on U. The set $\{x \in U / A(x) > 0\}$ is called the

support of a fuzzy set A and is denoted by supp(A).

Example

Let U = {a, b, c, d, e, f}

Consider A = {(a, 0.5), (b, 0.3), (f, 1)}

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Supp(A) = {a, b, f}

Remark 1.12

i. Core(A) $\subseteq$ Supp(A) for all fuzzy set A.

ii. Supp(A) is a crisp set on U for all fuzzy set A.

iii. Supp(A) = A for any crisp set A.

iv. Core(A) = Supp(A) only for crisp set A.

Definition 1.13

Every fuzzy set A on U associate with L(A), a crisp subset on I = (0,1) is called a **level set**.

$$L(A) = \{\alpha \in U / A(x) = \alpha \text{ for some } x \in U\}.$$

Definition 1.14

Given a fuzzy set A on U and any number $\alpha \in [0,1]$, the α-**cuts**

$$^{\alpha}A = \{x \in U; A(x) \geq \alpha\}.$$

Example

Let A = {(a, 0.8), (b, 1), (c, 0.3), (d, 0.1)}

Then $^{0.1}A$ = {a, b, c, d}

$\alpha -$ **FUZZY SUBGROUPS**

Definition: 2.1

A function $A : G \rightarrow [0,1]$ is called a **fuzzy subgroup** of G if

i. $A(xy) \geq \min\{A(x), A(y)\}$

ii. $A(x^{-1}) \geq A(x), \forall x, y \in G$

It is easy to show that a fuzzy subgroup of a group G satisfies $A(x) \leq A(e) \,\&\, A(x^{-1}) = A(x), \forall x \in G,$ where e is the identity element of G.

Definition: 2.2

Let A be a fuzzy subgroup of a group G, then it is called fuzzy normal subgroup of G if

$$A(xy) = A(yx), \forall x, y \in G$$

Definition: 2.3

Let A be a fuzzy subset of a group G. Let $\alpha \in [0,1]$. Then the fuzzy set A^{α} of G is called the $\alpha -$ fuzzy subset of G (with respect to fuzzy set A) and is defined as

$$A^{\alpha}(x) = \min\{A(x), \alpha\}, \forall x \in G.$$

Theorem: 2.4

If $A:G\to[0,1]$ is a $\alpha-$ fuzzy subgroupsof a group G, then

- $A^\alpha(x)\le A^\alpha(e),\forall x\in G,$ where e is the identity element of G
- $A^\alpha(xy^{-1})=A^\alpha(e)\Rightarrow A^\alpha(x)=A^\alpha(y),\forall x,y\in G,$

Proof:

(i) $A^\alpha(e)=A^\alpha(xx^{-1})$

$\ge\min\{A^\alpha(x),A^\alpha(x^{-1})\}$

$=\min\{A^\alpha(x),A^\alpha(x)\}$

$=A^\alpha(x)$

$$A^\alpha(x)\le A^\alpha(e),\forall x\in G$$

(ii) $A^\alpha(x)=A^\alpha(xy^{-1}y)$

$\ge\min\{A^\alpha(xy^{-1}),A^\alpha(y)\}$

$=\min\{A^\alpha(e),A^\alpha(y)\}$

$=A^\alpha(y)$

$=A^\alpha(yx^{-1}x)$

$\ge\min\{A^\alpha(yx^{-1}),A^\alpha(x)\}$

$\ge\min\{A^\alpha(xy^{-1}),A^\alpha(x)\}$

$=A^\alpha(x)$

Thus $A^\alpha(x)=A^\alpha(y),\forall x,y\in G$

$(\alpha,\beta)-$ ANTI FUZZY SUBGROUPS (NORMAL SUBGROUPS)

Definition: 3.1

A function $A:G\to[0,1]$ is an **anti fuzzy subgroup** of a group G if and only if

$$A(xy^{-1})\le\max\{A(x),A(y)\},\forall x,y\in G$$

Definition: 3.2

A fuzzy subgroup (or anti fuzzy subgroup) A of a group G is called fuzzy normal subgroup of G if and only if

