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Fuzzy Hypergraphs

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Abstract:- Graph theory has found many application area in science, engineering, and mathematics. In order to expand the application base, the notion of a graph was generalized to that of a hypergraph, that is, a set X of vertices together with a collection of subsets of X. In this chapter, we fuzzify the notation of a hypergraph and state some possible applications. In this article, we apply the concept of bipolar fuzzy sets to hypergraphs and investigate some basic theorems and some properties of bipolar fuzzy hypergraphs. Some basic concepts of bipolar fuzzy set are defined. It is shown that any bipolar fuzzy graph can be expressed as the bipolar fuzzy intersection graphs of some bipolar fuzzy sets.

Keywords::- *Fuzzy hypergraphs, bipolar fuzzy hypergraphs, bipolar fuzzy set, bipolar intersection graphs.*

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INTRODUCTION

Generally an undirected graph is a symmetric binary relation on a non- empty vertex set V. A fuzzy graph (undirected) is also a symmetric binary fuzzy relation on a fuzzy subset. Rosenfeld considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graphs in 1975.

Although the first definition of fuzzy graphs was given by Kaufman,R.T.Yeh and S.V.Bang have also introduced various connectedness concepts in fuzzy graphs during the same time. “Fuzzy set theory is a marvellous tool for modelling the kind of uncertainty associated with vagueness, with Imprecision and with a lack of information regarding to a particular element of the problem at hand”

Thus the idea of fuzziness is one of “enrichment” not of “replacement”.

Zadeh’s ideas have found application in computer science, artificial intelligence, decision analysis, information science, system science, control engineering, expert systems, pattern recognition, management science, operation research and robotics.

The idea of fuzzy set theory have been introduced in topology, abstract algebra, geometry graph and analysis.

In 1975, Rosenfeld introduced the concept of fuzzy graphs. In 2011, Akram introduced the bipolar fuzzy graphs and defined different operations on it. Bipolar fuzzy graph theory is now growing and expanding its applications.

The concept of bipolar fuzzy graphs can be applied in various areas of engineering, computer science: database theory, expert systems, neural networks, artificial networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks and medical diagnosis. In this paper, we define bipolar fuzzy intersection graphs and different properties.

In this dissertation work,

Chapter I refreshes some basic concepts in fuzzy set theory and fuzzy graph theory.

Chapter II deals with the concept of fuzzy hypergraphs and bipolar fuzzy hypergraphs and their properties.

Chapter III discusses thebipolar fuzzy line graph of a bipolar fuzzy hypergraphs.

Chapter IV illustrates thedefinition of Fuzzy intersection Graphs.

Chapter V illustrates the bipolar fuzzy intersection graphs.

This dissertation work consists of same basic concepts of fuzzy graphs and fuzzy hypergraphs. This dissertation also explains the concept of bipolar fuzzy hypergraphs and bipolar fuzzy intersection graphs.

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PRELIMINARIES

Fuzzy set theory was introduced by Zadeh in 1965.

Definition 1.1

The concept of a **fuzzy set** is an extension of the concept of a crisp set.

Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1}, a fuzzy set on a domain U is defined by its membership function from U to [0,1].

Let U be a non-empty set, to be called the universal set (or) the universe of discourse (or) simply a domain.

Then by a fuzzy set on U is meant a function, $A : U \to [0,1]$. 'A' is called the **membership function**, A(x) is called the **membership grade**

of x.

We also write,

$$A = \{x, A(x) : x \in U\}$$

Example

Let U = {1, 2, 3, 4, 5}.

A = The set of all numbers in U very close to 1.

Where $A(x) = \frac{1}{x}$

⇒ A = {(1, 1), (2, 0.5), (3, 0.3), (4, 0.25), (5, 0.2)}

Definition 1.2

Let σ and τ be two fuzzy sets of a set V. Then σ is said to be **fuzzy subset** of τ, written as $\sigma \subseteq \tau$, if $\sigma(x) \le \tau(x)$ for every $u \in V$.

Definition 1.3

A **fuzzy graph** $G : (\sigma, \mu)$ is a pair of functions $\sigma : S \to [0,1]$

And $\mu : S \times S \to [0,1]$,

we have, $\mu(x, y) \le \sigma(x) \wedge \sigma(y), \qquad \forall x, y \in S.$

Where σ is a subset of a non-empty set S and μ is a symmetric fuzzy relation on σ.