$A(y^{-1}xy)=A(x)$ or equivalently,

$A(xy)=A(yx), holds\forall x,y\in G$

Definition: 3.3

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Let A^{α} and A_{β} denote respectively the $\alpha-$ fuzzy set and $\beta-$ anti fuzzy set of the set X with respect to the fuzzy set A. Then the fuzzy set $A_{(\alpha,\beta)}$ defined by

$$A_{(\alpha,\beta)}(x) = Max\{(A^{\alpha})^{c}(x), A_{\beta}(x)\},\ for every x \in X,$$

is called $(\alpha,\beta)-$ anti fuzzy set of X (with respect to the fuzzy set A), where $\alpha,\beta \in [0,1]$ such that $\alpha+\beta \leq 1$

Theorem: 3.4

Let A be $\alpha-$ fuzzy subgroup as well as $\beta-$ anti fuzzy subgroup of a group G, then A is also $(\alpha,\beta)-$ anti fuzzy subgroup of G.

Proof:

Let x,y be any element of the group G, then

$$A_{(\alpha,\beta)}(xy^{-1}) = Max\{(A^{\alpha})^{c}(xy^{-1}), A_{\beta}(xy^{-1})\}$$

$$\leq Max\{Max\{(A^{\alpha})^{c}(x),(A^{\alpha})^{c}(y)\}, \max\{A_{\beta}(x), A_{\beta}(y)\}\}$$

$$= Max\{Max\{(A^{\alpha})^{c}(x), A_{\beta}(x)\}, \max\{(A^{\alpha})^{c}(y), A_{\beta}(y)\}\}$$

$$= Max\{A_{(\alpha,\beta)}(x), A_{(\alpha,\beta)}(y)\}$$

Thus $A_{(\alpha,\beta)}(xy^{-1}) \leq Max\{A_{(\alpha,\beta)}(x), A_{(\alpha,\beta)}(y)\}$

Hence A is $(\alpha,\beta)-$ anti fuzzy subgroup of G.

Homomorphism of Multi L-fuzzy subgroup

Definition: 4.1

The function $f: G \rightarrow G^{'}$ is said to be homomorphism if

$$f(xy) = f(x)f(y), \forall x, y \in G.$$

Definition: 4.2

The function $f: G \rightarrow G^{'}$ (G and G’ are not necessarily commutative) is said to be an anti homomorphism if

$$f(xy) = f(y)f(x), \forall x, y \in G.$$

Theorem: 4.3

Let G and G’ be any two groups. Let $f: G \rightarrow G^{'}$ be a homomorphism and onto. Let $\lambda: G \rightarrow L$ be a multi L-fuzzy subgroup of G. Then $f(\lambda)$ is a multi L-fuzzy subgroup of G’. If λ has sup property and λ is f-invariant.

Proof:

Let λ be a multi L-fuzzy subgroup of G.

(i) $$f(\lambda)(xy) = \vee\{\lambda(xy) / xy \in G, f(xy) = x_0 y_0\}$$

$$= \lambda(x_0 y_0)$$

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$$\geq \lambda(x_0) \wedge \lambda(y_0)$$

$$\geq (\vee\{\lambda(x) / x \in G, f(x) = x_0\}) \wedge (\vee\{\lambda(y) / y \in G, f(y) = y_0\})$$

$$\geq (f(\lambda)(x)) \wedge (f(\lambda)(y))$$

$$f(\lambda)(xy) \geq (f(\lambda)(x)) \wedge (f(\lambda)(y))$$

(ii) $f(\lambda)(x^{-1}) = \vee\{\lambda(x^{-1}) / x^{-1} \in G, f(x^{-1}) = x_0\}$

$$= \vee\{\lambda(x) / x \in G, f(x) = x_0\}$$

$$= \lambda(x_0)$$

$$= \vee\{\lambda(x) / x \in G, f(x) = x_0\}$$

$$= f(\lambda)(x)$$

$$f(\lambda)(x^{-1}) = f(\lambda)(x)$$

Hence $f(\lambda)$ is a multi L-fuzzy subgroup of G'.

CONCLUSION

In this paper, we have introduced the concept of $\alpha -$ fuzzy subgroups and used it to introduce the concept of $\alpha -$ fuzzy normal subgroups and discussed various related properties. We will formulate the concept of $(\alpha, \beta) -$ fuzzy subgroups and applied it to study some properties.

In the studies, we have also studied the concept of homomorphism and anti homomorphism in multi L-fuzzy subgroups are discussed and also the anti L-fuzzy normal subgroup are established using homomorphism and anti homomorphism.

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