Definition 1.4

A fuzzy graph $H : (\gamma, t)$ is called a **fuzzy subgraph** of $G : (\sigma, \mu)$ if

$$\gamma(v) \le \sigma(v), \qquad \forall v \in S,$$
$$t(x, y) \le \mu(x, y), \qquad \forall x, y \in S.$$

Definition 1.5

A fuzzy graph $G : (\sigma, \mu)$ is called **strong fuzzy graph** if

$$\mu(u, v) = \sigma(u) \wedge \sigma(v), \quad \forall (u, v) \in S \times S.$$

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Definition 1.6

A fuzzy graph $G:(\sigma,\mu)$ is called **complete fuzzy graph** if

$$\mu(u,v)=\sigma(u)\wedge\sigma(v), \qquad \forall(u,v)\in S.$$

Definition 1.7

Given a fuzzy graph $G:(\sigma,\mu)$ with the underlying set S, the **order of a fuzzy graph G** is denoted as P and it is defined as,

$$p=\sum_{x\in s}\sigma(x)$$

Definition 1.8

For a fuzzy graph $G:(\sigma,\mu)$ the **size of a fuzzy graph G** is denoted by q and it is defined as,

$$q=\sum_{x,y\in s}\mu(x,y)$$

Definition 1.9

The **complement of a fuzzy graph** $G:(\sigma,\mu)$ is a fuzzy graph $\overline{G}:(\overline{\sigma},\overline{\mu})$.

Where, $\overline{\sigma}=\sigma,$

$$\overline{\mu}(u,v)=\sigma(u)\wedge\sigma(v)-\mu(u,v), \qquad \forall u,v\in S.$$

Definition 1.10

Let $G:(\sigma,\mu)$ be a fuzzy graph with underlying graph $G:(\sigma,\mu)$.

The **fuzzy line graph of G** is $L(G):(w,\lambda)$ with the underling graph (z,W) where the node set is Z and edge set W,

$$Z=\{S_x=\{x\}\cup\{u_x,v_x\}x\in E,u_x,v_x\in V,x=(u_x,v_x)\}$$

$$w=\{(S_x,S_y)/S_x\cap S_y\neq\phi,x,y\in E,x\neq y\}$$

$$\omega(S_x)=\mu(x), \qquad \forall S_x\in z\ \&$$

$$\lambda(S_x,S_y)=\mu(x)\wedge\mu(y), \qquad \forall(S_x,S_y)\in W.$$

BIPOLAR FUZZY HYPERGRAPHS

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Definition 2.1

A **fuzzy hypergraph** H=(X,E) is a m tempered fuzzy hypergraph of a crisp hypergraph H*=(X,E) if there exists a fuzzy set A(X,m) such that $m: X \to (0,1]$ and $E = \{\gamma_{E_i} / E_i \in E\}$where

$$\gamma_{E_i}(x) = \begin{cases} \min\{m(e) / e \in E_i, if x \in E_i \\ 0, if otherwise \end{cases}$$

Definition 2.2

Let X be a finite set and let ξ be a finite family of **bipolar fuzzy subsets** B on X (or subsets of X) such that $X = \bigcup_{B\in\xi} \sup p(B).$

Definition 2.3

The pair H=(X, ξ) is called a **bipolar fuzzy hypergraph** (on X) and ξ is called edge set of H, which are bipolar fuzzy sets on subsets of X.

Definition 2.4

A bipolar fuzzy hypergraph H=(X, ξ) is **simple** if ξ has no repeated bipolar fuzzy edges and whenever $A, B \in \xi$ and $A \subseteq B,$ then A=B.

Definition 2.4

A bipolar fuzzy hypergraph H=(X, ξ) is **support simple** if whenever $A, B \in \xi$ and $A \subseteq B,$ andsupp(A)=supp(B), then A=B.

Theorem 2.5

A bipolar fuzzy hypergraphH=(X, ξ) is a $\{m^+, m^-\}$ tempered bipolar fuzzy hypergraph of some crisp hypergraph H* then H is elementary, support simple and simply ordered.

Proof:

Let H=(X, ξ) is a $\{m^+, m^-\}$ tempered bipolar fuzzy hypergraph of some crisp hypergraph H*.

As it is $\{m^+, m^-\}$ tempered, the positive membership values and negative membership values of bipolar fuzzy edges of H are constant.

Hence it is elementary.

Clearly if support of two bipolar fuzzy edges of the $\{m^+, m^-\}$ tempered bipolar fuzzy hypergraph are equal.

Then the bipolar fuzzy edges are equal.

Hence it support simple.

Let $C(H) = \{H_{(r_1,s_1)}, H_{(r_2,s_2)}, ..., H_{(r_k,s_k)}\}$

Since H is elementary, it is ordered.

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Now we are to show that it is simple.

Let $E \in \{H_{(r_{i+1},s_{i+1})} \setminus H_{(r_i,s_i)}\}$

Then there exists $x^* \in E$ such that

$$\mu^+(x^*) = r_{i+1} \text{ and } \mu^-(x^*) = s_{i+1}$$

Since $r_{i+1} < r_i, s_{i+1} > s_i$, it follows that

$$x^* \notin X_{(r_i,s_i)} \text{ and } E \not\subset X_{(r_i,s_i)}$$

Hence H is simply ordered.

Hence the proof.

BIPOLAR FUZZY LINE GRAPH OF A BIPOLAR FUZZY HYPERGRAPHS

Definition: 3.1

Let L(G*)=(Z,W) be a line graph of a simple graph G*=(A_1,B_1) be the bipolar fuzzy graph of G*. We define a **bipolar fuzzy line graph** L(G)=(A_2,B_2) of a bipolar fuzzy graph G as follows:

(i) A_2 and B_2 are bipolar fuzzy sets of Z and W respectively;

(ii) $\mu_{A_2}{}^P(S_x) = \mu_{B_1}{}^P(u_x v_x);$

(iii) $\mu_{A_2}{}^N(S_x) = \mu_{B_1}{}^N(u_x v_x);$

(iv) $\mu_{B_2}{}^P(S_x S_y) = \min(\mu_{B_1}{}^P(x), \mu_{B_1}{}^P(y);$

(v) $\mu_{B_2}{}^N(S_x S_y) = \max(\mu_{B_1}{}^N(x), \mu_{B_1}{}^N(y), \forall S_x, S_y \in Z, S_x, S_y \in W.$

Definition: 3.2

The Line graph L(G*) of a simple graph G* is another graph L(G*) that represents the adjacencies between the edges of G*. Given a graph G*, its line graph L(G*) is a graph such that:

(i) each vertex L(G*) represents an edge of G*, and
(ii) two vertices of L(G*) are adjacent if and only if their corresponding edges share a common endpoint.

Definition: 3.3

Let L(H*)=(S,T) be a line graph of a simple huypergraph. H*=(X,E). Let H=(X, ξ) be a bipolar fuzzy hypergraphs of H*. We define a bipolar fuzzy line graph L(H)=(A_1,B_1) where A_1 is the vertex set of L(H) and B_1 is the edge set of L(H) as follows:

(i) A_1 and B_1 are bipolar fuzzy sets of S and T respectively;

(ii) $\mu_{A_1}{}^P(E_i) = \min_{x \in E_i}(\mu_{E_i}{}^P(x);$

(iii) $\mu_{A_1}{}^N(E_i) = \max_{x \in E_i}(\mu_{E_i}{}^N(x), E_i \in \xi;$

(iv) $\mu_{B_1}{}^P(E_j E_k) = \min_i(\min(\mu_{E_j}{}^P(x_i), \mu_{E_k}{}^P(x_i)));and$

(v) $\mu_{B_1}{}^N(E_j E_k) = \max_i(\max(\mu_{E_j}{}^N(x_i), \mu_{E_k}{}^N(x_i))), where, x_i \in E_j \cap E_k, j,k = 1,2,....., n.$

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Proposition: 3.4

If L(H) is a bipolar fuzzy line graph of the bipolar fuzzy hypergraph H then L(H*) is the line graph of H*.

Proof:

Since H=(X, ξ) is a bipolar fuzzy hypergraph and L(H) is a bipolar fuzzy line graph, we have

$$\mu_{A_1}{}^{P}(E_i) = \min_{x \in E_i}(\mu_{E_i}{}^{P}(x),$$

$$\mu_{A_1}{}^{N}(E_i) = \max_{x \in E_i}(\mu_{E_i}{}^{N}(x), E_i \in \xi;$$

and so $E_i \in S \Leftrightarrow x \in E_i$ and $E_i \in \xi$

Aslo, $\mu_{B_1}{}^{P}(E_j E_k) = \min_i(\min(\mu_{E_j}{}^{P}(x_i), \mu_{E_k}{}^{P}(x_i)));and$

$$\mu_{B_1}{}^{N}(E_j E_k) = \max_i(\max(\mu_{E_j}{}^{N}(x_i), \mu_{E_k}{}^{N}(x_i))),$$

$where, x_i \in E_j \cap E_k, j,k = 1,2,....., n.$ for all $E_j E_k \in S$ and so

$$T\{E_j E_k : E_j \cap E_k \neq \phi, E_j E_k \in \xi, j \neq k\}.$$

This completes the proof.

FUZZY INTERSECTION GRAPHS

The Intersection graph of a family of crisp sets F is the graph G=(F,E), where for each A_i and A_j in F. $A_i, A_j \in E$ if $A_i \cap A_j$ is nonempty. Generally loops are suppressed; that is, i*j is required when forming the intersections.

If the family F is the edge set of a hypergraph H, then the intersection graph of F is called the line graph of H.

Given a finite family of fuzzy set, McAllister defines two structures which together are called a fuzzy intersection graph.

Definition: 4.1

Let $\Im = \{\alpha_1, ..., \alpha_n\}$ be a finite family of fuzzy sets on a set X and consider $\Im$ as a crisp vertex set. The fuzzy intersection graph of $\Im$ is the fuzzy graph $Int(\Im) = (\sigma, \mu), where$

$\sigma : \Im \to [0,1]$ by $\sigma(\alpha_1) = h(\alpha_1)$ and

$\mu : \Im \times \Im \to [0,1]$ is defined by

$$\mu(\alpha_i, \alpha_j) = \begin{cases} h(\alpha_i \wedge \alpha_j), if i * j \\ o, if i = j \end{cases}$$

An edge $\{\alpha_i, \alpha_j\}$ has zero strength if and only if $\alpha_i \wedge \alpha_j$ is the zero function (empty intersection) or i=j (no loops).

Theorem: 4.2

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Let $G=(\sigma,\mu)$ be a fuzzy graph without loops. Then there exists a family of fuzzy sets $\Im$ where $G=Int(\Im)$.

Proof:

The proof is a generalization of Marczewski's crisp result.

Let $G=(\sigma,\mu)$ be a fuzzy with fuzzy vertex set $\sigma: X \to [0,1]$ and

symmetric edge membership function $\mu: X\times X \to [0,1]$

We must find a family of fuzzy sets $\Im=\{\alpha_x / x\in X\}$, where

(i) for each $x\in X, h(\alpha_x)=\sigma(x)$
(ii) for each $x * y\in X, h(\alpha_x\wedge\alpha_y)=\mu(x,y)$

For each $x\in X$ define the symmetric fuzzy set $\sigma_x : X\times X \to [0,1]$ by

$$\alpha_x(y,z)=\begin{cases}\sigma(x), if y=x \,\&\, z=x\\ \mu(x,z), if y=x \,\&\, z\neq x\\ \mu(y,x), if y\neq x \,\&\, z=x\\ o, if y\neq x, if y\neq x \,\&\, z\neq x\end{cases}$$

We show that $\Im=\{\alpha_x / x\in X\}$ is the desired family of fuzzy sets. Fix $x\in X$ and $y\in X$ and $z\in X$ be arbitrary. By the definition of fuzzy graph, $\sigma(x)\geq\mu(x,y)\geq 0$ for each $y\in X$. Therefore $\sigma(x)\geq\alpha_x(y,z)$.

Computing $\alpha_x(x,x)=\sigma(x)$ we have $h(\alpha_x)=\sigma(x)$ as required.

Let $x\neq y$ be fixed elements of X,z and w be arbitrary elements of X and consider the value of $(\alpha_x\wedge\alpha_y)(z,w)=\alpha_x(z,w)\wedge\alpha_y(z,w)$.

If $x\neq z$ and $x\neq w$, then $\alpha_x(z,w)=0$

Similarly $y\neq z$ and $y\neq w$, implies $\alpha_x(z,w)=0$

Therefore a nonzero value is possible only if x=z and y=z and =w).

By definition

$$(\alpha_x\wedge\alpha_y)(x,y)=\alpha_x(x,y)\wedge\alpha_y(x,y)=\mu(x,y).$$

Thus $h(\alpha_x\wedge\alpha_y)=\mu(x,y)$ as required.

CONCLUSION

Graph theory is an extremely useful tool in solving the combinatorial problems in different areas including geometry, algebra, number theory, topology, operations research, optimization and computer science. The bipolar fuzzy sets constitute a generalization of Zadeh's fuzzy set theory. The bipolar fuzzy models give more precision, flexibility and comparability to the system as compared to the classical and fuzzy models. We have introduced the concept of bipolar fuzzy hypergraphs, bipolar fuzzy intersection graph is characterized here. Instead of using a hyper graph, the line graph of a hyper graph is useful in hyper-networks. The concept of the hypergraphs can be applied in various areas of engineering, computing science: database theory, expert systems, neural networks, artificial networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks and medical diagnosis.

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